Colliding Kinky Branes

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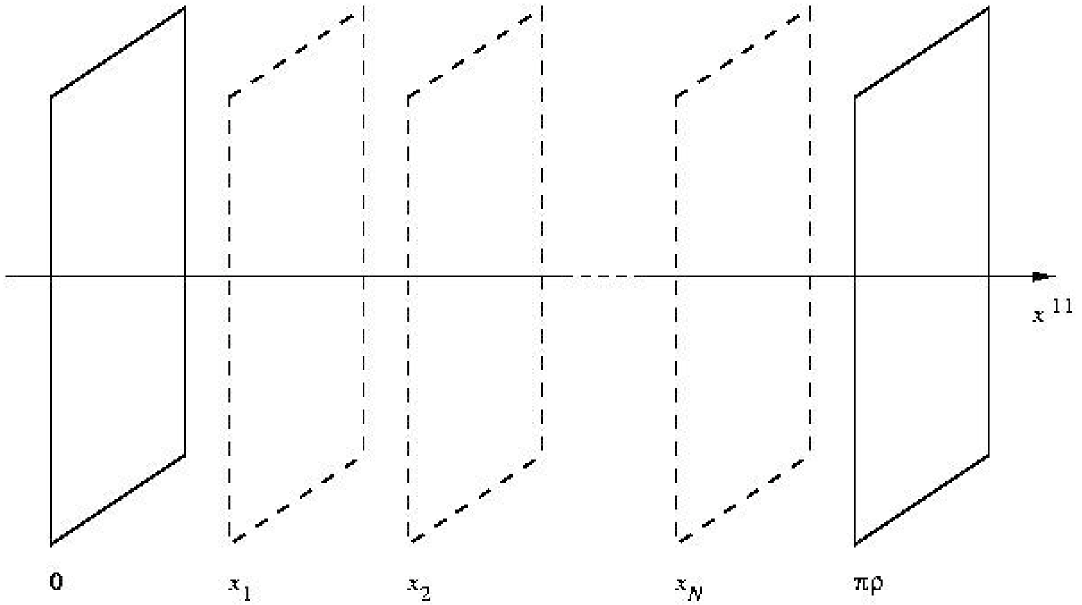
1-Introduction



11d fundamental theory M-theory → 5d brane-world theory: Low-energy theory in 4d, is determined by the charges of the boundary branes (α_1, α_2) and the total bulk brane charge α_3.

Question: Is “fundamental” physics in 4d unique?

- No, all choices are allowed, we just happen to live in a Universe with one specific realization.
- Yes, there may be a dynamical mechanism that selects a single “vacuum”

Can the number of branes in the bulk change dynamically? Is topology change allowed?

We investigate a simple process, the collision of a bulk brane with a boundary brane.

2-Scalar field model of branes

To describe brane dynamics: model branes as kinks of an auxiliary Z_2 scalar field theory.

Simplified model with single scalar field: $S = S_{\rm blk} + S_{\rm bd}$

$$S_{\rm blk} = \int {\rm dt} \int_{-L}^{L} {\rm dx} \left\{ \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 - V(\phi) \right\} , \quad \phi(x) = \phi(-x)$$

$$S_{\rm bd} = - \int {\rm dt} \int_{-L}^{L} {\rm dx} \left\{ 2\delta(x-0) W(\phi) - 2\delta(x-L) W(\phi) \right\}$$

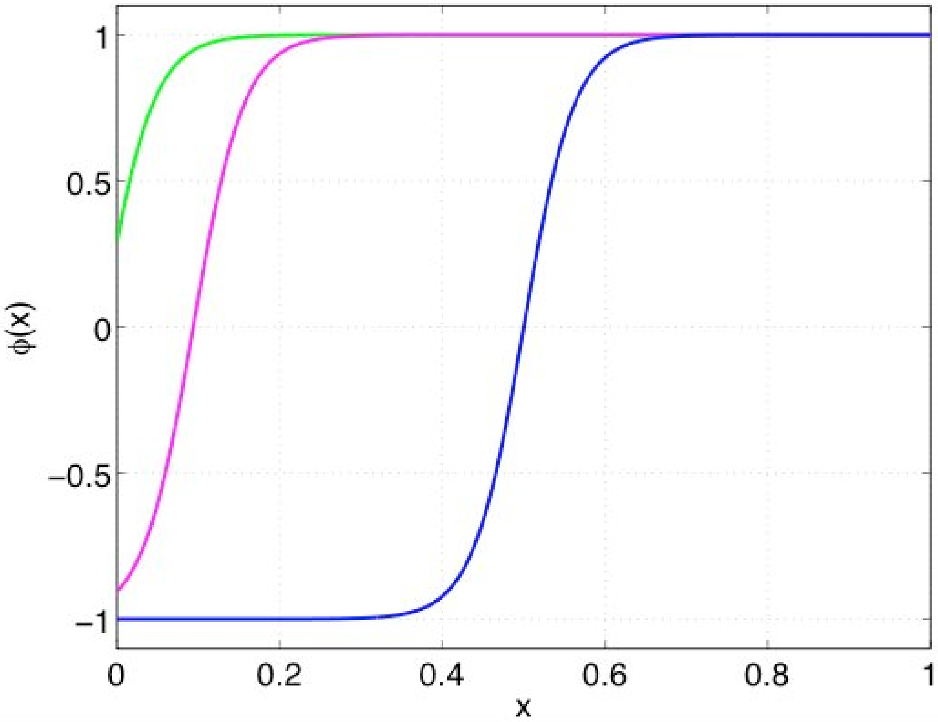
Equation of motion: $$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial x^2} - V'(\phi), \quad x \in]0, L[$$

Boundary condition: $$\frac{\partial \phi}{\partial x} = W'(\phi), \quad x = 0, L$$

The brane potential is chosen so that the theory allows for BPS kink solutions: $W'(\phi) = \sqrt{2V(\phi)}$

The static kink obeys the BPS equation: $\phi'_{\rm K} = \sqrt{2V(\phi_{\rm K})}$

As an example we use a quartic potential: $V(\phi) = \frac{1}{2}g^2(\phi^2 - v^2)^2$

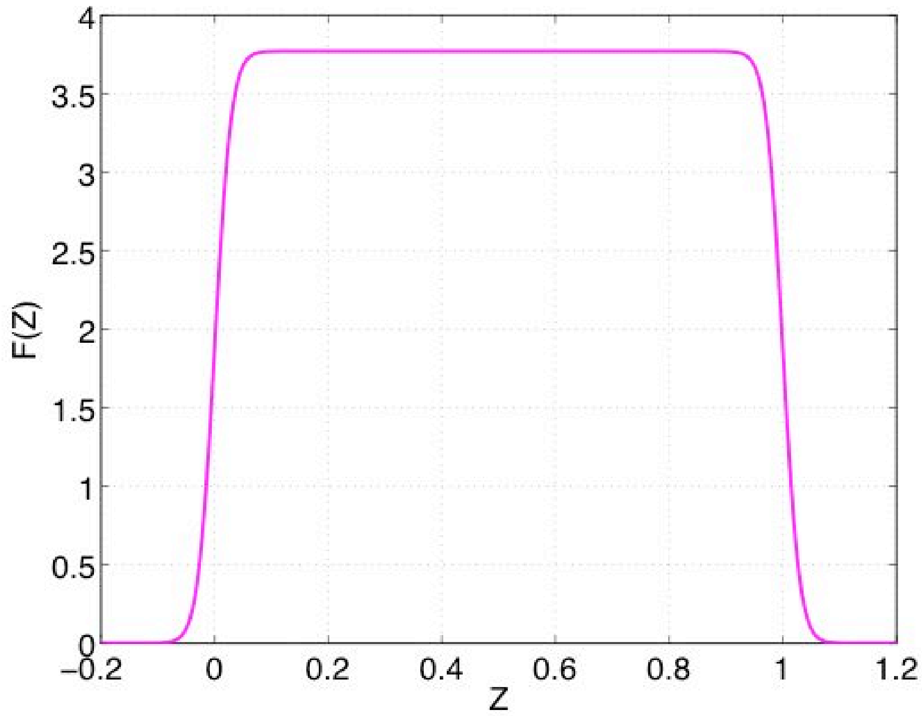


- The core of the kink can be place anywhere, both inside and outside the bulk.
- All these solutions have the same energy $E = 0$.

3-Moduli approximation

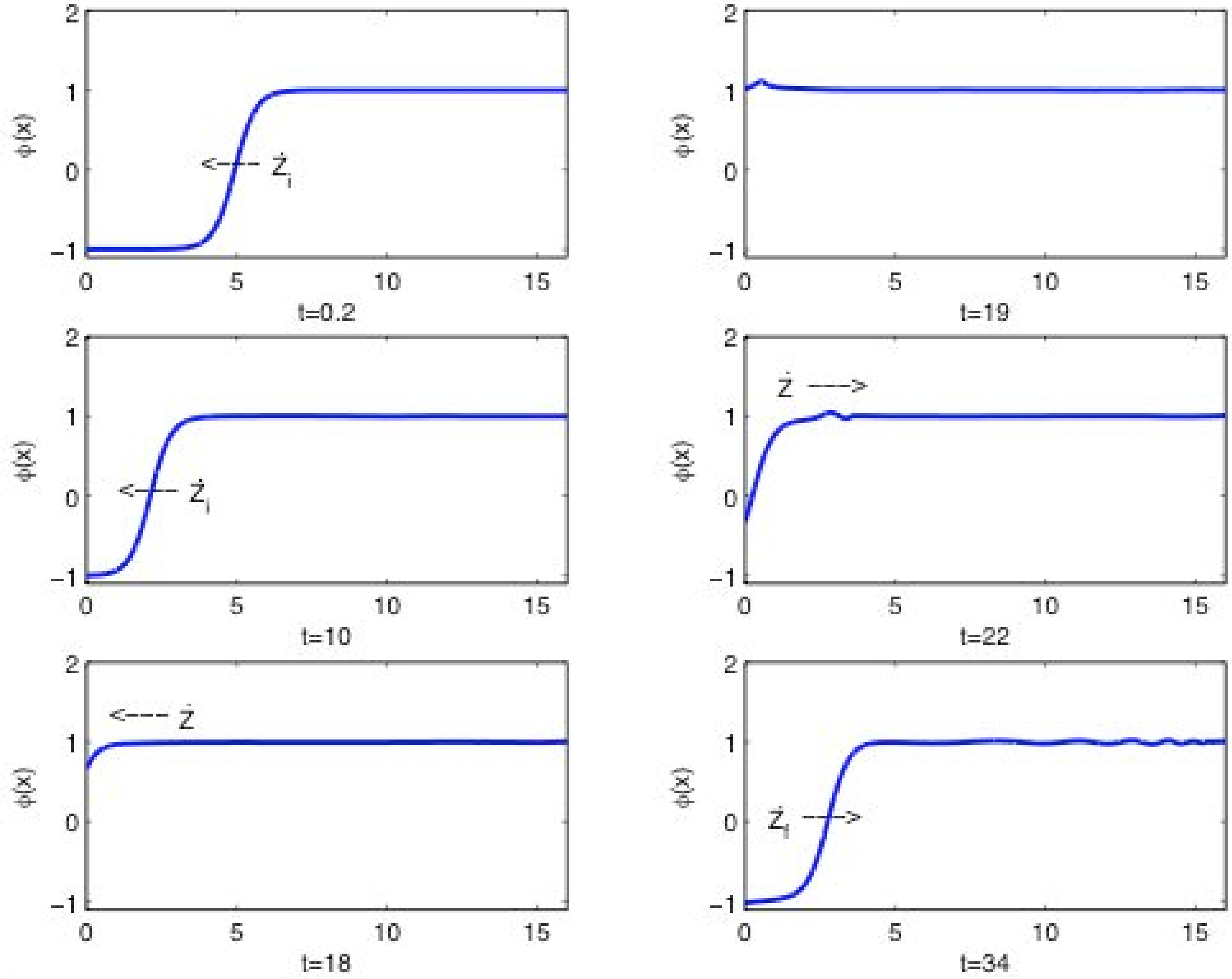
Introduce a coordinate $Z(t)$ for the position of the core of the kink, replace $\phi_{\mathrm{K}}(x - Z(t))$ into the original action $\rightarrow$

Effective Lagrangian: $\mathcal{L} = F(Z)\dot{Z}^2$



- Well inside the bulk $F = const$, the kink moves with constant velocity.
- The energy is conserved. As the kink approaches the boundary, it accelerates, as $F(Z)$ decreases.
- As the kink crosses the boundary, $F(Z) \rightarrow 0$ and the kink accelerates to infinity in finite time.

4-Numerical simulation $\dot{Z}_i = 0.3$



5-Collision as perturbation of the vacuum

As $Z \to -\infty$ $\phi(x)$ is everywhere close to the vacuum v → use linear perturbation around v.

- Define $\phi = v + \chi$. Linearized equation of motion and boundary condition:

$$\ddot{\chi} = \chi'' - m^2\chi, \quad x \in]0, L[; \qquad \chi' = m|\chi|, \quad x = 0, L \qquad\qquad m = gv$$

- As the field goes above the vacuum, $\chi(t, 0) > 0$ and the boundary condition is $\chi' = m\chi$. The corresponding mode basis is:

$$\chi_k \propto e^{i\omega t}\left[\cos(kx) + \frac{m}{k}\sin(kx)\right]$$

- When the field oscillates back below v, the boundary condition changes and there is a new basis of solutions:

$$\chi_k \propto e^{i\omega t}\left[\cos(kx) - \frac{m}{k}\sin(kx)\right] \qquad \chi_0 = (a + bt)e^{-mx}$$

The zero mode grows into the incoming kink → the velocity of the reflected kink is proportional to the amplitude of the zero mode component of the field. We calculate:

- Reflection coefficient:

$$R \equiv \frac{\dot{Z}_f}{\dot{Z}_i} = \frac{16m^3}{\pi} \int_0^\infty \mathrm{dk} \frac{k^2}{\omega^6} \cos(\omega t_R)$$

- Reflection time:

$$\int_0^\infty \mathrm{dk} \frac{k^2}{\omega^5} \sin(\omega t_R) = 0$$

- After rescaling the integrals can be evaluated:

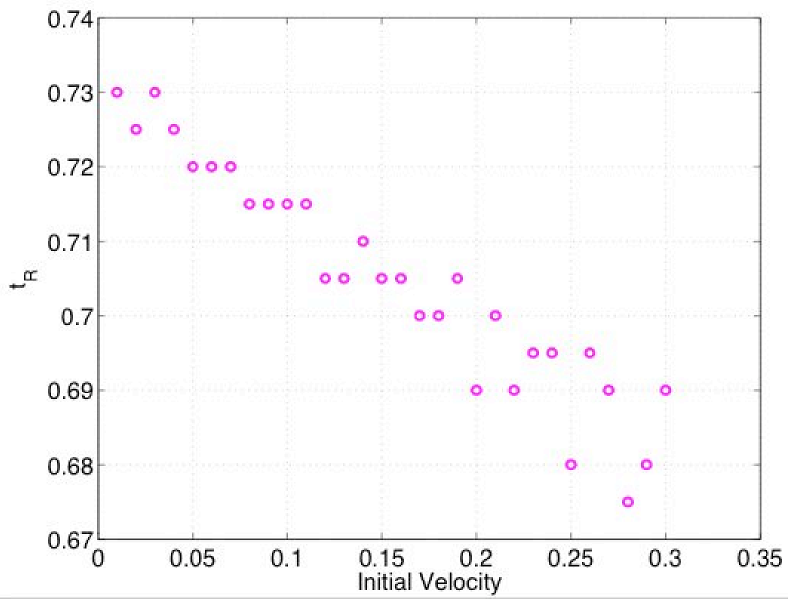
$$t_R = \frac{2.06}{m}, \qquad R = 0.63$$

Numerical reflection time and reflection coefficient for

$$g = 2\sqrt{2},\ v = 1,\ m = 2\sqrt{2}$$

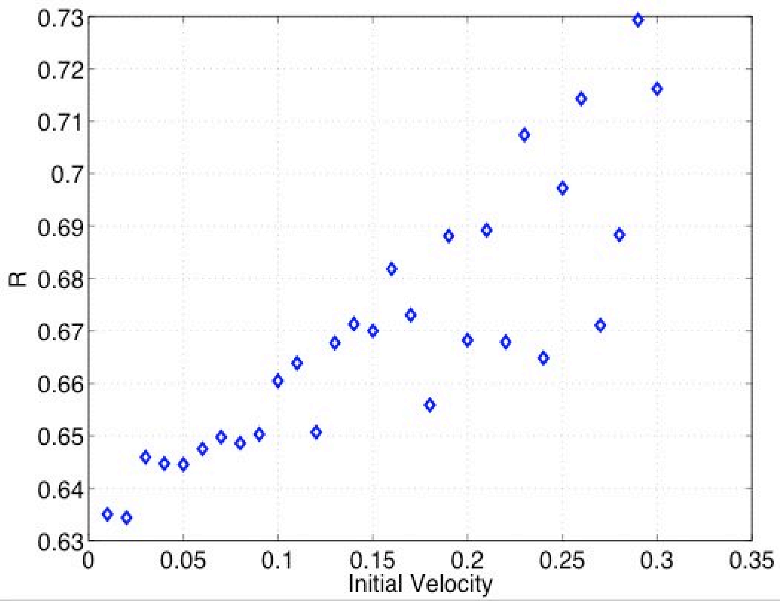
Analytical prediction:

$$t_R = 0.73$$



Analytical prediction:

$$R = 0.63$$



6-Generic topology change

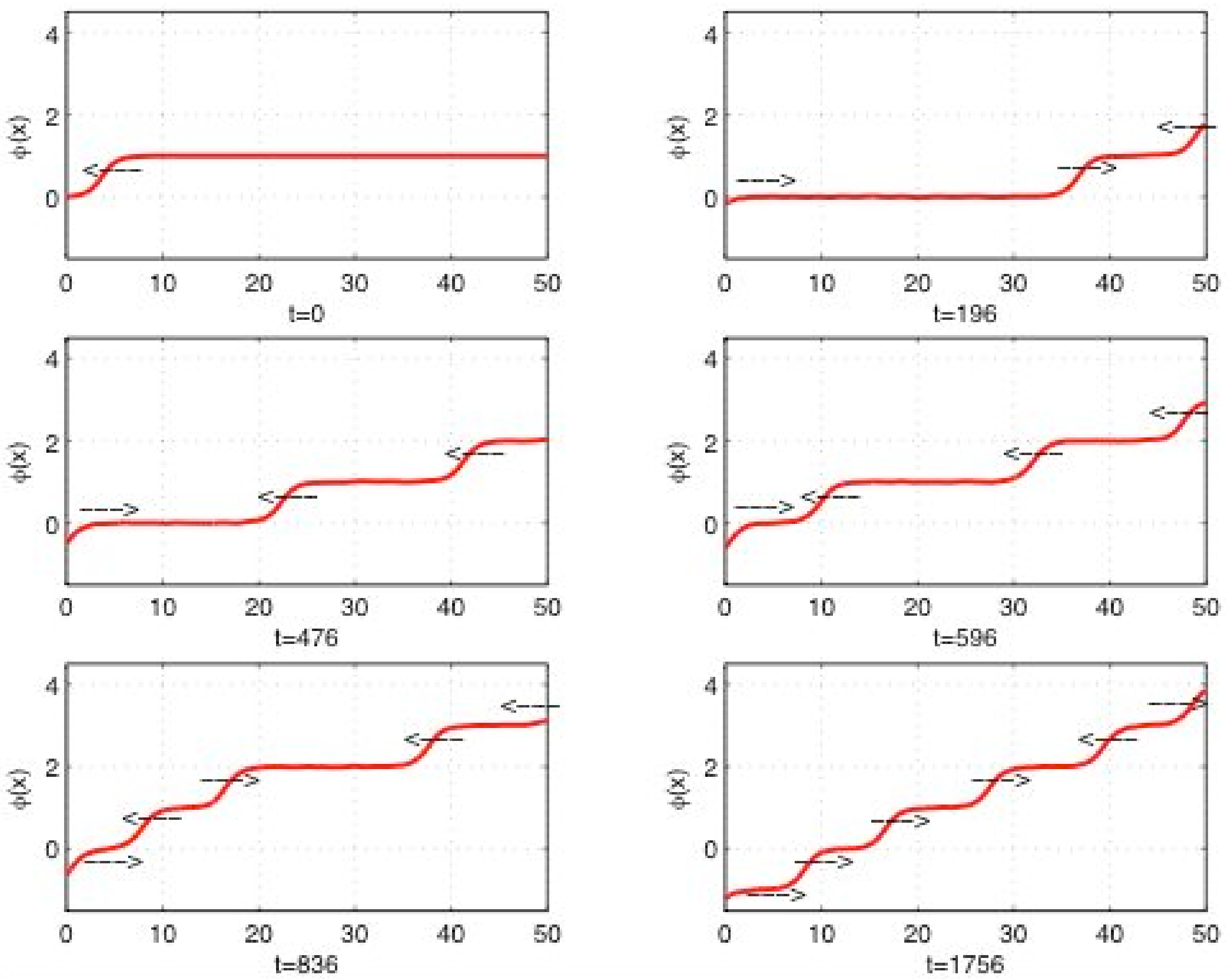
The kink is not absorbed by the boundary → is topology change ruled out?

On the contrary:

- Any configuration near the vacuum will be likely to excite the zero mode.
- Kinks can be extracted from the boundary very easily. Apart from the final kinetic energy, it costs no energy to produce a kink.
- Topology change is very likely!

For a potential with multiple vacuua, increasing numbers of kinks can be produced. Ex: Sine-Gordon

7-Long time evolution for kink collision



8-Conclusions

- In a boundary collision the brane is always reflected inelastically. The fraction of energy lost in the collision is model independent.

- Topology change is a natural process, in fact it is hard to avoid!

- Conjecture: For long times, a brane gas will from, its density limited by the repulsive kink-kink interaction. The value of the final density should be related to the energy of the initial condition → a well-defined final state is singled-out from the general initial field configurations.