Universal Behavior of Cosmological Singularities & String Theory

Mónica Borunda
Neuchâtel University

Work done in collaboration with:
Matthias Blau (Neuchâtel U.)
Martin O'Loughlin (SISSA)
George Papadopoulous (King's College)

One of the major problems in String Theory and an issue of great interest in String Cosmology is

☛ A better understanding of String Theory in non-trivial and possible singular time-dependent backgrounds

Possible approach $\Rrightarrow$ Penrose limit

Motivations

☛ Plane waves are exact (no α' corrections) and, in principle, exactly solvable string backgrounds: in the light cone gauge the bosonic action is quadratic ⇨ classical string mode equations are linear.

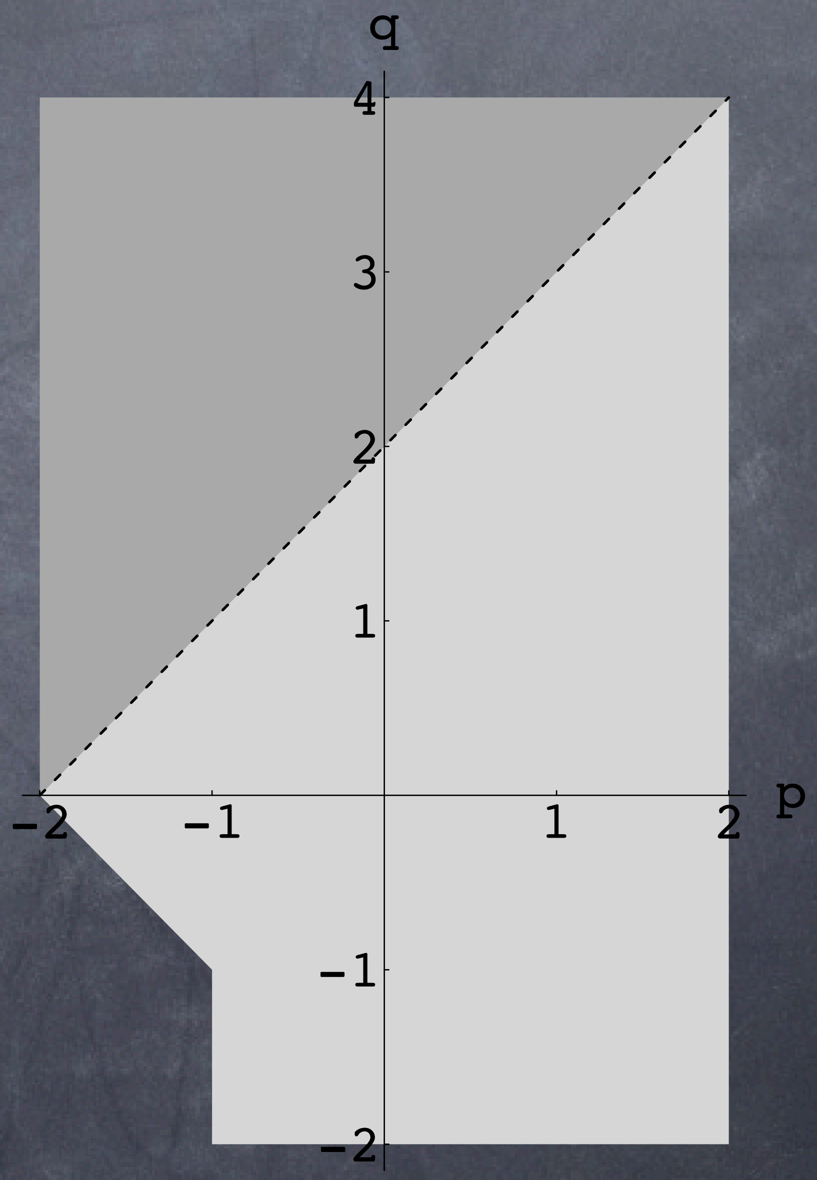
☛ To any metric and choice of null geodesic one can associate a plane wave metric by means of the Penrose limit. (Penrose)

☛ The Penrose limit metric encodes covariant information about the growth of curvature along a null geodesic of the original space-time metric. (Blau, M.B., O'Loughlin, Papadopoulos)

Compute & study the Penrose limit of cosmological singularities ☺

The Penrose limit of reasonable metrics with cosmological singularities shows a universal behavior near the singularity.

$$ds^2 = -x^p dw dz + x^q d\Omega_d^2$$



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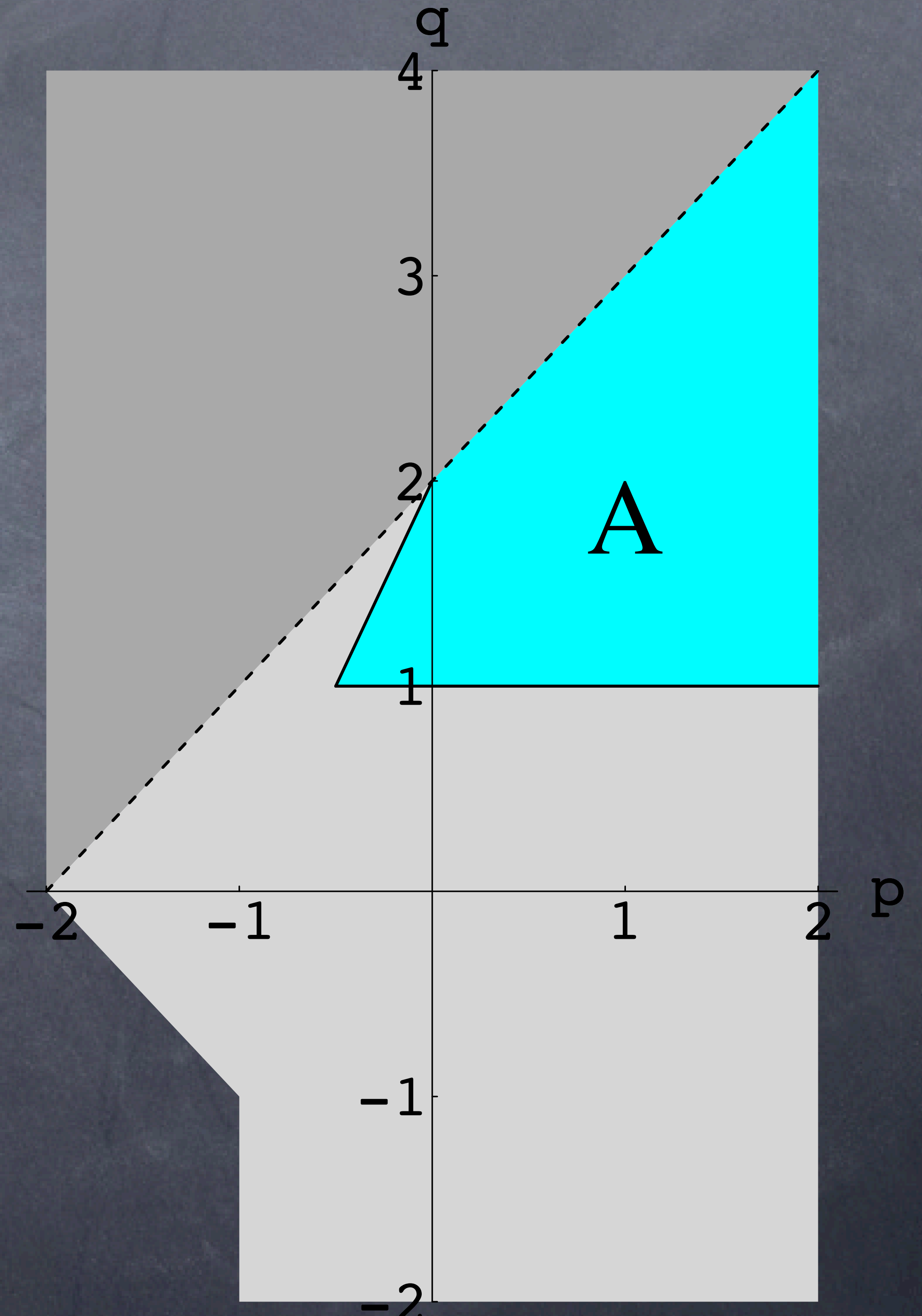

Dominant Energy Condition:

For any observer the local energy density is non-negative and the energy flux causal.

The typical power-like singularity behavior,

$$ds^2 = -x^p dwdz + x^q d\Omega_d^2 \qquad \left(\begin{array}{l}\text{SS: } p=-1/2, q=1 \\ \text{FRW(rad): } p=q=2\end{array}\right)$$

leads to homogeneous scale invariant singular plane waves

$$ds^2 = 2dudv - \lambda(u)\vec{x}^2 du^2 + d\vec{x}^2 \, , \quad \lambda(u) = k/u^2$$

which exhibit scale invariance under $(u,v) \to (cu, c^{-1}v)$ (contrary to BFHP plane waves, $\lambda(u)$=const , which are translational invariant ($u \to u$+const)).

(Blau, M.B., O'Loughlin, Papadopoulos)

String theory in HPW backgrounds

In the light-cone gauge, $U(\sigma,\tau)=2\alpha' p_v t$, ($p_v = p^u$ the light-cone momentum) the dynamics of the transverse string coordinates $X^a(\sigma,\tau)$ in the plane wave metric,

$$ds^2 = 2dudv - \frac{k}{u^2}\vec{x}^2 du^2 + d\vec{x}^2\,,$$

is governed by the quadratic light-cone Hamiltonian

$$H_{lc}(t) = \frac{1}{8\pi\alpha'^2 p_v}\int_0^\pi d\sigma(\dot{\vec{X}}^2 + \vec{X}'^2 + \frac{k}{t^2}\vec{X}^2)\,.$$

Notice that H_{lc} is independent of p_v as a consequence of scale invariance!

The Fourier modes of the string are harmonic oscillators with time dependent frequencies given by

$$w_n^a(t) = \sqrt{n^2 + \frac{k}{4t^2}}\,,$$

whose diagonalized light-cone Hamiltonian operator is

$$H_{lc}(t) = \frac{1}{\alpha' p_v}[H_0(t) + \sum_{n=1}^{\infty}\sum_{a=1}^{d}\frac{w_n^a(t)}{n}(\alpha_n^{a\,\dagger}(t)\alpha_n^a(t) + \tilde{\alpha}_n^{a\,\dagger}(t)\tilde{\alpha}_n^a(t))] + h(t)$$

The oscillators satisfy

$$[\alpha_n^a(t), \alpha_m^{b\,\dagger}(t)] = [\tilde{\alpha}_n^a(t), \tilde{\alpha}_m^{b\,\dagger}(t)] = n\delta_{n+m}\delta^{ab}\,,$$

and h(t) is a normal ordering function.

(Papadopoulos, Russo, Tseytlin)

Now that we have established this universal behavior of the metric, it is interesting to investigate the consequences of the resulting scale invariance for String Theory.

As an example of a calculation in this background one can look at the Thermodynamics.

For this time-dependent toy model we use the formalism for treating non-equilibrium thermodynamics.

☞ We consider the density operator for time-dependent systems:

$$\hat{\rho}_I = \frac{e^{-\beta \hat{I}}}{\mathrm{tr}\, e^{-\beta \hat{I}}},$$

where $\hat{I}$ is an invariant operator,

$$\frac{d}{dt}\hat{I} = \frac{\partial}{\partial t}\hat{I} + i[H, \hat{I}] = 0,$$

such that it satisfies the quantum counterpart of the classical Liouville theorem for the phase space density, namely the Liouville-von Neumann equation:

$$\frac{d}{dt}\hat{\rho} = \frac{\partial}{\partial t}\hat{\rho} + i[H, \hat{\rho}] = 0; \qquad \mathrm{tr}\,\hat{\rho} = 1.$$

(Lewis, Riesenfeld)

We choose the initial ensemble such that

$$I(t_0) = H(t_0)\ ; \quad \lim_{\mathrm{flat}} I(t) = H_{\mathrm{flat}}\,.$$

(note that for large $t \Rightarrow$ system varies slowly: adiabaticity)

The non-equilibrium thermodynamical description breaks down at a parameter which value corresponds to that of the Hagedorn temperature in flat space!

Scale invariance

The functional behavior of the density matrix exhibits a Hagedorn like behavior as the one of strings in flat space (contrary to BFHP plane waves) (Blau, M.B., O'Loughlin)

What did we learn?

The universal behavior of cosmological singularities near the singularity ➔ scale invariance.

This scale invariance could make interacting physics near the singularity simpler than what we thought!