Nonequilibrium dynamics in scalar hybrid models

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J.B., Andreas Heinen, Phys. Rev. **D69** 083523 (2004)

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Outline

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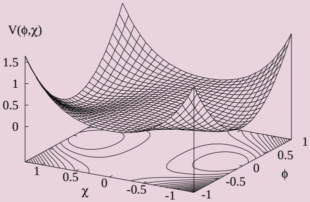
Introduction

The Hybrid model

A. D. Linde, Phys. Lett. **B249**, 18 (1990)

Classical potential:

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 \quad (1)$$



$\phi \longrightarrow$ Inflaton field

$\chi \longrightarrow$ symmetry breaking field

Idea: Inflation terminates by a phase transition. Subsequent reheating done by the symmetry breaking field.

Complication: Coupled dynamics of classical fields and quantum fluctuations!

Questions

We study the **transition from the metastable phase to the broken symmetry phase** $\rightarrow$ preheating after cosmic inflation.

- Spinodal regime (χ field has a tachyonic mass) $\rightarrow$ stabilization?
- Treatment of the quantum fluctuations ($\rightarrow$ coupled channels, approximation, renormalization)?
- Effect of back reaction of the fluctuations onto themselves (not included in previous simulations of other groups)?
- Which modes become classical ($\rightarrow$ decoherence time)?

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Effective Action

Effective action in the two-particle point-irreducible (2PPI) formalism

Resummation of local self energy insertions.

$$\Gamma[\phi,\chi,\Delta_{\phi\phi},\Delta_{\phi\chi},\Delta_{\chi\chi}]$$
$$= S[\phi,\chi] + \Gamma^{2\mathrm{PPI}}\left[\phi,\chi,\mathcal{M}^2_{\phi\phi},\mathcal{M}^2_{\phi\chi},\mathcal{M}^2_{\chi\chi}\right] \quad (2)$$
$$+\frac{3\lambda}{4}\int d^Dx\Delta^2_{\chi\chi}(x) + \frac{g^2}{2}\int d^Dx\left(\Delta_{\phi\phi}(x)\Delta_{\chi\chi}(x)+2\Delta^2_{\phi\chi}(x)\right),$$

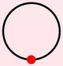
$$\Gamma^{2\mathrm{PPI}}\left[\phi,\chi,\mathcal{M}^2_{\phi\phi},\mathcal{M}^2_{\phi\chi},\mathcal{M}^2_{\chi\chi}\right] = \quad (3)$$

H. Verschelde, M. Coppens, Phys. Lett. **B287**, 133 (1992); Z. Phys. **C58**, 319 (1993)

J.B., Andreas Heinen, Phys. Rev. **D67** 105020 (2003); *ibid.* **68** 127702 (2003)

Here: One-loop bubble-resummation

$$\Gamma^{2\mathrm{PPI}} \approx \frac{i}{2}\mathrm{Tr}\ln\left[\mathcal{G}^{-1}\right] \quad \longrightarrow \quad \Delta_{ij}(x) = -2\frac{\delta\Gamma^{2\mathrm{PPI}}}{\delta\mathcal{M}^2_{ij}(x)} \rightarrow \quad (4)$$



Renormalization – a brief sketch

H. Verschelde, Phys. Lett. **B497**, 165-171 (2001)

Full Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi_i\partial^\mu\Phi_i - \frac{1}{2}m^2_{ij}\Phi_i\Phi_j - \frac{1}{4}\lambda_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l + \frac{1}{2}\delta Z_{ij}\partial^\mu\Phi_i\partial^\mu\Phi_j - \frac{1}{2}\delta m^2_{ij}\Phi_i\Phi_j - \frac{1}{4}\delta\lambda_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l \ . \quad (5)$$

$$(\delta m^2)^{ij} = \delta Z_m^{ij;kl} m^2_{kl} \quad (6)$$

$$\delta\lambda^{ij;kl} = \lambda^{ij}{}_{pq}\delta Z_m^{pq;kl} \quad (7)$$

$$\delta Z_m^{ij;kl} = \lambda^{ij}{}_{pq}\delta\zeta^{pq;kl} \quad (8)$$

$$\delta E_{\rm vac} = \frac{1}{2}\mathcal{M}^2_{ij}\mathcal{M}^2_{kl}\,\delta\zeta^{ij;kl} \quad (9)$$

Multiplicative renormalization in a **mass independent renormalization scheme** → all renormalization constants are derived from a vacuum counter term $\delta E_{\rm vac}$.

$$\mathcal{M}^2_{\mathrm{R},\,ij} = m^2_{ij} + \delta m^2_{ij} + \frac{1}{2}(\lambda_{ijkl} + \delta\lambda_{ijkl})\left(\phi^k\phi^l + \Delta^{kl}\right) \ . \quad (10)$$

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Explicitly:

$$\delta E_{\rm vac} = -\delta\xi \left[(\mathcal{M}^2_{\phi\phi})^2 + 2(\mathcal{M}^2_{\phi\chi})^2 + (\mathcal{M}^2_{\chi\chi})^2 \right] \quad (11)$$

$$\delta\xi = -\frac{1}{64\pi^2}\left(\frac{2}{\epsilon} - \gamma + 1 + \ln 4\pi\right) \quad (12)$$

$$\mathcal{M}^2_{\mathrm{R},\,\phi\phi}(t) = m^2 + g^2 \left[\chi^2(t) + \Delta_{\chi\chi}(t)\right] - 4g^2\,\delta\xi\mathcal{M}^2_{\mathrm{R},\chi\chi}(t) \quad (13)$$

$$\mathcal{M}^2_{\mathrm{R},\,\chi\chi}(t) = -\lambda v^2 + g^2\left[\phi^2(t) + \Delta_{\phi\phi}(t)\right] + 3\lambda\left[\chi^2(t) + \Delta_{\chi\chi}(t)\right] - 4g^2\,\delta\xi\mathcal{M}^2_{\mathrm{R},\,\phi\phi}(t) - 12\lambda\,\delta\xi\mathcal{M}^2_{\mathrm{R},\,\chi\chi}(t) \quad (14)$$

$$\mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t) = 2g^2\left[\phi(t)\chi(t) + \Delta_{\phi\chi}(t)\right] - 8g^2\,\delta\xi\mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t)\,. \quad (15)$$

- The 2PPI-formalism helps a lot in determining the counter terms!
- Freedom to choose an appropriate regularization scheme, e.g. **dimensional regularization** → no cut-offs needed
- $(\mathcal{M}^2_{\mathrm{R},\,\phi\phi}, \mathcal{M}^2_{\mathrm{R},\,\chi\chi}, \mathcal{M}^2_{\mathrm{R},\,\phi\chi})$ form a 3×3 system of (linear) equations → coefficient matrix from the **finite renormalization constants**

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Equations of motion

Classical fields:

$$\ddot{\phi}(t) + \mathcal{M}^2_{\mathrm{R},\,\phi\phi}(t)\phi(t) + \mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t)\chi(t) - 2g^2\chi^2(t)\phi(t) = 0$$
$$\ddot{\chi}(t) + \mathcal{M}^2_{\mathrm{R},\,\chi\chi}(t)\chi(t) + \mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t)\phi(t) - 2\lambda\chi^3(t) - 2g^2\phi^2(t)\chi(t) = 0$$

Mode functions (quantum fluctuations):

$$\ddot{f}^{\alpha}_{\phi}(t;\mathbf{p}) + \mathbf{p}^2 f^{\alpha}_{\phi}(t;\mathbf{p}) + \mathcal{M}^2_{\mathrm{R},\,\phi\phi}(t) f^{\alpha}_{\phi}(t;\mathbf{p}) + \mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t) f^{\alpha}_{\chi}(t;\mathbf{p}) = 0$$
$$\ddot{f}^{\alpha}_{\chi}(t;\mathbf{p}) + \mathbf{p}^2 f^{\alpha}_{\chi}(t;\mathbf{p}) + \mathcal{M}^2_{\mathrm{R},\,\chi\chi}(t) f^{\alpha}_{\chi}(t;\mathbf{p}) + \mathcal{M}^2_{\mathrm{R},\,\phi\chi}(t) f^{\alpha}_{\phi}(t;\mathbf{p}) = 0 \tag{16}$$

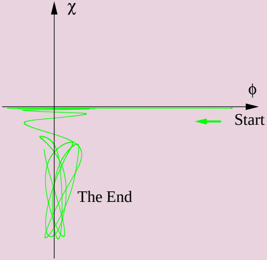
Greens function (factorized in mode functions):

$$G_{ij}(t,t';\mathbf{p}) = \sum_{\alpha=1}^{2} \frac{1}{2\omega_\alpha} f^{\alpha}_{i}(t;\mathbf{p}) f^{*\,\alpha}_{j}(t';\mathbf{p}) \tag{17}$$

ω_α self-consistently calculated at $t = 0$ ($\rightarrow$ Fock space)

Results from numerical simulations

Initial conditions

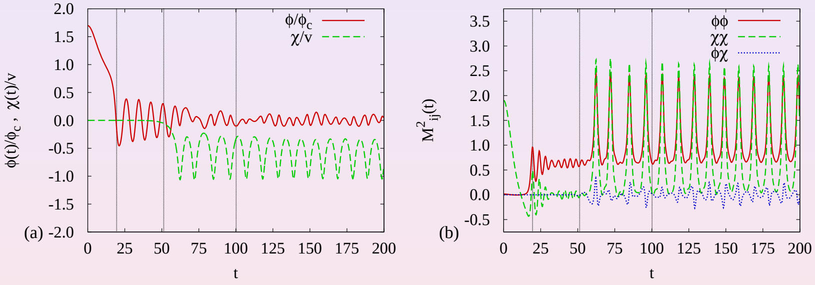


Start point:
$\phi(0) \neq 0$
$\chi(0) \approx 0 \quad$ (e.g. 10^{-7})

End point:
$\phi(0) \approx 0$
$\chi(0) \neq 0$

- The Inflaton field ϕ rolls down its potential and oscillates around the metastable point
- The symmetry breaking field χ rolls down the double-well potential to one of the stable minima

Evolution of zero modes and effective masses



Parameters $g^2 = 2\lambda = 2, m^2 = 0, v = 1, \phi(0) = 1.2, \chi(0) = 10^{-7}$

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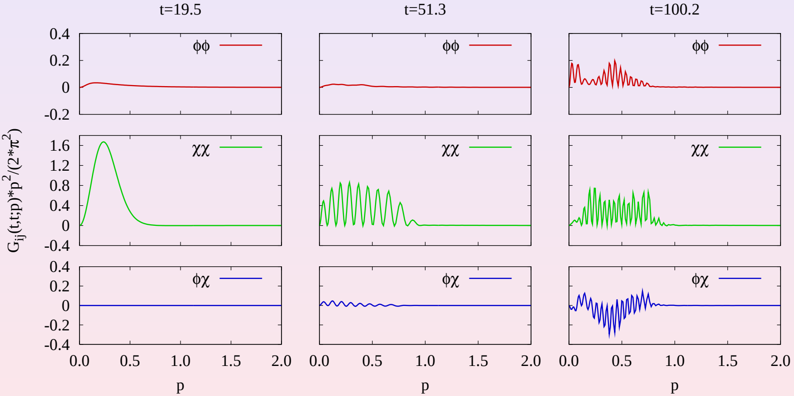
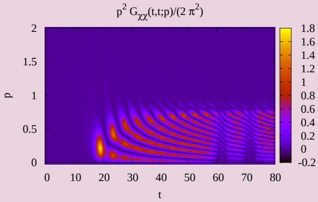
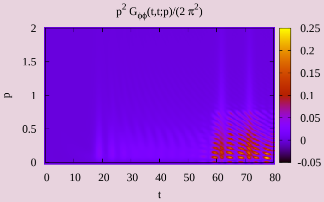
Evolution of momentum spectra



Momentum spectra





- The two caracteristic times are clearly visible in the momentum spectra, i.e. $p^2 G_{ij}(t, t; \mathbf{p})$.
- Fixed momentum band at late times, no thermalization in this approximation.

Parameters: $m^2 = 0$, $v = 1$, $g^2 = 2\lambda$, $\lambda = 1$, $\phi(0) = 1.2$, $\chi(0) = 10^{-7}$

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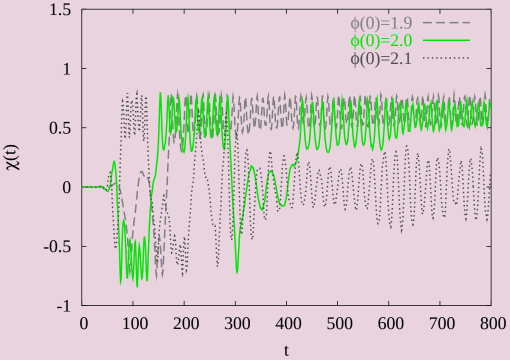
Dependence on $\phi(0)$,"phase transition"



- System ends up with $\chi(\infty) = 0$ or $\chi(\infty) = \pm v$, symmetric or broken final state
- $\phi(0)$ plays the rôle of a temperature in this "phase transition"

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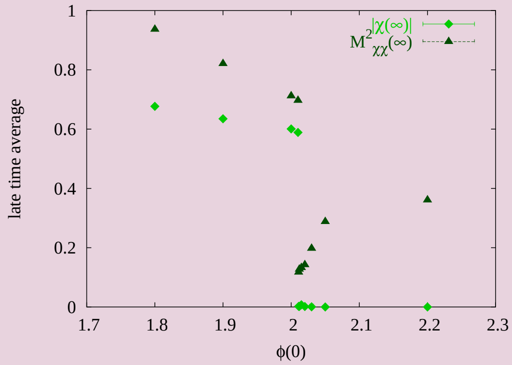
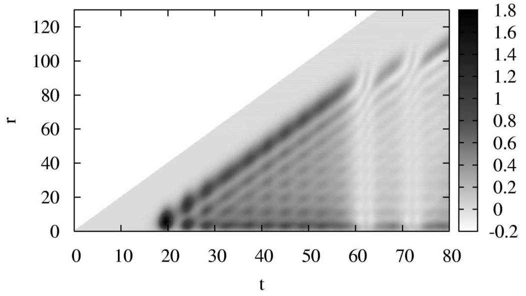
Dependence on $\phi(0)$,"phase transition"



- ... looks like first order

Correlations

$$C_{\chi\chi}(t,r) = \frac{1}{2\pi^2 r}\int_0^\infty dp\,p\sin(pr)\,G_{\chi\chi}(t,t;\mathbf{p}) \qquad (18)$$



Spatial correlations of Higgs fluctuations

- correlations build up in the spinodal regime and decrease after the phase transition
- propagate by twice the speed of light (opposite space directions)

Parameters: $m^2 = 0,\ v = 1,\ g = 2\lambda,\ \lambda = 1,\ \phi(0) = 1.2,\ \chi(0) = 10^{-7}$

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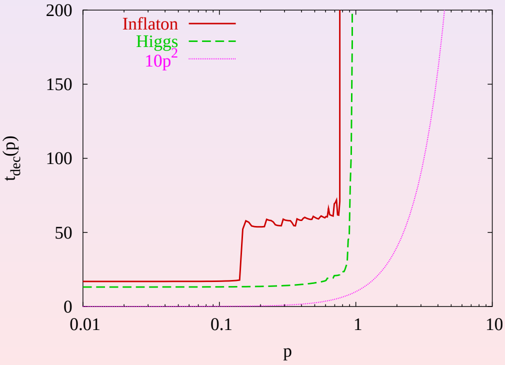
Decoherence time

Modes become classical when

$$|F_{ii}| \gg 1 \quad \text{with} \quad F_{ij}(t,\mathbf{p}) = \mathrm{Im}\left[\sum_\alpha \frac{f_i^{\alpha *}(t,p)\dot{f}_j^\alpha(t,p)}{2\omega_\alpha(p)}\right] \qquad (19)$$



- If one assumes $\mathcal{M}^2_{\chi\chi} \propto (t_0 - t)$ then $t_{\rm dec} \propto p^2 \longrightarrow$ quite different with back reaction

Summary & Conclusions

- We have simulated the transition from the unstable "vacuum" to the stable vacuum.
- Dynamical stabilization in the classical unstable (spinodal) regime due to quantum fluctuations and their back reaction
- Different asymptotic regimes: while on the tree level one expects broken symmetry final state, quantum fluctuations can cause the system to be in a symmetric state ("phase transition") at late times
- One-loop resummations lack of an efficient mechanism for dissipation $\longrightarrow$ no scattering, no memory kernels $\longrightarrow$ has to be improved

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Thank you for your attention!