Non-Gaussianity from a subdominant component

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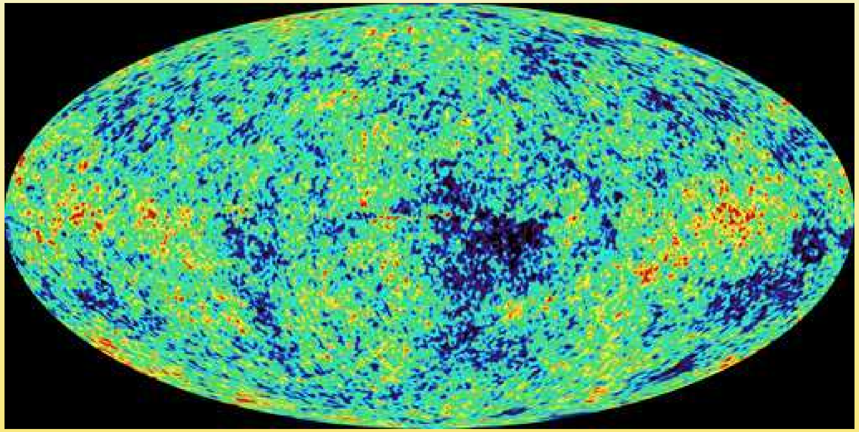
Based on:

- LB and D.H. Lyth , `astro-ph/0504046`

2-point Statistics of CMBR



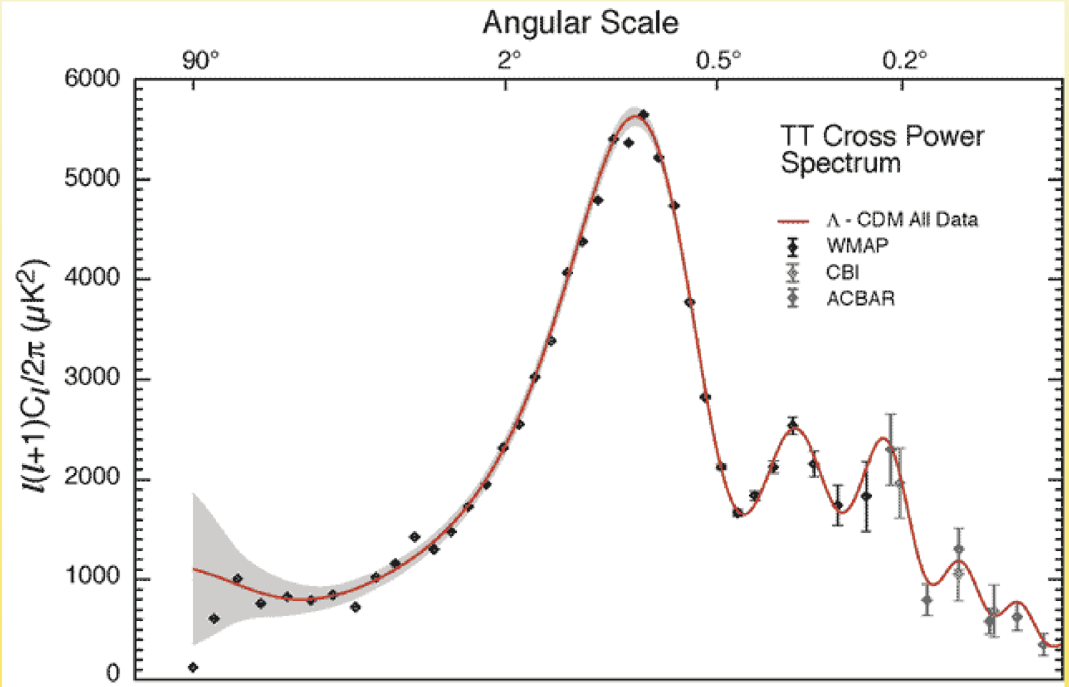
The observed temperature anisotropies are related to the primordial curvature perturbation ζ through the Sachs-Wolfe effect

$$\frac{\delta T}{T} = -\frac{\zeta}{5}$$

The amplitude of the power spectrum of the curvature perturbation is defined as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \left(\frac{2\pi^2}{k_1^3} \right) \mathcal{P}_\zeta(k_1)$$

with $\mathcal{P}_\zeta(k) \propto (k/a_0 H_0)^{n_s - 1}$.



$$\frac{\ell(\ell+1)C_\ell^{TT}}{2\pi} = \int d\ln k\, \mathcal{P}_\zeta(k)\left[g_\ell^T(k)\right]^2$$

Latest measurements of WMAP gave

$$\begin{aligned} \mathcal{P}_\zeta^{1/2} &\simeq 5\times 10^{-5} \\ |n_s - 1| &\lesssim 0.05 \end{aligned}$$

Inflationary Paradigm

Inflation is a dominant paradigm for the early Universe.

- Solves the standard cosmological problems.

→ scalar field ϕ *(inflaton)*, with slowly varying potential

$$a(t) \sim e^{H\,t}, \text{ with } H^2 \simeq \frac{V(\phi)}{3M_P^2}$$

.

- Provides the seed for structure formation.

$$\text{Quantum fluctuations of the inflaton } \delta\phi \simeq \frac{H}{2\pi}$$

$$\Downarrow$$

$$\text{Classical curvature perturbation } \zeta.$$

Single field inflation	Predictions	Experiments
$\frac{\delta\rho}{\rho}$	$\sim \frac{V^{3/2}}{M_P^3 V'}$	$\sim 10^{-5}$
GW	$r \equiv \frac{A_T}{A_S} \sim \frac{V}{(4 \times 10^{16} \mathrm{GeV})^4}$	$\lesssim 0.46$
Scalar tilt	$\lvert n_s - 1 \rvert \sim \frac{1}{N_e}$	$\lesssim 0.05$
Nature of $\frac{\delta\rho}{\rho}$	only adiabatic no isocurvature Almost Gaussian	$\frac{A_{\mathrm{iso}}}{A_S} \lesssim 0.1$ $\left(\frac{\delta\rho}{\rho}\right)_{\mathrm{NG}} \lesssim 10^{-3}$

Alternative Scenarios for Curvature Perturbation

Despite this striking success, several alternatives for the generation of primordial curvature perturbation have been put forward.

Lyth & Wands
Enqvist & Sloth
Moroi & Takahashi

- The curvaton scenario

$$\frac{\delta\rho}{\rho} \sim \frac{\delta\sigma}{\sigma_*}$$

- Inhomogeneous reheating

Dvali, Gruzinov & Zaldarriaga

$$\frac{\delta\rho}{\rho} \sim \frac{\delta\Gamma_\phi}{\Gamma_\phi}$$

- Other scenarios (DBI, Ghost inflation, ...)

These scenarios are based on the hypothesis that there are other light field besides the inflaton during inflation.

They have distinctive features in the CMB: Non-Gaussianity, correlations, ... and typically no GW.

3-point Statistics of CMBR

The curvature perturbation $\zeta(\mathbf{x})$ can be expanded to 2nd order as Komatsu & Spergel

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) - \frac{3}{5} f_{\rm NL} \left(\zeta_g^2(\mathbf{x}) - \langle \zeta_g^2 \rangle\right)$$

where the coefficient $f_{\rm NL}$ measures the strength of the quadratic term.

The *bispectrum* $B_\zeta(k_1, k_2, k_3)$ is defined as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$$

The non-linearity parameter $f_{\rm NL}$ can thus be defined as

$$f_{\rm NL} = -\frac{5}{6} \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_1)P_\zeta(k_3) + P_\zeta(k_2)P_\zeta(k_3)}$$

which is generally scale-dependent.

- Prediction of $f_{\rm NL}$.

Scenario	$f_{\rm NL}$ (k-independent part)
Single-field inflation	$-\frac{5}{12}(n_s - 1)$
Curvaton scenario	$\sim \frac{1}{r_\sigma}; \quad r_\sigma \equiv \left(\frac{\rho_\sigma}{\rho_{\rm tot}}\right)_{\rm decay}$
Inhomogeneous decay scenario	$O(1)$ simplest case
Other scenarios	$\gtrsim O(50)$

- Experimental constraints on $f_{\rm NL}$

$$-58 < f_{\rm NL} < 134 \quad 95\% \text{ C.L.} \quad \text{WMAP}$$

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$$|f_{\rm NL}| \sim O(5) \quad \text{PLANCK} \sim 2008$$

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- Cosmic strings.
- Magnetic fields.
- Axion CDM. (More later in this talk)
- Any other light scalar field (curvaton, fluctuating decay rate...).

Framework

- Assume the curvature perturbation is sourced by two sources: the first one is Gaussian ζ_g (say the inflaton) and the second one χ^2-distributed:

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle\sigma^2\rangle$$

- The two sources are assumed to be completely uncorrelated i.e. $\langle\zeta_g\,\sigma\rangle = 0$, and σ is Gaussian i.e. $\langle\sigma\rangle = 0$.

- The inflaton ζ_g is assumed to dominate the power spectrum

$$\mathcal{P}_\zeta = \mathcal{P}_{\zeta_g} + \mathcal{P}_{\sigma^2} \simeq \mathcal{P}_{\zeta_g} \gg \mathcal{P}_{\sigma^2}$$

- For simplicity assume ζ_g and σ have flat spectrum.

Since the 2 sources are uncorrelated the bispectrum is

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What is the level of non Gaussianity in such a scenario?

The Bispectrum

We can compute the bispectrum of σ^2, giving

$$B_{\sigma^2}(k_1, k_2, k_3) = (2\pi)^3 \mathcal{P}_\sigma^3 \int d^3p \, \frac{1}{p^3|\mathbf{p} - \mathbf{k}_1|^3|\mathbf{p} + \mathbf{k}_2|^3}$$

with $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$.

As for the power spectrum, we must regularize the divergences at $\mathbf{p} = 0$, $\mathbf{p} = \mathbf{k}_1$ and $\mathbf{p} = -\mathbf{k}_2$ by working in a finite volume. We get

$$B_{\sigma^2}(k_1, k_2, k_3) \simeq \frac{6(2\pi)^4}{k^6} \mathcal{P}_\sigma^3 \ln(kL) \, ,$$

from which we can read off $f_{\rm NL}$

$$\begin{aligned} f_{\rm NL} &\simeq -\frac{20}{3} \frac{\mathcal{P}_\sigma^3}{\mathcal{P}_\zeta^2} \ln(kL) \\ &= -\frac{1}{\sqrt{\ln(kL)}} \left(25.5 \frac{\mathcal{P}_{\sigma^2}^{1/2}}{\mathcal{P}_\zeta^{1/2}} \right)^3 \end{aligned}$$

Observational Consequences

The current WMAP bound $|f_{\rm NL}| \lesssim 100$ gives

$$\left(\frac{\mathcal{P}_{\sigma^2}}{\mathcal{P}_\zeta}\right)^{1/2} \lesssim 0.18$$

PLANCK will be able to go down to

$$\left(\frac{\mathcal{P}_{\sigma^2}}{\mathcal{P}_\zeta}\right)^{1/2} \lesssim 0.04$$

A Gaussian Squared Isocurvature Perturbation

- The axion solves the strong CP problem.
- It also provides an attractive and economical candidate for CDM.
- It can have isocurvature perturbations

$$S_a = \frac{\delta n_a}{n_a} - \frac{\delta n_\gamma}{n_\gamma}\,,$$

that can feed into the CMB temperature anisotropies through the Sachs-Wolfe formula

$$\left(\frac{\Delta T}{T}\right)_{\rm SW} = \left(\frac{\Delta T}{T}\right)_{\rm A} + \left(\frac{\Delta T}{T}\right)_{\rm S} = -\frac{\zeta_{\rm g}}{5} - \frac{2}{5} S_a$$

There are two possible regimes

Lyth '91

$$n_a \sim m_a^2 a^2 \Rightarrow \frac{\delta n_a}{n_a} = \frac{\delta a}{a} + \frac{1}{2}\left(\frac{\delta a}{a}\right)^2$$

- $\delta a \ll a \Rightarrow S_a \sim \dfrac{\delta a}{a}$ (Gaussian).
- $\delta a \gg a \Rightarrow S_a \sim \left(\dfrac{\delta a}{a}\right)^2$ (Highly non-Gaussian).

Applying the previous bounds, one gets

$$S_a \lesssim 0.18 \qquad \text{WMAP}$$

$$S_a \lesssim 0.04 \qquad \text{PLANCK}$$

Experimental Smoking Gun

The scenario can be tested by combining data on gravitational waves and non-Gaussianity.

- If the B-modes (tensors) are detected at a level expected from inflation, then primordial curvature perturbation is likely to be dominated by inflaton fluctuations.

- If the amount of measured NG is significant i.e. $f_{\mathrm{NL}} \gg |n_s - 1|$, then this will be attributed to a subdominant non-Gaussian source, not to the inflaton.

The scenario can be confirmed or ruled out as accuracy of CMB experiments increases.

Conclusions

• We studied a new possible source of non-Gaussianity, sourced by a subdominant uncorrelated component of ζ

$$\zeta = \zeta_g(\mathbf{x}) + \sigma^2(\mathbf{x}) - \langle \sigma^2 \rangle$$

$$\sigma \neq \zeta_g \qquad \mathcal{P}_{\zeta_g} \gg \mathcal{P}_{\sigma^2}$$

• We computed the non-linearity parameter

$$f_{\rm NL} \simeq \frac{\mathcal{P}_{\sigma^2}^{3/2}}{\mathcal{P}_\zeta^2}$$

• Can be applied to (uncorrelated) axion CDM.

• Future observations can test such a scenario.