Non-Gaussianity of Cosmic Magnetic Fields

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Motivation

- Non-Linearity
  - Standard cosmological model linear and Gaussian
  - Non-linearity = non-Gaussianity – 3-point correlations
  - Higher order correlations between scalar/vector/tensor components?
  - Observable correlations between E and B modes?

- Magnetic fields?
  - Inherently non-linear source
  - Galactic fields, ~μG, l_c≈kpc; cluster fields, ~ nG-μG, l_c≈Mpc; inter-cluster fields, ~nG? μG?

Motivation

- Generation: early-time / continuous / recombination / late-time
  - e.g. Berezhiani and Dolgov, astro-ph/0305595; Widrow, astro-ph/0207240; Mataresse et. al., astro-ph/0410687; Gopal and Sethi, astro-ph/0411170
- Impact on plasma physics $\Rightarrow$ impact on microwave background
  - CAMB (vector and tensor): Lewis, astro-ph/0406096
  - CMBFast (scalar): Giovannini, astro-ph/0312614
  - Faraday rotation: e.g. Loeb and Kosowsky, astro-ph/9601055, Kosowsky et. al., astro-ph/0409767
  - Non-Gaussianity from turbulence: e.g. Chen et. al., astro-ph/0403695
  - Reviews: Grasso and Rubenstein, astro-ph/0009061; Mack et. al. astro-ph/0105504; Giovannini, astro-ph/0508544

Motivation

- Aim:
  - Construct simulated primordial magnetic fields
  - Generate statistics of the stress-energy tensor
    - 1-Point: probability distribution function, skewness, kurtosis
    - 2-Point: auto- and cross- power spectra
    - 3-Point: bispectra
  - Wrap spectra and bispectra onto microwave sky
  - While model-specific, techniques and results could be relevant elsewhere – defect models?

Tangled Magnetic Field

– Infinite conductivity ⇒ E=0
– First order stress-energy tensor

$$\tau_{ab}(\mathbf{k}) = \frac{1}{2}\delta_{ab}\tilde{\tau}_{ii}(\mathbf{k}) - \tilde{\tau}_{ab}(\mathbf{k}), \;\; \tilde{\tau}_{ab}(\mathbf{k}) = \int B_a(\mathbf{q})B_b(\mathbf{k}-\mathbf{q})d^3\mathbf{q}.$$

– Large-scale tangled fields ⇒
  - cut-off scale $k_{c,}$ damping purely with universal expansion

– Gaussian statistics on the field – test case

$$\langle \mathbf{B}_a(\mathbf{k})\mathbf{B}_b^*(\mathbf{k}')\rangle = \mathcal{P}(k)P_{ab}(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}')\,,\; P_{ab}(\mathbf{k}) = \delta_{ab} - \hat{k}_a\hat{k}_b$$

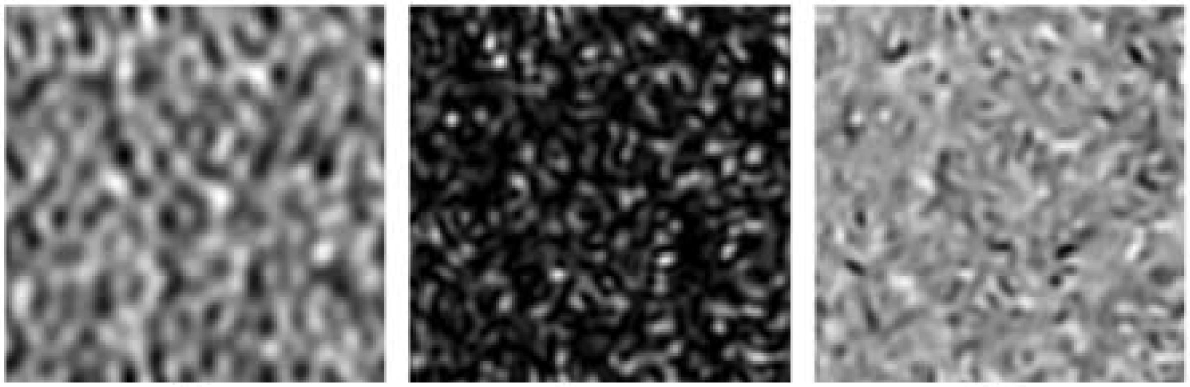
– Power law spectrum; nucleosynthesis bounds ⇒ n≈-3
  - Consider n=0, n=-2.5

Simulations

- Automatic ultraviolet cutoff; infrared cutoff inavoidable
- Solenoidal fields $\Rightarrow$

$$\mathbf{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \mathbf{R} \cdot \mathbf{C}, \quad \mathbf{R} = \frac{\mathcal{P}(k)^{1/2}}{\sqrt{\hat{k}_x^2 + \hat{k}_y^2}} \begin{pmatrix} \hat{k}_x \hat{k}_z & \hat{k}_y \\ \hat{k}_y \hat{k}_z & -\hat{k}_x \\ -\left(\hat{k}_x^2 + \hat{k}_y^2\right) & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

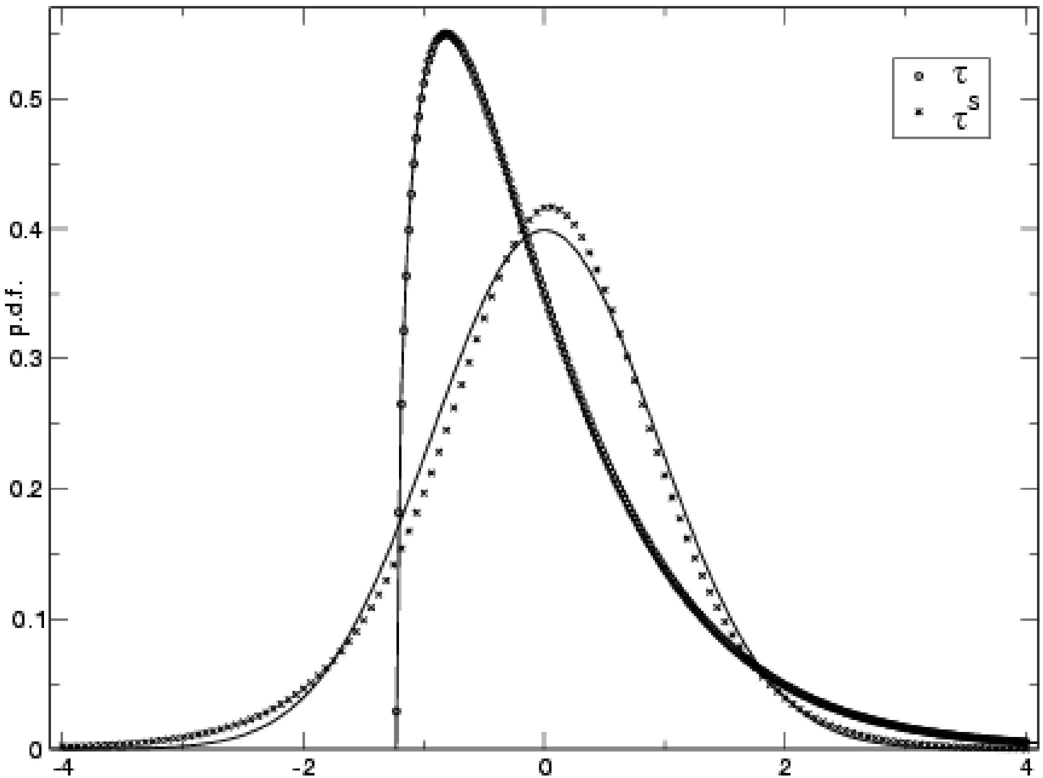
- Gaussian magnetic field; iso- and aniso-tropic pressures:



$B_x|_{z=0}$ $\qquad$ $\tau|_{z=0}$ $\qquad$ $\tau^S|_{z=0}$

1-Point Statistics

- Scalar probability distribution functions



1-Point Statistics

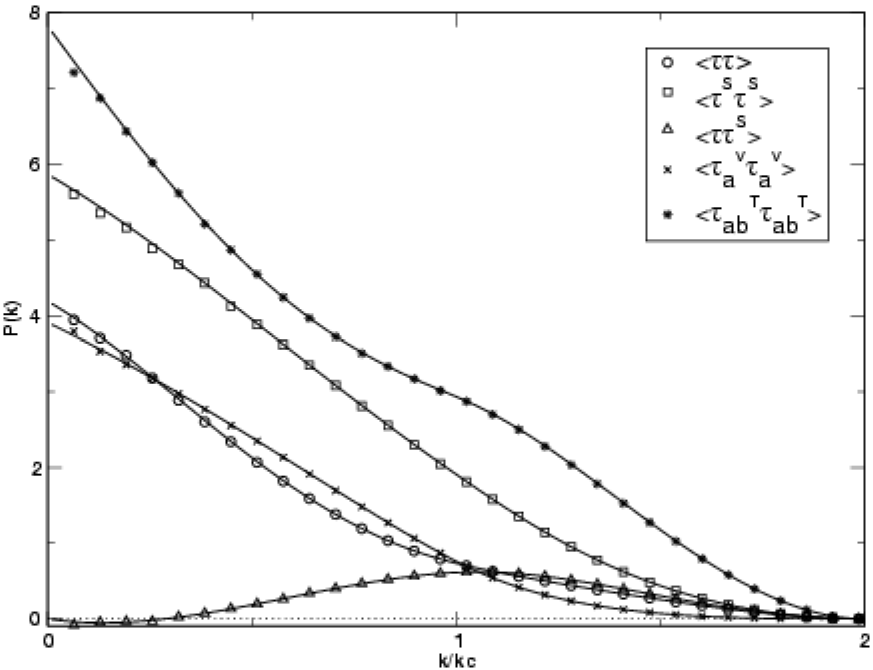
- Skewness and kurtosis:
  - Measures of deviation from Gaussian
  - Skewness = ‘tilt’, kurtosis = ‘curvature’
  - 20 realisations
- τ:
  - $\gamma_1 \approx 1.63 \pm 0.01$, $\gamma_2 = 3.99 \pm 0.05$ (insensitive to n)
  - expect $\gamma_1 \approx 1.63$, $\gamma_2 = 4$
- τ^s:
  - $\gamma_1 \approx -0.24 \pm 0.03$, $\gamma_2 = 1.10 \pm 0.01$ (n=0)
  - $\gamma_1 \approx -0.38 \pm 0.01$, $\gamma_2 = 0.86 \pm 0.02$ (n=-2.5)

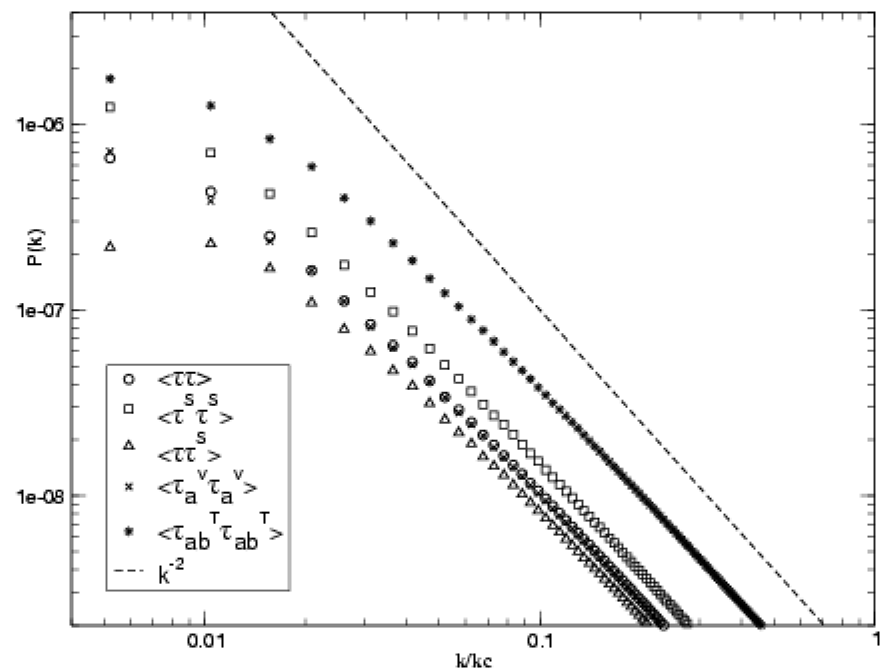
2-Point Statistics

– Calculate $$\langle \tilde{\tau}_{ab}(\mathbf{k})\tilde{\tau}^*_{cd}(\mathbf{p})\rangle = \delta(\mathbf{k}-\mathbf{p})\int d^3\mathbf{k}'\mathcal{P}(k')\mathcal{P}(|\mathbf{k}-\mathbf{k}'|)$$
$$\times\left(P_{ac}(\mathbf{k}')P_{bd}(\mathbf{k}-\mathbf{k}')+P_{ad}(\mathbf{k}')P_{bc}(\mathbf{k}-\mathbf{k}')\right)$$

– Extract invariant measures:

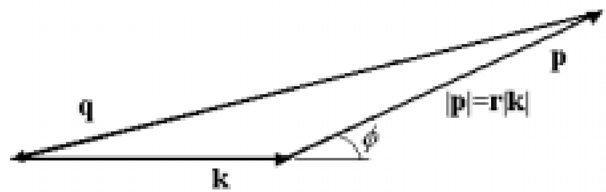
- $<\tau\tau>, <\tau^s\tau^s>, <\tau_a{}^v\tau_a{}^v>, <\tau_{ab}{}^t\tau_{ab}{}^t>, <\tau\tau^s>$





3-Point Statistics

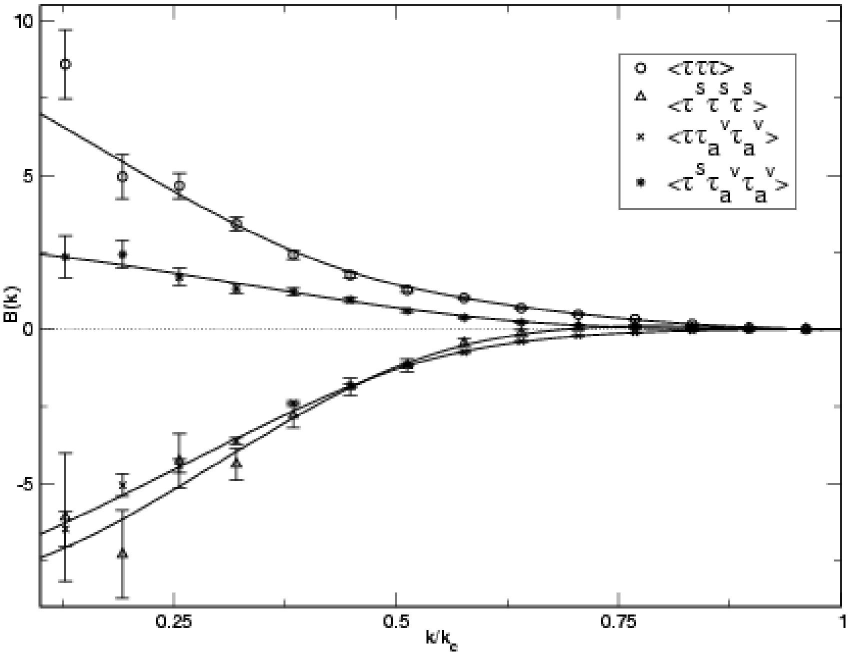
- Bispectra – three wavevectors **k**, **p**, **q**
- Statistical isotropy $\Rightarrow$ triangular formations
  - Colinear (degenerate)
    - r=1, φ=0
  - Equilateral
    - r=1, φ=120

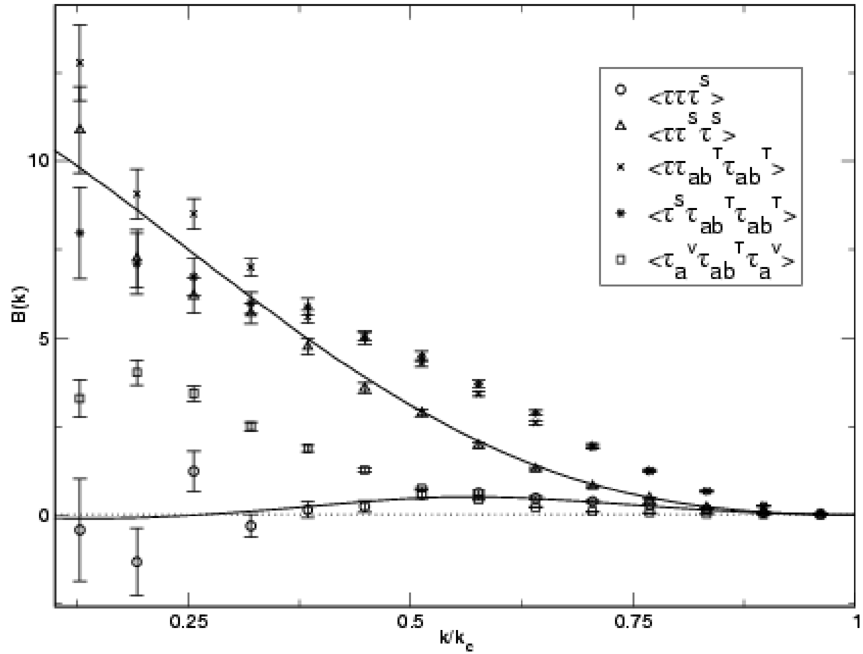


- Invariant: $<\tau\tau\tau>$, $<\tau\tau\tau^s>$, $<\tau\tau^s\tau^s>$, $<\tau^s\tau^s\tau^s>$, $<\tau\tau^v{}_a\tau^v{}_a>$, $<\tau^s\tau^v{}_a\tau^v{}_a>$, $<\tau\tau^T{}_{ab}\tau^T{}_{ab}>$, $<\tau^s\tau^T{}_{ab}\tau^T{}_{ab}>$, $<\tau_a{}^v\tau_{ab}{}^T\tau_b{}^v>$, $<\tau_{ab}{}^T\tau_{bc}{}^T\tau_{ac}{}^T>$
  - Symmetries of projection and full correlation operators remove full tensor correlation in colinear case
  - Realisations heavily compromised by sparse mode-selection
  - Average 1,500 runs at l_{dim}=192

3-Point Statistics

– Colinear, n=0:



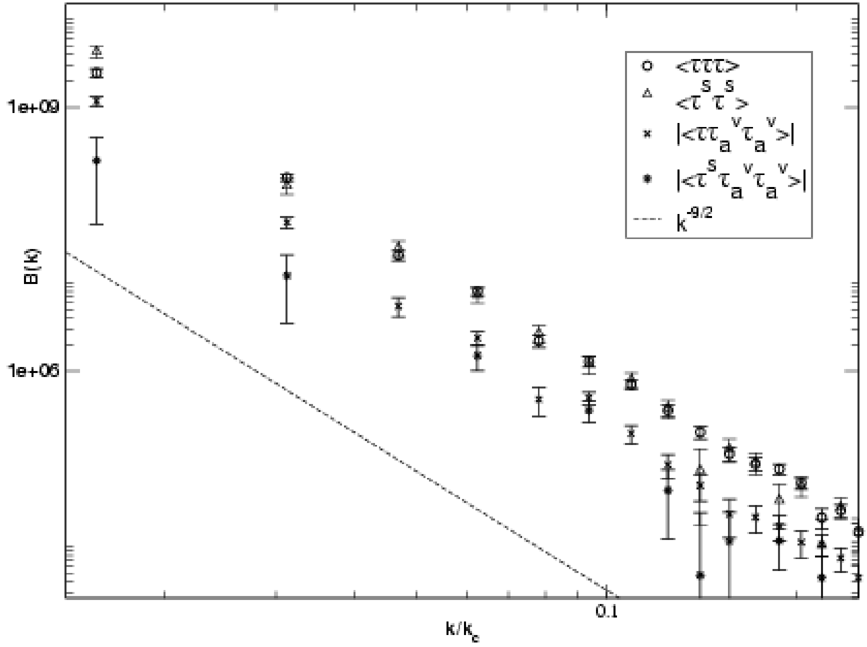


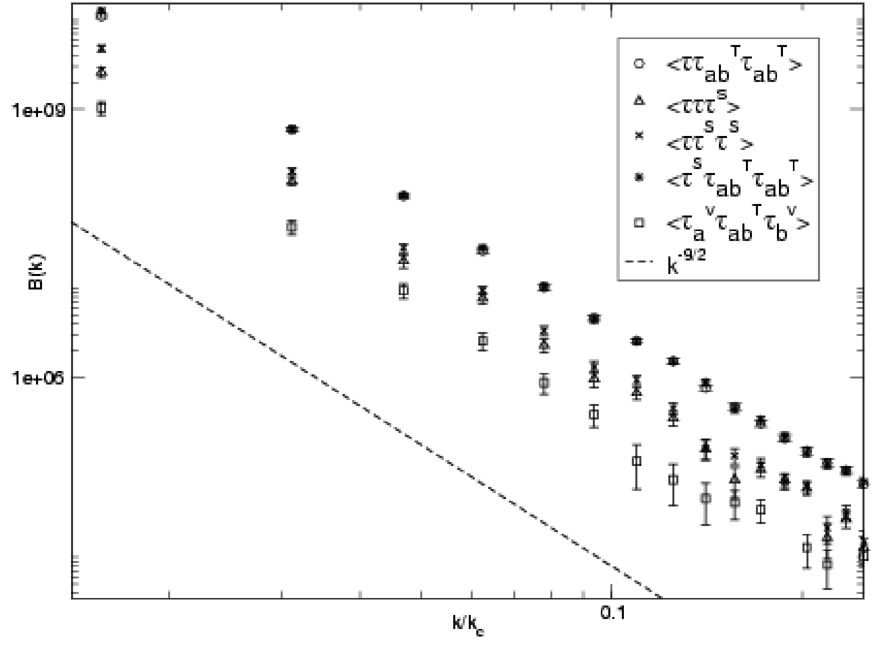
– Analysis and simulations agree very well

3-Point Statistics

– Colinear, n=-2.5:

- naively expect $\propto k^{-3(n+1)} = k^{-4.5}$





CMB Statistics

– In progress

– 2-Point statistics:

$$C_{AB,l} = \int_k \mathcal{P}(k)\Delta_{A,l}(k,\eta_0)\Delta^*_{B,l}(k,\eta_0)k^2 dk$$

– 3-Point scalar statistics:

$$B_{ll'l''} = \sqrt{\frac{(2l+1)(2l'+1)(2l''+1)}{4\pi}} \begin{pmatrix} l & l' & l'' \\ 0 & 0 & 0 \end{pmatrix} \int_k \int_p \int_q \mathcal{B}(k,p,q)$$
$$\times \Delta^S_{T,l}(k,\eta_0)\Delta^S_{T,l'}(p,\eta_0)\Delta^S_{T,l''}(q,\eta_0)\, J_{ll'l''}(k,p,q) q^2 dq p^2 dp k^2 dk$$

– Transfer functions from CMBFast, CAMB

– Large-scale approximations in, e.g., Mack et. al.

Conclusions and Prospects (1)

- Non-linear effects modify the statistics of the CMB
- Presented an informative test case
- Statistics will help constrain a primordial magnetic field
- Robust and general code generated
- 1-Point:
  - Non-Gaussian (χ^2) pressure, n-dependent anisotropic pressure
- 2-Point:
  - Uncorrelated $<\tau\tau^s>$ signal, $\approx$0.7 for n=-2.5, $\approx$0 for n=0
- 3-Point:
  - Strong non-Gaussianities in all signals at large-scales
  - Calculations of cross-correlations
  - Excellent agreement between theory and simulations

Conclusions and Prospects (2)

– Wanted:
  - CMB!
  - Faster realisations of non-degenerate bispectra
  - Analytic tensor modes
  - Non-Gaussian magnetic fields - χ^2 field?
  - Helicity
  - Techniques and analysis not necessarily magnetic: defect models?