Inflation and Primordial Spectrum from Background Free Quantum Gravity

Ken-ji Hamada
KEK

Based on
T. Yukawa and K.H., astro-ph/0401070
and
Review article invited for publication in
"Focus on Quantum Gravity Research"
by S. Horata, T. Yukawa and K. H.
(Nova Science Publishers, NY)

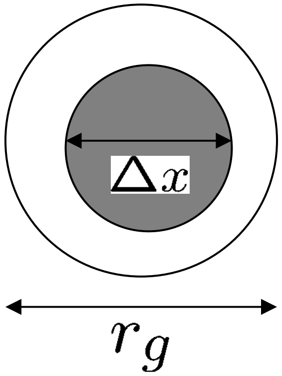
1. Introduction

Problems in Einstein theory

- *Spcetime singularities, Non-renormalizability*
- *Elementary excitations with the Planck mass => Black holes*

Compton wave length < Schwarzshild radius

Particle information is concealed inside B.H.



Key idea to resolve singularities/divergences

Background-metric independence

because no fixed scale and no special point in space.

Can break the wall of the Planck scale !

In very early universe, spacetime fluctuations are so great that geometry lose its classical meaning, and background-metric independent picture will emerge.

On the other hand, in the present universe, the metric acquires a physical significance for measuring time and distance.

A Novel Dynamical Scale separating these phases

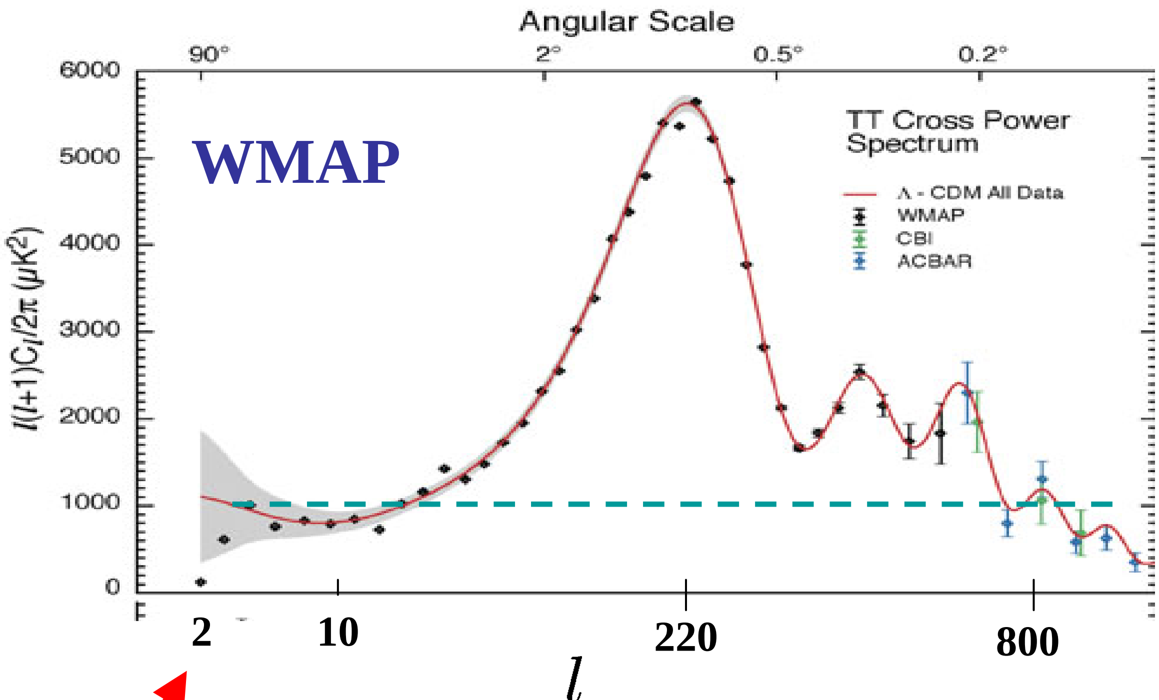
In very early universe, there is a spcetime transition at this scale:

Quantum Spacetime
(Background Free)

Classical Spacetime

There is a possibility to observe the instance of the transition, because we can trace the past guided by the known physical laws as far as the classical spacetime exists.

Primordial spectrum ($\simeq$ H-Z spectrum) → **Deformed by dynamics in the history of universe**



Scale inv (HZ) spectrum

Dynamical scale of quantum gravity

Acoustic peaks

Collisional damping (Silk damping)

2. Renormalizable Quantum Gravity

$$I = \int d^4x\sqrt{-g}\left\{\underbrace{-\frac{1}{t^2}C^2_{\mu\nu\lambda\sigma} - bG_4}_{\text{conformal invariant}} + \frac{M_P}{2}R - \Lambda\right\} + I_{\mathrm{Matter}}$$

$\mathrm{sgn} = (-1, 1, 1, 1)$

Perturbation about conformal flat : $C_{\mu\nu\lambda\sigma} = 0$

No restriction on conf. mode

$$g_{\mu\nu} = e^{2\phi}\left(\hat{g}_{\mu\nu} + th_{\mu\nu} + \cdots\right), \qquad tr(h) = 0$$

Conformal mode and traceless-tensor mode are treated separately

Dynamics of the traceless-tensor mode $h_{\mu\nu}$ indicates

Asymptotically Free

$$\alpha_G = \frac{t_r^2(p)}{4\pi} = \frac{1}{2\pi\beta_0}\frac{1}{\log(p^4/\underline{\underline{\Lambda_{\mathrm{QG}}^4}})}, \qquad \beta = -\beta_0 t_r^3$$

Renormalization

Beta functions (QED + gravity)

$$\beta_t = -\left(\frac{n_F}{40}+\frac{10}{3}\right)\frac{t_r^3}{(4\pi)^2} - \frac{7n_F}{72}\frac{e_r^2 t_r^3}{(4\pi)^4} + o(t_r^5)$$

$$\beta_e = \frac{4n_F}{3}\frac{e_r^3}{(4\pi)^2} + \left(4n_F - \frac{8}{9}\frac{n_F^2}{b_1}\right)\frac{e_r^5}{(4\pi)^4} + o(e_r^3 t_r^2)$$

where $b_1 = \frac{11n_F}{360} + \frac{40}{9}$: coeff. of WZ action of type $\phi\hat{\Delta}_4\phi$

New WZ actions (=new vertices) like $\phi^{n+1}\hat{\Delta}_4\phi,\ \phi^n C^2_{\mu\nu\lambda\sigma},\ \phi^n F^2_{\mu\nu}$
are induced at higher orders of α_G

Conformal mode is not renormalized at all order:

$$Z_\phi = 1$$

K.H., hep-th/0203250

Asymptotic freedom for the traceless mode

The $C_{\mu\nu\lambda\sigma} = 0$ configuration dominates at very high energy, and thus singularities with $R_{\mu\nu\lambda\sigma} = \infty$ are removed quantum mechanically.

On the other hand, the singular configuration cannot be excluded in the Einstein theory, because such a configuration has the vanishing scalar curvature so that its quantum weight in the path integral is unity: $\exp\left(i\int d^4x\sqrt{g}R\right) \sim O(1) \quad \text{for } R = 0$

Fluctuations of the conformal mode dominate, and thus the spacetime dynamics is described by CFT at very high energies.

t_r **: a deviation from CFT**

Conformal mode is treated non-perturbatively

The partition function

Distler-Kawai, David,
Antoniadis-Mazur-Mottola.
Sugino-K.H., K.H.

$$
\begin{aligned}
Z &= \int [dg \cdots]_{\underline{\underline{g}}} \exp(iI) \\
&= \int [d\phi dh \cdots]_{\underline{\underline{\hat{g}}}} \exp(i\underline{\underline{S(\phi)}} + iI)
\end{aligned}
$$

Jacobian = Wess-Zumino action

Dynamics of conformal mode is induced from the measure, where

$$
S(\phi) = -\frac{b_1}{(4\pi)^2}\int d^4x\sqrt{-\hat{g}}\left\{2\phi\hat{\Delta}_4\phi + \left(G_4 - \frac{2}{3}\hat{\nabla}^2\hat{R}\right)\phi\right\} + O(\phi^3)
$$

Higher order of the coupling α_G

$\alpha_G \to 0$

Conformal Field Theory (CFT)

conformal inv. : $\phi \to \phi + \omega$, and thus

$$
Z(e^{2\omega}\hat{g}) = Z(\hat{g})
$$

: Background Free !

Physical States are governed by Conformal Invariance

Antoniadis-Mazur-Mottola
Horata-K.H., K.H.

Conformal algebra (on cylinder $R \times S^3$):

$$[Q_M, Q_N^\dagger] = 2\delta_{MN} H + 2R_{MN}$$

special conf. transfs. Hamiltonian rotation on S^3

M, N = vector index of SO(4)

Physical state conditions:

$$Q_M|\mathrm{phys}\rangle = H|\mathrm{phys}\rangle = R_{MN}|\mathrm{phys}\rangle = 0$$

mixes positive-metric and negative-metric modes

Physical operators: $e^{\gamma_0 \phi}$ cosmological const.

$\mathcal{R}(\partial\phi)e^{\gamma_2 \phi}$, scalar curvature

$\cdots$

Conformal charge: $\gamma_n = 2b_1 - 2\sqrt{b_1^2 - (4-n)b_1} = 4 - n + 0(1/b_1)$

Conformal inv. vacuum = physical state satisfying $Q_M^\dagger|\Omega\rangle = 0$

3. Quantum Gravity Scenario of Inflation

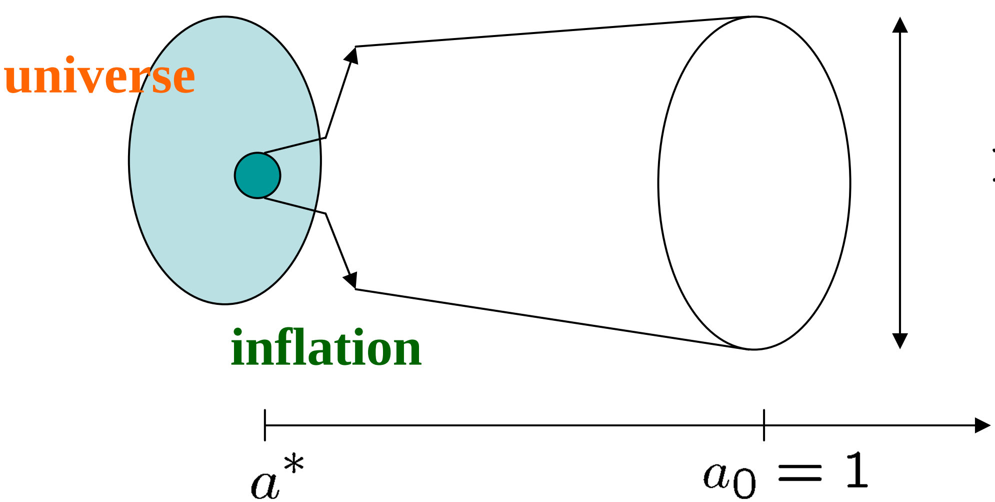
If we believe the idea of inflation, CMB anisotropies provide us information about dynamics beyond the Planck scale.

Initial conditions would be given by quantum gravity

Trans-Plankian problem: $L_P = a^* \times (1/H_0) \rightarrow a^* = 6.3 \times 10^{-59}$

This value is necessary to solve flatness problem

$$
\begin{aligned}
1/H_0 &= 2998/h \text{ Mpc } (h = 0.72) \\
&= 1.3 \times 10^{28} \text{ cm}
\end{aligned}
$$



a : scale factor

Evolution of the universe:

Consider the orderings $M_P \gg \Lambda_{QG}$

Effective action (WZ + Einstein) suggests

Conformal symmetry starts to be broken at the Planck scale

$\Longrightarrow$ Inflationary universe with $H \sim M_P$

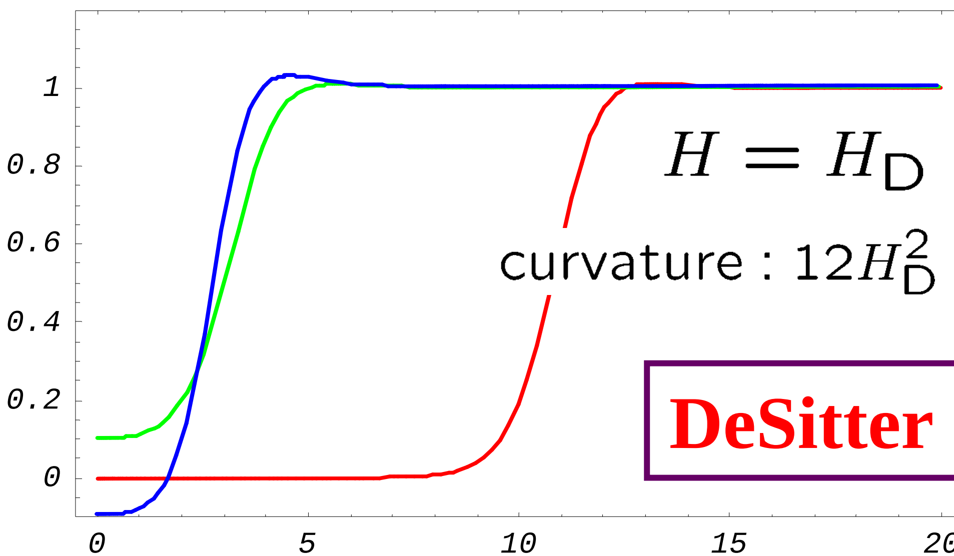
(No logarithmic violation at M_P , because dynamics is still described by CFT)

$$b_1\partial_\eta^4\phi - 24\pi^2 M_P^2 e^{2\phi}\left\{\partial_\eta^2\phi + (\partial_\eta\phi)^2\right\} + o(\cancel{\alpha_G}) = 0$$

WZ action

H

Hubble parameter



τ : proper time

Tachyonic instability

$$H_D = \sqrt{8\pi^2/b_1}\,M_P$$

$$d\tau = e^{\phi}d\eta$$

DeSitter solution is stable!

Conformal symmetry is completely broken at the dynamical scale

$\Longrightarrow$ Friedmann universe

Our transition scenario:

1. At energies gets lower to the dynamical scale Λ_{QG}, the running coupling will diverges and higher-derivative actions disappear.

2. Then, extra degrees of freedom in higher-derivative gravitational fields transfer to matter degrees of freedom, and cause Big Bang.

3. Below this energy, the Einstein gravity governs the dynamics.

A Model of Spacetime Phase Transition

For simplicity, we approximate the running coupling operator by its time-dependent average $\langle t_r^2(\tau)\rangle$ defined by

$$-\tau\frac{d}{d\tau}\langle t_r^2(\tau)\rangle = \beta\left(\langle t_r^2(\tau)\rangle\right)$$

A correction to the coefficient in front of WZ action:

$$b_1 \to b_1(1-a_1 t_r^2+\cdots) \sim \underline{\underline{\frac{b_1}{1+a_1 t_r^2}}} \qquad (a_1>0)$$

Running coupling effect is introduced by the replacement:

$$t_r^2 \to \langle t_r^2(\tau)\rangle = \frac{-1}{2\beta_0 \log(\tau\Lambda_{QG})}$$

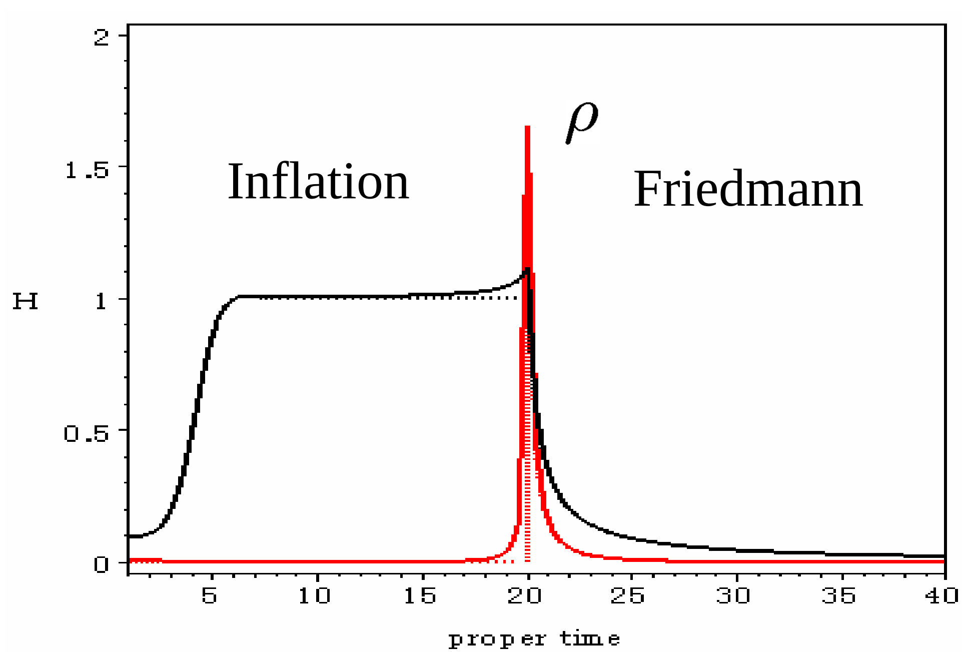
This dynamical coeff. vanishes at $\tau = 1/\Lambda_{QG}\ (\equiv \tau_\Lambda)$

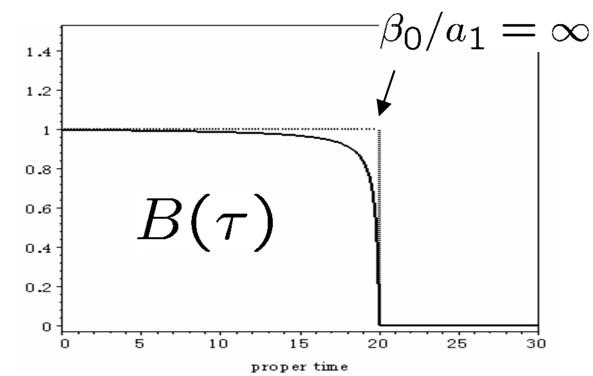
Evolution Equation and Energy Conservation (in proper time rep.)

$$\begin{cases} b_1 B(\tau)\left(\dddot{H} + 7H\,\ddot{H} + 4\,\dot{H}^2 + 18H^2\,\dot{H} + 6H^4\right) - 24\pi^2 M_P^2\left(\dot{H} + 2H^2\right) = 0 \\ b_1 B(\tau)\left(2H\,\ddot{H} - \dot{H}^2 + 6H^2\,\dot{H} + 6H^4\right) - 24\pi^2 M_P^2 H^2 + 8\pi^2\rho = 0 \end{cases}$$

where $B(\tau) = \dfrac{1}{1 + a_1\langle t_r^2(\tau)\rangle}$ $\quad H = \dot{a}(\tau)/a(\tau) = \dot{\phi}(\tau)$

Gravitational degrees of freedom decay to matter fields ρ at $\tau = \tau_\Lambda$





Number of e-foldings:

$$\mathcal{N}_e = \log\frac{a(\tau_P)}{a(\tau_\Lambda)} \simeq \frac{H_D}{\Lambda_{QG}}$$

4. Primordial Power Spectrum

WMAP observes quantum fluctuations of scalar curvature just before quantum spacetime transits to classical spacetime at $\tau_\Lambda = 1/\Lambda_{\mathrm{QG}}$.

$$\left.\frac{\delta R}{R}\right|_{\tau_{\mathrm{P}}} \Longrightarrow \left.\frac{\delta R}{R}\right|_{\tau_\Lambda^-} \Longrightarrow \left.\frac{\delta\rho}{\rho}\right|_{\tau_\Lambda^+} \Longrightarrow \left.\frac{\delta T}{T}\right|_{\mathrm{today}}$$

preserved during inflation

transition to classical spacetime

spectrum deformed by some scales

$$\left\langle \frac{\delta T}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle \sim \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k^4} \left\langle\left\langle \frac{\delta R}{R}(\mathbf{k}) \frac{\delta R}{R}(-\mathbf{k}) \right\rangle\right\rangle\Big|_{\tau=\tau_{\mathrm{P}}} e^{i\mathbf{k}\cdot(\mathbf{n}-\mathbf{n}')x_{\mathrm{ls}}}$$

Use Sachs-Wolfe effect

$k = |\mathbf{k}|$ **: comoving wavenumber**

Scaling exponents can be calculated using CFT

Power Spectrum: $C_l = \int_\lambda^\infty \frac{dk}{k} j_l^2(kx_{\mathrm{ls}}) P(k)$

$$P(k) = A \left(\frac{k}{m_\lambda} \right)^{n-1+\frac{v}{\log(k^2/\lambda^2)}} \qquad (v > 0)$$

Running coupling effect

where

$$n = 5 - 8 \frac{1-\sqrt{1-2/b_1}}{1-\sqrt{1-4/b_1}} = \underline{1} + 2/b_1 + 4/b_1^2 + O(1/b_1^3)$$

spectral index determined by CFT.

$$\left\{ \begin{array}{l} b_1 = \frac{1}{360}\left(N_{\mathrm{X}} + \frac{11}{2}N_{\mathrm{W}} + 62 N_{\mathrm{A}}\right) + \frac{769}{180} \\ \lambda = a(\tau_{\mathrm{P}})\Lambda_{\mathrm{QG}} \\ m_\lambda = a(\tau_{\mathrm{P}}) H_{\mathrm{D}} \end{array} \right.$$

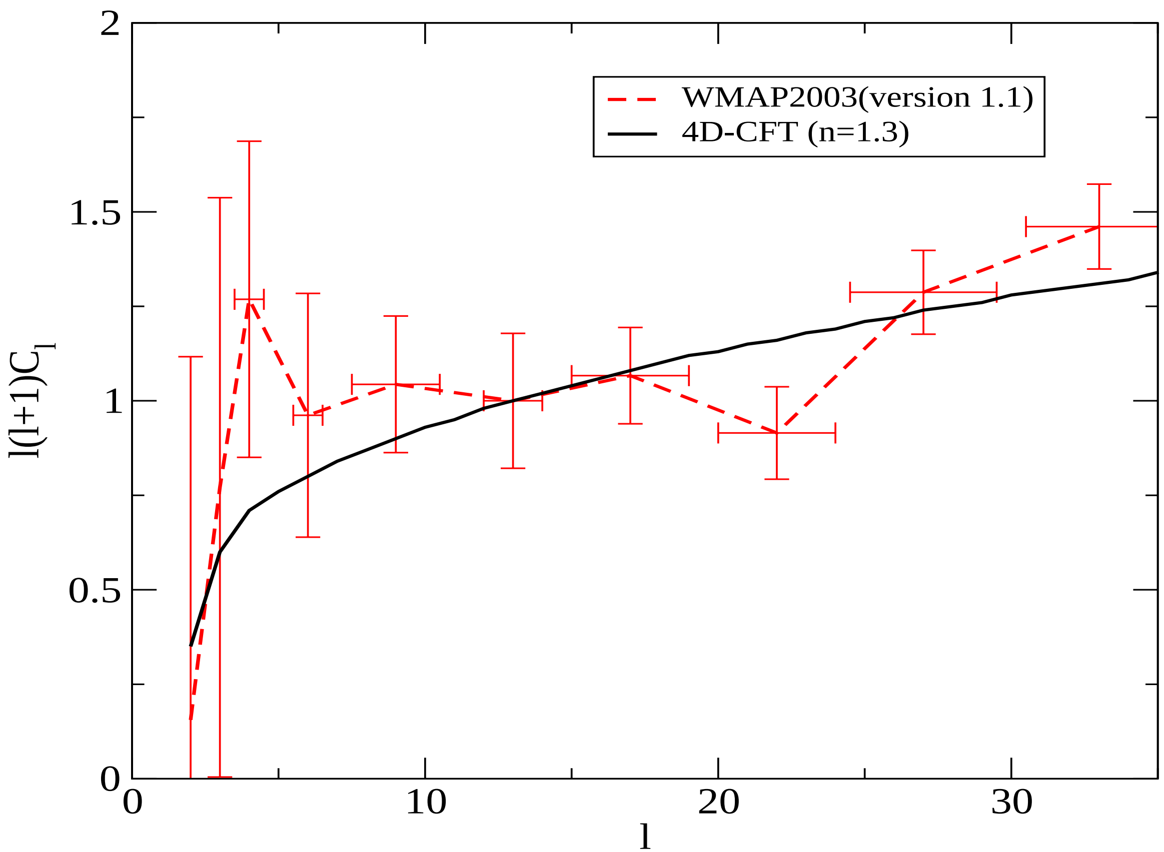
- b_1 **: coeff. of WZ action**
- λ **: comoving dynamical scale**
- m_λ **: comoving Planck constant**

Number of e-foldings : $\mathcal{N}_e = \log \frac{a(\tau_{\mathrm{P}})}{a(\tau_\Lambda)} \simeq \frac{H_{\mathrm{D}}}{\Lambda_{\mathrm{QG}}} = \frac{m_\lambda}{\lambda}$

Sharp damping at l=3 → comoving dynamical scale

$$\lambda = 3/x_{\rm ls} = 0.0002\ {\rm Mpc}^{-1}\ (x_{\rm ls} = 14000\ {\rm Mpc})$$

If we take the number of e-folding as $\mathcal{N}_e = 100$



n=1.3

$v = 0.001$

multi-poles

Naive Estimation of amplitude

Scalar curvature in deSitter spacetime: $R = 12H_{\mathrm{D}}^2$

Near transition point, the running coupling gets large, and so $\nabla^2 \sim \Lambda_{\mathrm{QG}}^2$. Since scalar curvature has two derivatives, the size of fluctuation is estimated to be the order of the square of dynamical scale: $\delta R \sim \Lambda_{\mathrm{QG}}^2$

$$\sqrt{\left\langle \left(\frac{\delta R}{R} \right)^2 \right\rangle} \Bigg|_{\tau=\tau_\Lambda^-} \sim \frac{\Lambda_{\mathrm{QG}}^2}{12 H_{\mathrm{D}}^2} \simeq \frac{1}{12 \mathcal{N}_e^2} \sim 10^{-5}$$

5. Conclusion

•Asymptotic freedom for the traceless tensor mode

$\Rightarrow$ • Spacetime singularities removed quantum mechanically.
• Physical states are changed, which are governed by CFT.

No singularity and changing of physical states in quantum spacetime are preferable features to resolve information loss problem.

• Quantum gravity scenario of inflation without any additional fields

$$M_{\rm P} \sim 10^{19}\,\mathrm{GeV} \xrightarrow[\mathcal{N}_e = 100]{} \Lambda_{\rm QG} \sim 10^{17}\,\mathrm{GeV}$$

We proposed that a spacetime phase transition occurs at $\Lambda_{\rm QG}$ scale, and then extra degrees of freedom in higher-derivative gravitational fields may decay to matter degrees of freedom causing Big Bang.

•Angular power spectrum

Primordial fluctuations dominated by conformal mode (CFT).

- Sharp damping of the power spectrum at low multi-poles was explained by dynamical scale of quantum gravity.
- Large blue spectrum at large angle was predicted.
- Tensor/scalar ratio is negligible, because of the conformal mode dominance in inflation era.
- There are multi-point correlations, because of CFT.

To obtain full spectrum, the details of phase transition are necessary.

•Discussions

- Repulsive effect in quantum gravity casts light on the question why the universe has been expanding since it was born.
- The repulsive effect may prevent black hole from collapsing to a point, and play an important role to release matters in black hole outside.

Reduction of the traceless field to ordinary graviton and classical background occurs about Λ_{QG} .

Full propagator of the traceless mode :

No tachyon condition

$$\frac{1}{p^2 M(p^2)} \qquad p^2 > \Lambda_{QG}^2 \qquad \left(M_P > \sqrt{4\beta_0/e}\,\Lambda_{QG} \right)$$

graviton pole

correction
(no ghost pole)

These modes decay
to matter fields

$$M(p^2) = M_P^2 + 2\beta_0 p^2 \log \frac{p^4}{\Lambda_{QG}^4}$$

Our Evolution Scenario

E

Background-metric independent space-time without singularity
Described by CFT (long range)

Quantum Space-time

M_{P}

Inflation era (still long range)

Λ_{QG}

The traceless-mode coupling α_G gets large, correlation becomes short range: $1/\Lambda_{\mathrm{QG}}$

Big Bang

Quantum fluctuations will percolate to ordinary particles:
graviton, gauge bosons, fermions.

Classical Friedmann Space-time

$\Lambda_{\mathrm{COS}}^{1/4}$

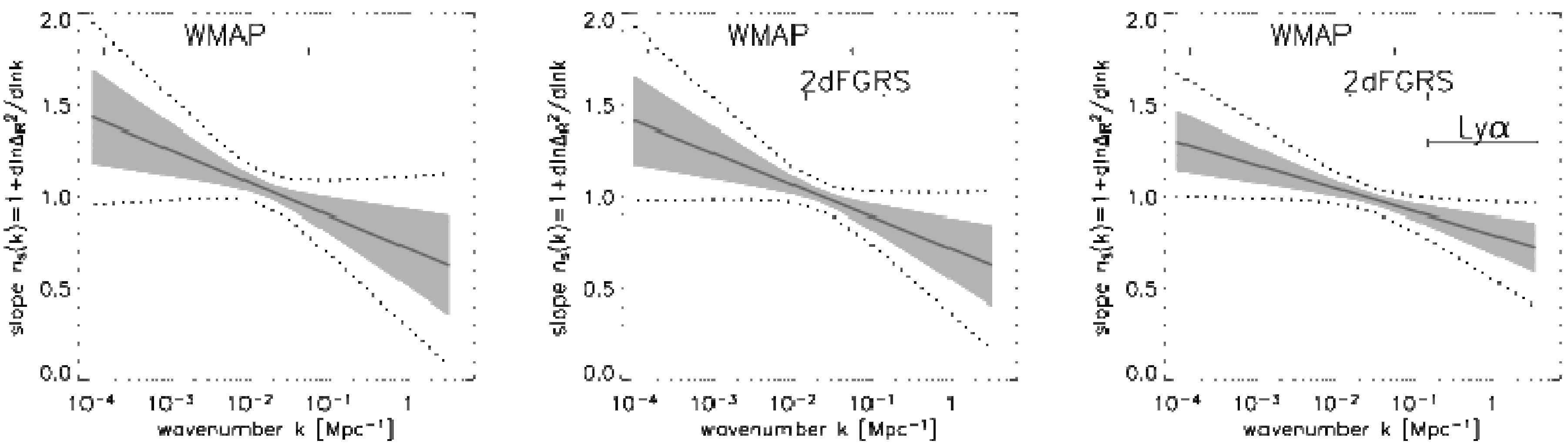
De Sitter again

WMAP suggests red spectrum (n < 1) at small angle

We have assumed that the unique decoupling time, τ_Λ , for entire momentum range.

However, if there is a time lag in the phase transition, short scale delay will change m_λ to be an increasing function of the comoving wavenumber.

$$m_\lambda \to m_\lambda(k)$$



Peiris et al astro-ph/0302225