CMB power spectrum from cosmic strings

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N. Bevis, MH, M. Kunz[a] astro-ph/0403029

N. Bevis, MH, M. Kunz, A. Liddle, J. Urrestilla in progress.

[a]U. de Genève

University of Sussex

Introduction

String defects[a] may be formed at phase transitions in the early Universe

- Thermal[b]
- End of hybrid inflation[c]
- (D-$\bar{D}$)-brane collisions[d]

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[a] Hindmarsh & Kibble (1994); Vilenkin & Shellard (1994); Kibble (2004)
[b] Kibble (1976); Zurek (1996); Rajantie (2002)
[c] Yokoyama (1989); Kofman, Linde, Starobinski (1996)
[d] Jones, Stoica, Tye (2002); Sarangi & Tye (2003); Copeland, Myers, Polchinski (2003)

... introduction

Observational consequences:

- Gravitational perturbations (scalar, vector & tensor)[a]
- Baryon asymmetry[b]
- Cosmic rays[c]

Network dynamics not well understood:

Energy lost to (loops $\longrightarrow$ gravitational waves) OR (classical radiation $\longrightarrow$ particles)?

Significant uncertainty in quantitative calculations (not often acknowledged)

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[a] Zel'dovich (1980); Vilenkin (1981); Kaiser & Stebbins (1984); ...

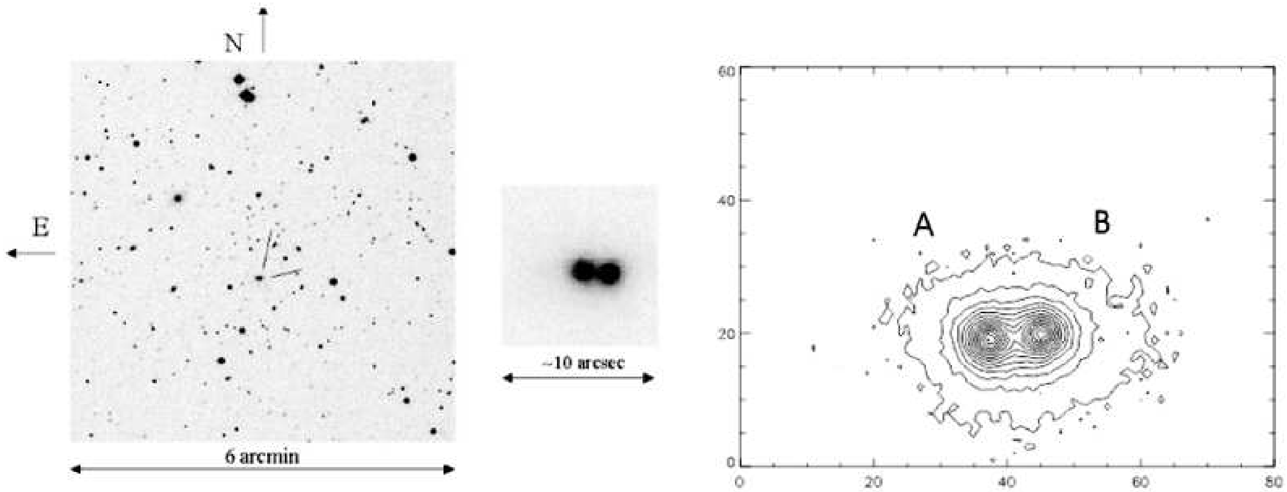
[b] Bhattarcharjee, Kibble, Turok (1982); Brandenburger, Davis, Hindmarsh (1991); Brandenburger, David, Trodden (1994); Sahu, Bhattarcharjee, Yajnik (2004)

[c] Bhattarcharjee (1990); Sigl (1996); Protheroe (1996); Berezhinksi (1997); Vincent, M.H., Sakellariadou (1998)

New strings for old

Recent revival of interest:

- Cosmic F- and D- strings: M-theory allows many types of string network [a]
- CSL1:[b]



[a] Jones, Stoica, Tye (2002); Sarangi & Tye (2003); Copeland, Myers, Polchinski (2003)
[b] Sazhin et al (2003)

Topological defects in cosmology

- Global “defects”:[a]
  - Spontaneous breaking of global symmetry
  - Low-energy dynamics: non-linear sigma model
  - Gravitational perturbations sourced by Goldstone modes
- Cosmic strings[b]
  - Spontaneous breaking of global or gauge symmetry
  - Topology of vacuum manifold allows string-like solitons in 3D
  - e.g. Abelian Higgs model (gauge cosmic strings)

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[a] Dürrer, Kunz, MelchiorriarXiv:astro-ph/0110348]

[b] MH & Kibble [arXiv:hep-ph/9411342]; Kibble arXiv:astro-ph/0410073; Polchinski arXiv:hep-th/0412244

Global defects: temperature power spectrum

O(4) non-linear sigma model[a]

Normalisation:

$$C_\ell^{(\mathrm{NLSM})} = 0.13 C_\ell^{(\mathrm{ad.scal.})}$$
$$C_\ell^{(\mathrm{tensor})} = 0.19 C_\ell^{(\mathrm{scal.})}.$$

Cosmological parameters:

$\Omega = 1$

$h = 0.70$

$\Omega_\mathrm{b} h^2 = 0.022$

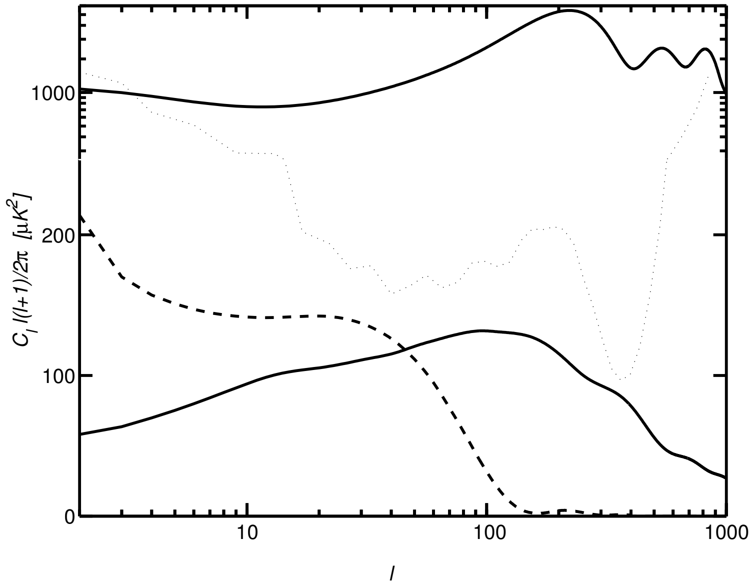
$\Omega_m = 0.30$

$\tau = 0.10$ or $z_\mathrm{r} = 13$

Primordial adiabatic scalar

Tensor

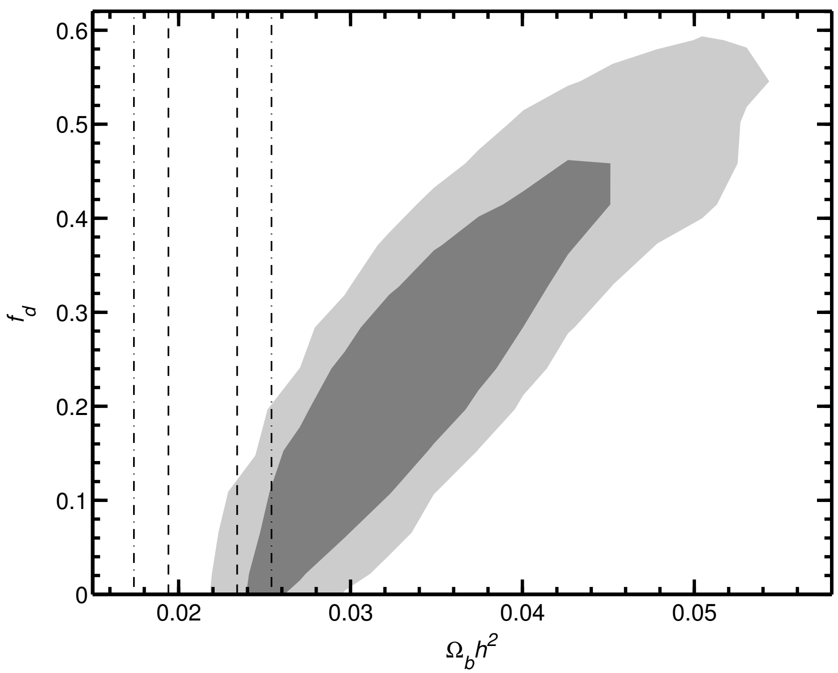
Defect



[a] Bevis, Hindmarsh, Kunz [arXiv:astro-ph/0403029]

Lifting the $\Omega_{\rm b}h^2$-$f_{\rm d}$ degeneracy with Big Bang Nucleosynthesis

- Marginalised likelihoood from WMAP
- 68% and 95% confidence levels.
- Vertical lines: 68% and 95% confidence limits on $\Omega_{\rm b}h^2$(Kirkman *et al*)

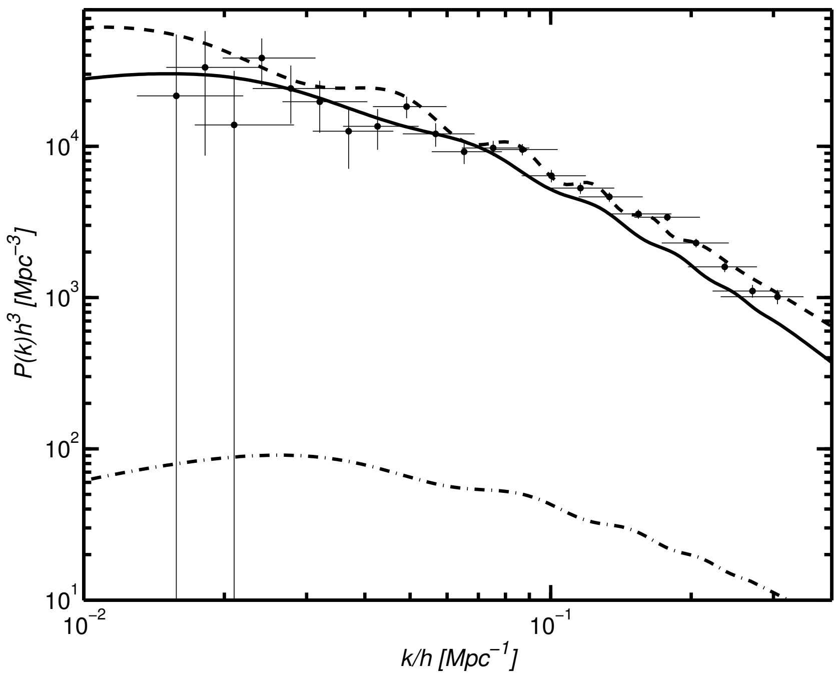


SDSS and global defects: matter doesn't matter

Solid: Adiabatic HZ scalar

Dashed: Scalar + $f_d = 0.4$

Dot-dashed: defect contribution



Global defects: headline result

Global defects contribute less than 13%[a] at $\ell = 9$ (95% confidence).

[a](theoretical uncertainty 2%)

Calculating perturbations from defects: UETC method

$h_\alpha(\tau, k)$: linear perturbation (metric, matter, temperature ...)

$S_\alpha(\tau, k)$: source (energy-momentum, separately conserved)

$D_{\alpha\beta}(\tau, k)$: time dependent differential operator

Perturbation equation: $\mathcal{D}_{\alpha\beta}(\tau, k) h_\beta(\tau, k) = S_\alpha(\tau . k)$

Power spectrum:[a] $\left\langle |h_\alpha(\tau_0, k)|^2 \right\rangle = \int \int \mathcal{D}^{-1}\mathcal{D}^{-1} \left\langle S_\alpha(\tau, k) S^*_\alpha(\tau', k) \right\rangle$

Need unequal-time correlators (UETCs) of source or energy-momentum tensor

$$C_{\mu\nu\rho\lambda}(k, \tau, \tau') = \left\langle T_{\mu\nu}(k, \tau) T^*_{\rho\lambda}(k, \tau') \right\rangle$$

[a] Pen, Seljak, Turok (1997); Dürrer, Kunz, Melchiorri (1998,2002)

... UETC method

Isotropy + EM conservation + parity: $C_{\mu\nu\rho\lambda}$: 3 scalar, 1 vector, 1 tensor

(S,V,T) correlators can be diagonalised: $C(k\tau, k\tau') = \sum_n \lambda_n v_n(k\tau) v_n^*(k\tau')$

Eigenvectors $v_n(k\tau)$ as sources: $h_\alpha^n(\tau, k) = \int_{\tau_i}^{\tau} \mathcal{D}_{\alpha\beta}^{-1}(\tau, \tau', k) v_n(\tau', k)$

Reconstruct complete power spectrum:

$$C_\ell = \sum_n \lambda_n^{(S)} C_\ell^{(S)n} + \sum_n \lambda_n^{(V)} C_\ell^{(V)n} + \sum_n \lambda_n^{(T)} C_\ell^{(T)n}$$

UETC method for power spectrum: summary

Calculate UETCs

$\downarrow$

Diagonalise UETCs

$\downarrow$

Solve perturbation equations with eigenfunctions as sources

$\downarrow$

Square (e.g.) $(\Delta T)_\ell^{(S,V,T)n}$ and sum

Energy momentum tensor decomposition

Scalar (S), Vector (V) and Tensor (T) under 3D rotation group[a]

$$T^{(S)}_{00} = v^2 f_\rho \qquad T^{(S)}_{j0} = i v^2 k_j f_v \qquad T^{(S)}_{jl} = v^2 \left[(f_p + \frac{1}{3} k^2 f_\pi)\delta_{jl} - k_j k_l f_\pi \right]$$

$$T^{(V)}_{j0} = v^2 w^{(v)}_j \qquad T^{(V)}_{jl} = i v^2 \frac{1}{2} \left(k_j w^{(\pi)}_l + k_l w^{(\pi)}_j \right)$$

$$T^{(T)}_{jl} = v^2 \tau^{(\pi)}_{ij}$$

v is v.e.v. of scalar field: $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$

[a] Notation of Dürrer, Kunz, Melchiorri [arXiv:astro-ph/0110348]

Linearised Einstein equations for defect “seeds”

Gauge invariant formalism[a]

Scalar: $\Phi^{(s)}$, $\Psi^{(s)}$ Vector: $\Sigma_i^{(s)}$ Tensor: $H_{ij}^{(s)}$

$$\begin{aligned}
k^2\Phi^{(s)} &= \epsilon(f_\rho + 3\frac{\dot a}{a} f_v) \\
\Phi^{(s)} + \Psi^{(s)} &= -2\epsilon f_\pi \\
-k^2\Sigma_i^{(s)} &= 4\epsilon w_i^{(v)} \\
\ddot H_{ij}^{(s)} + 2\frac{\dot a}{a}\dot H_{ij}^{(s)} + k^2 H_{ij}^{(s)} &= 2\epsilon\tau_{ij}^{(\pi)} \,.
\end{aligned}$$

$\epsilon = 4\pi G v^2$, where v is v.e.v. of scalar field

[a] Notation of Dürrer, Kunz, Melchiorri [arXiv:astro-ph/0110348]

Unequal time correlators: scaling

Do not have to trace entire history of network

Defect networks exhibit scaling

Scaling: correlators depend only on time τ and wavenumber k

E.g. scalar:[a] ($x = k\tau$)

$$\begin{aligned} \langle \Phi_s(\mathbf{k},\tau)\Phi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}}C_{11}(x,x') \\ \langle \Phi_s(\mathbf{k},\tau)\Psi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}}C_{12}(x,x') \\ \langle \Psi_s(\mathbf{k},\tau)\Psi_s^*(\mathbf{k},\tau')\rangle &= \frac{1}{k^4\sqrt{\tau\tau'}}C_{22}(x,x') \end{aligned}$$

NB scaling fails during a change in expansion rate

[a] Notation of Dürrer, Kunz, Melchiorri [arXiv:astro-ph/0110348]

CMB power spectrum from (gauge) cosmic strings: previous work

Authors	*String model*	COBE $G\mu$
Perivolaropoulos (1995)	Moving segment model	$\sim 1.6 \times 10^{-6}$
Allen et al (1997)	Ideal strings (FRW)	$1.05^{+0.35}_{-0.20} \times 10^{-6}$
Albrecht, Battye, Robinson (1997)	Moving segment model	$\sim 2 \times 10^{-6}$
Contaldi, Hindmarsh, Magueijo (1998)	Ideal strings (Minkowski)	–
Landriau & Shellard (2004)	Ideal strings (FRW)	$\sim 0.7 \times 10^{-6}$
Wyman, Pogosian, Wasserman (2005)	Moving segment model	$\sim 2 \times 10^{-6}$

MSM: moving segments of increasing length with random velocities[a]

Ideal Strings: 1D objects, Nambu-Goto action

[a] Vincent, Hindmarsh, Sakellariadou (1997); Albrecht, Battye, Robinson (1997)

Approximations

Quantum field theory

$\downarrow$ (Large occupation number, bosons)

Classical field theory

$\downarrow$ (Smooth, low curvature string configurations)

“Ideal” (Nambu-Goto) strings

$\downarrow$ (Phenomenological)

Moving segment model

Abelian Higgs model

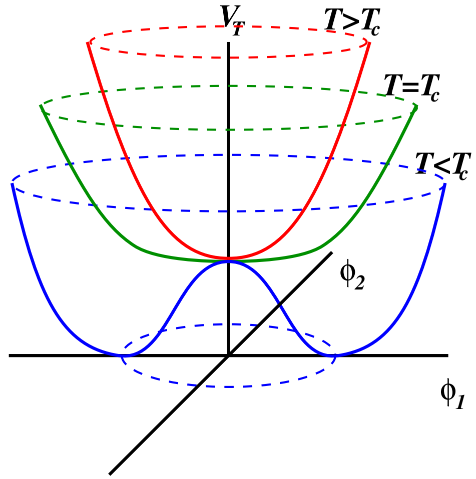
$$S = -\int d^4x \sqrt{-g}\left(g^{\mu\nu}D_\mu\phi^* D_\nu\phi + V(\phi) + \frac{1}{4e^2}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right),$$

Complex scalar field $\phi(\mathbf{x},t)$, vector field $A_\mu(\mathbf{x},t)$

Covariant derivative $D_\mu = \partial_\mu - iA_\mu$.

Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

Metric $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$



Temporal gauge ($A_0 = 0$) field equations (index raised with Minkowski metric).

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^2(|\phi|^2 - v^2)\phi = 0,$$

$$\partial^\mu\left(\frac{1}{e^2}F_{\mu\nu}\right) - ia^2(\phi^* D_\nu\phi - D_\nu\phi^*\phi) = 0,$$

Abelian Higgs model: shrinking string problem

Comoving width of string shrinks as a^{-1} ($a \sim \tau^{-2}$ in matter era)

Modify field equations[a]

$$\begin{aligned} \ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^{2s}(|\phi|^2 - v^2)\phi &= 0, \\ \partial^\mu \left(\frac{a^{2(1-s)}}{e^2} F_{\mu\nu} \right) - ia^2(\phi^* D_\nu \phi - D_\nu \phi^* \phi) &= 0, \end{aligned}$$

Physical width of string “fattens” if $s < 1$

Preserves Gauss’s Law (current conservation) but violates EM conservation

Take $s = 0.3$ (check with $s = 0$)

[a] Press, Ryden, Spergel (1989); Bevis et al in prep.

Parallel simulations of field theories: LATfield

- C++ library of objects for classical lattice simulation[a]
- Inspired by MDP/FermiQCD[b]
- Objects:

  Lattice: Takes care of boundary conditions and domain decomposition

  Field: Template - can have real, complex, user-defined object.

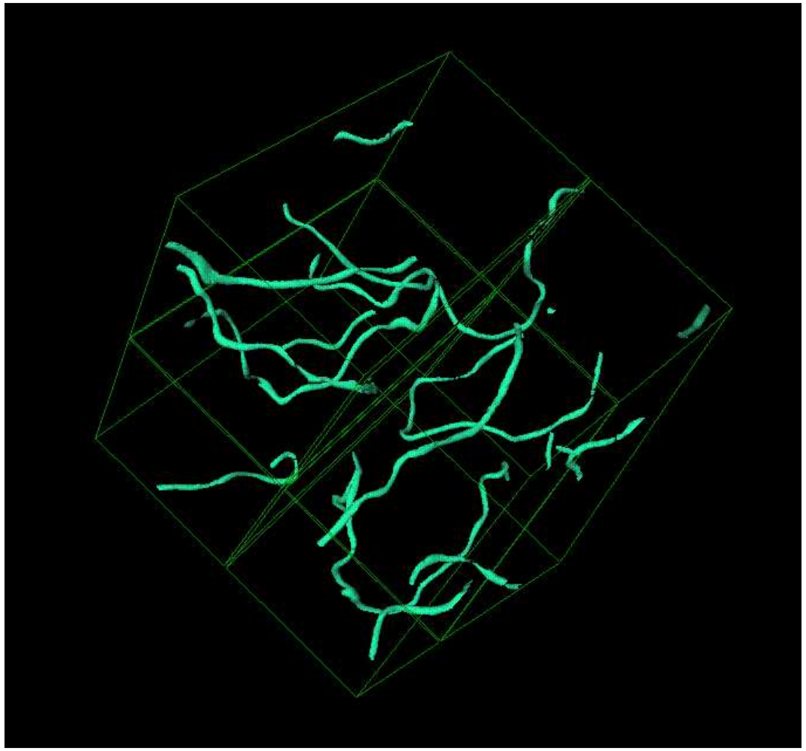
  Site: Accesses elements of field
- Parallelisation: compiler switch `-DPARALLEL_MPI`

[a] Bevis & Hindmarsh, in preparation

[b] Massimo di Pierro http://www.fermiqcd.net/

Strings in 3D (Minkowski space)

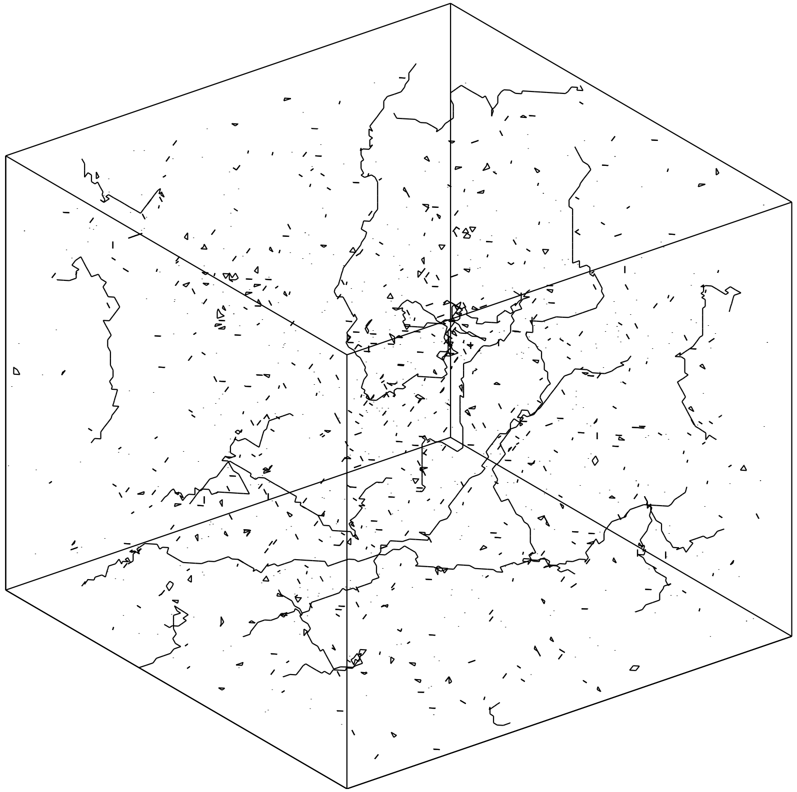
Classical Abelian Higgs model



Isosurfaces of constant ϕ

($\phi = 0$ at string centre).

Nambu-Goto strings



Abelian Higgs model simulations 1: string length scale

Total length of string: L

Scaling: $L/V \propto \tau^{-2}$

Network scale: $\xi = \sqrt{(V/L)} \propto \tau$

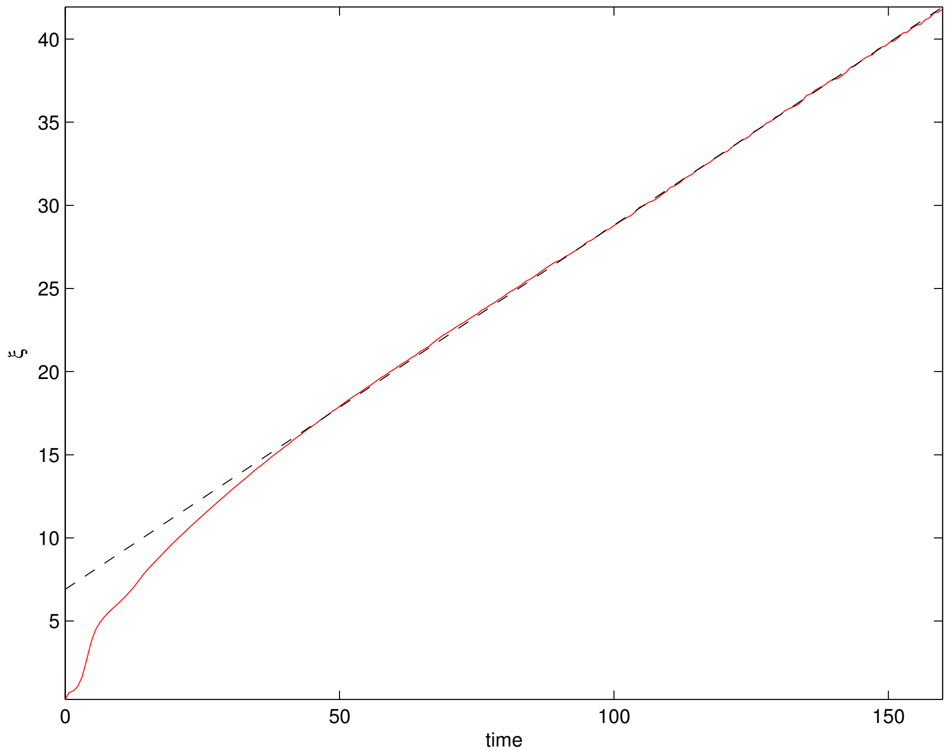
Lattice spacing: $\Delta x = 0.5$

Volume 512^3

Coupling constants: $\lambda = e = 1$

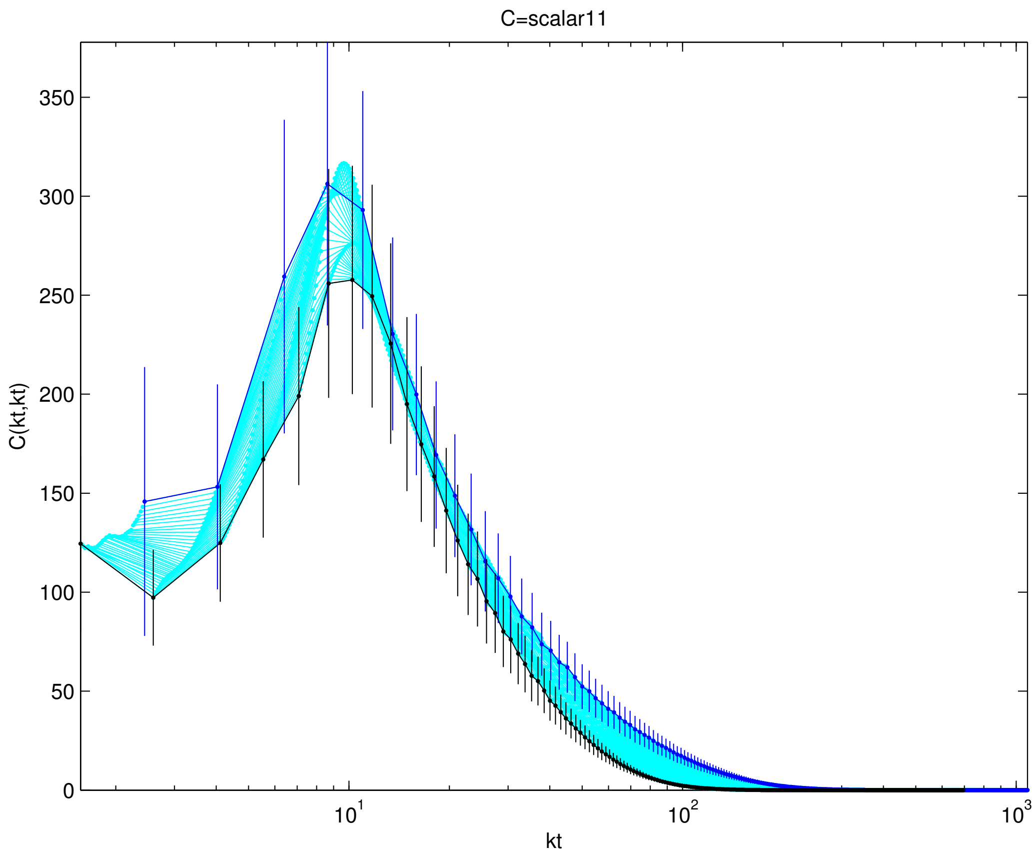
Matter era, "fattening" string algorithm.

Note $\xi = x(\tau + \tau_o)$



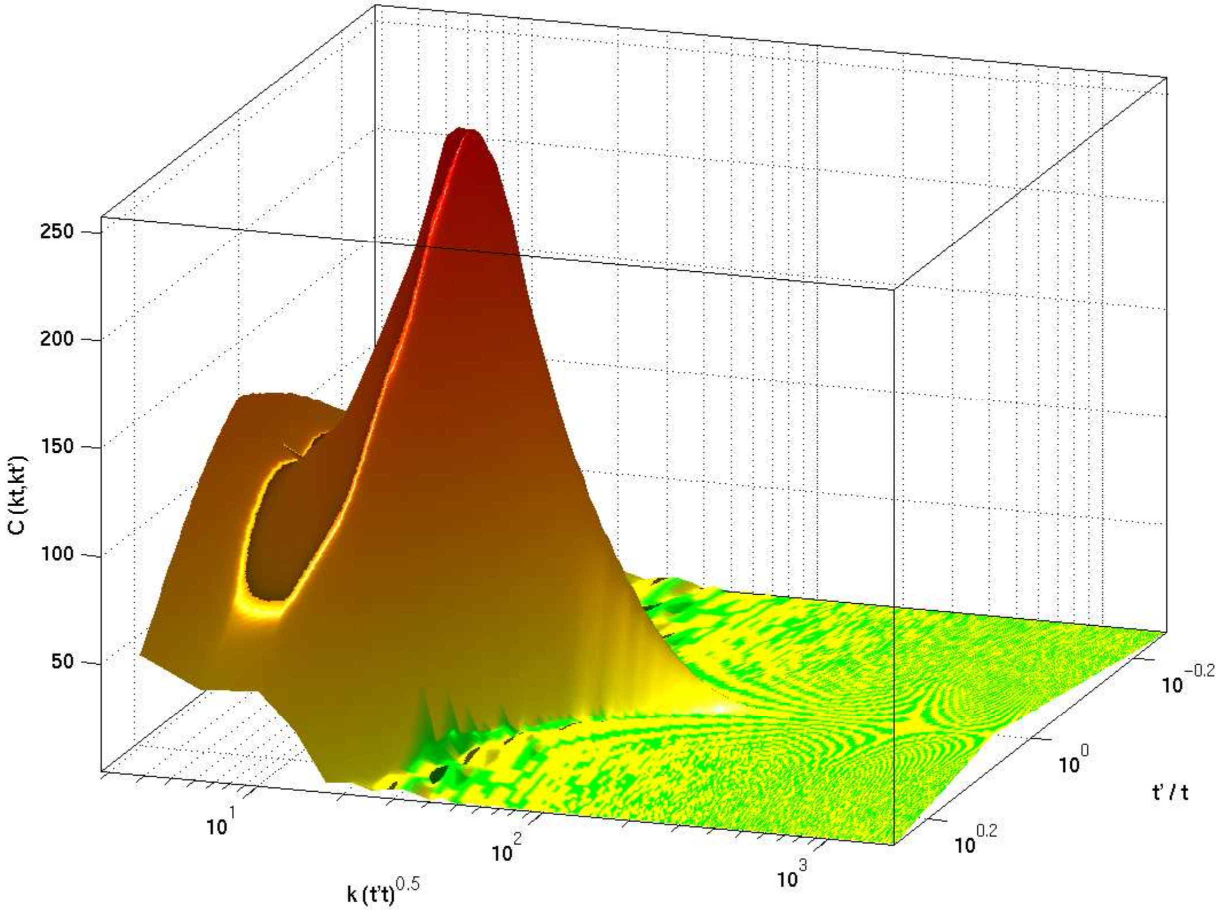
$\Phi\Phi$ equal time correlator: apply time offset $\tau_{\rm o}$, start at $\tau = 64$

$$C_{11}(k(\tau+\tau_{\rm o}), k(\tau+\tau_{\rm o})) = k^4(\tau+\tau_{\rm o})\langle\Phi_s(\mathbf{k},\tau)\Phi_s^*(\mathbf{k},\tau)\rangle$$

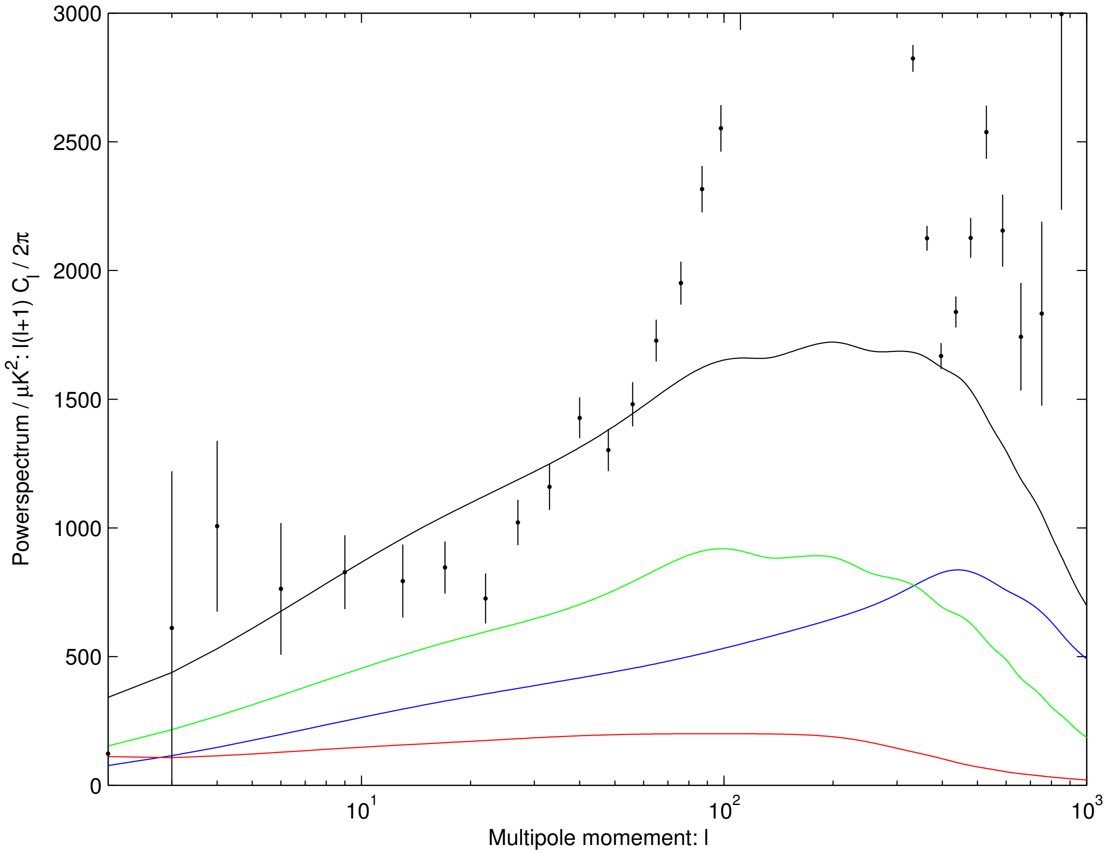


Simulations 2: $\Phi\Phi$ unequal time correlator

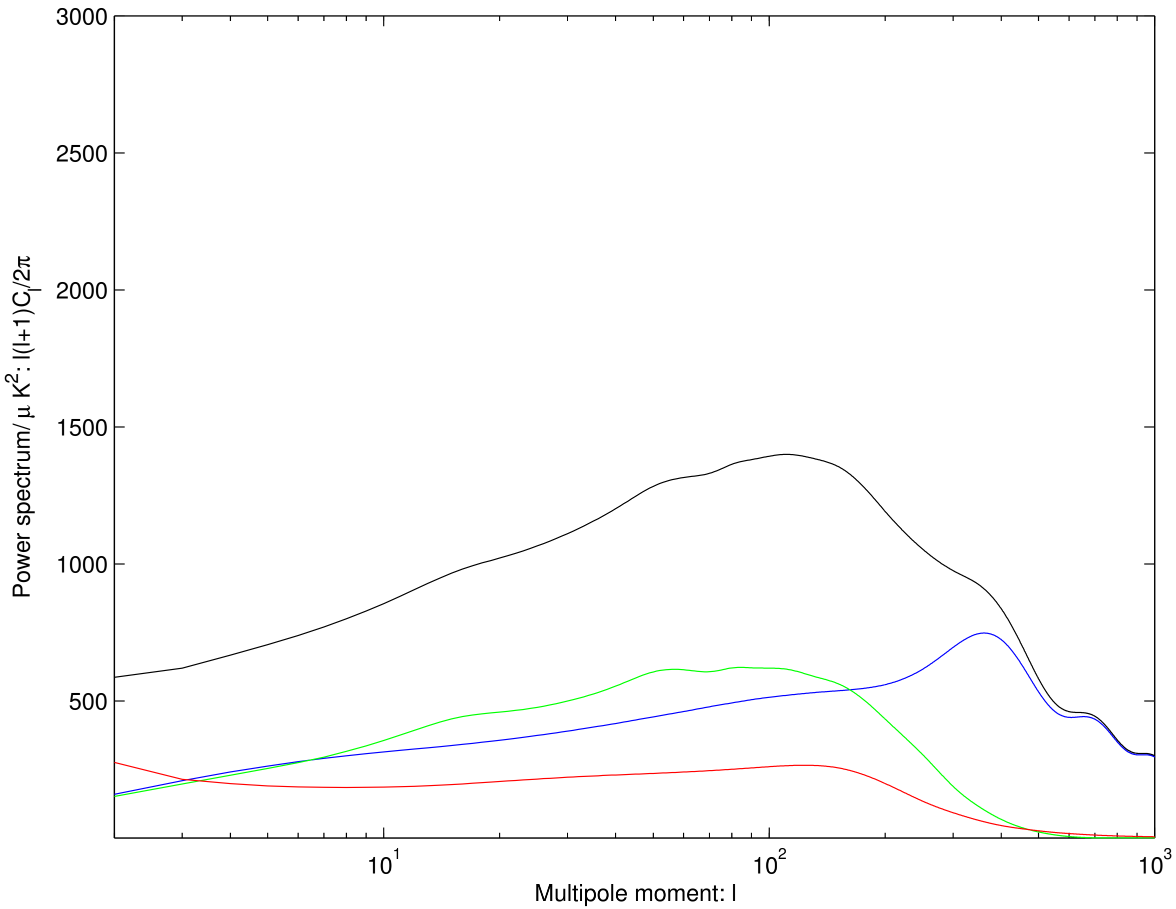
Plot against $k\sqrt{\tau\tau'}$, τ'/τ instead of $k\tau$, $k\tau'$.



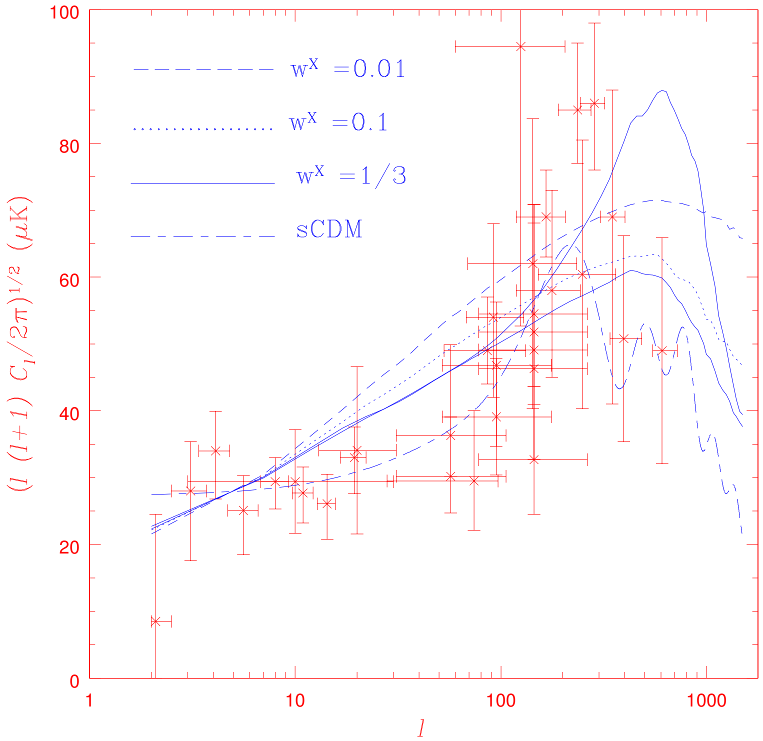
WMAP-normalised C_ℓs from strings PRELIMINARY!



WMAP-normalised C_ℓs from O(4) NLSM

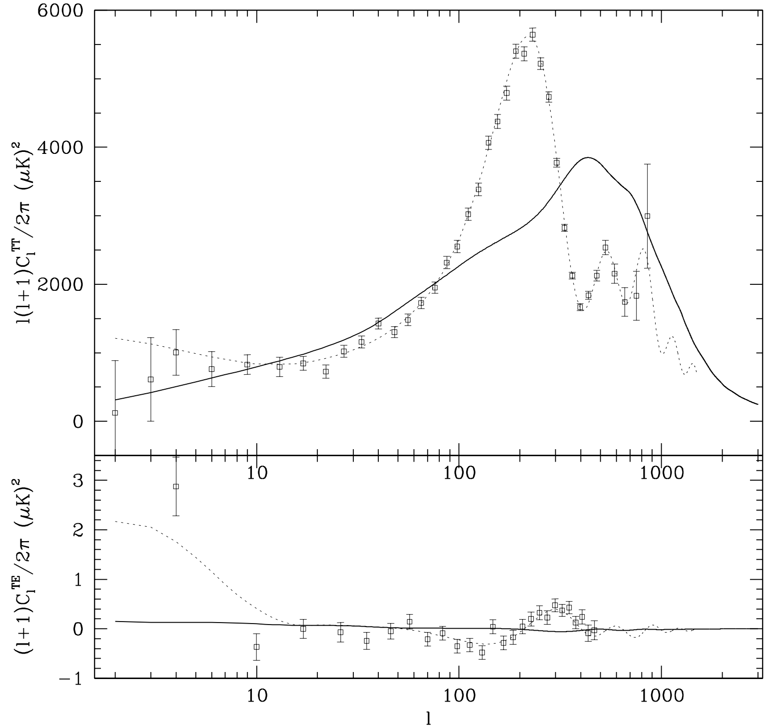


Previous calculations of cosmic string C_ℓs



Contaldi, Hindmarsh, Magueijo (1998)

Minkowski Nambu-Goto



Wyman, Pogosian, Wasserman (2005)

Moving segment model

Conclusions

- Cosmic strings may be detected as a subdominant contribution to CMB signal
- NEW: field theory simulations:
  - less modelling, more physics
  - include contributions to EM tensor from decay products
- Technology spin-offs:
  - parallel N-dimensional field theory simulations: LATfield
  - codes for UETCs and sourced perturbations (modified CMBeasy)
- Preliminary results:
  - Larger vector contribution
  - broader, lower, peak
- To be done:
  - MCMC parameter estimation to bound symmetry-breaking scale
  - Statistical, finite size, and model error estimates