Acceleration without Dark Energy

COSMO–05 Bonn, August 2005

Rocky Kolb, Fermilab & Chicago

Without dark energy!!!!!

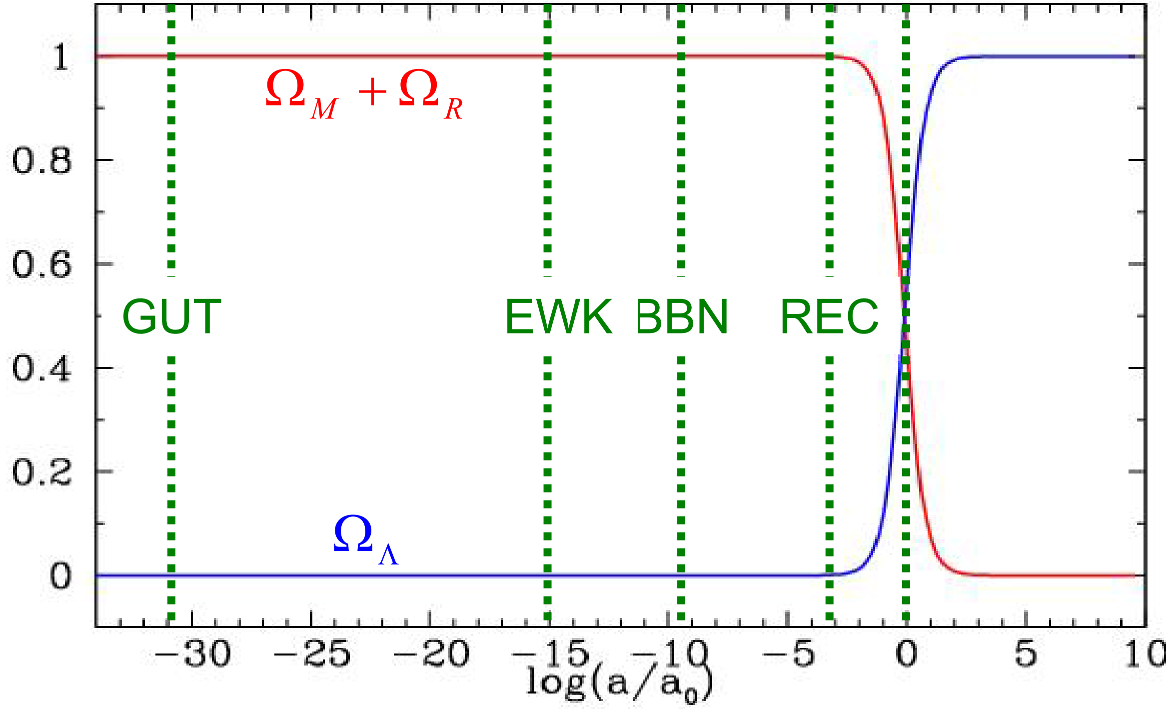
Cosmo-illogical constant

The magnitude

$$\rho_\Lambda \simeq 10^{-30}\,\mathrm{g\ cm^{-3}} \simeq \left(10^{-4}\ \mathrm{eV}\right)^4 \simeq \left(10^{-3}\ \mathrm{cm}\right)^{-4}$$

$$\Lambda = 8\pi G\rho_\Lambda \simeq \left(10^{29}\,\mathrm{cm}\right)^{-2} \simeq \left(10^{-33}\,\mathrm{eV}\right)^2$$

What's so special about today?



Do we “know” there is dark energy?

- Assume model cosmology:
  - Friedmann model: $H^2 + k/a^2 = 8\pi G\rho / 3$
  - Energy (and pressure) content: $\rho = \rho_M + \rho_R + \rho_\Lambda + \ldots$
  - Input or integrate over cosmological parameters: H_0, *etc*.
- Calculate observables $d_L(z), d_A(z), \ldots$
- Compare to observations
- Model cosmology fits with ρ_V, but not without ρ_V
- All evidence for dark energy is indirect: observed $H(z)$ is *not* described by $H(z)$ calculated from the Einstein-de Sitter model

Evolution of $H(z)$ is a key quantity

Many observables based on the comoving distance r

$$\left.\begin{array}{l}\sin^{-1} r(z)\\ r(z)\\ \sinh^{-1} r(z)\end{array}\right\} = \int_0^z \frac{dz'}{H(z')}$$

- Luminosity distance

  Flux = Luminosity $/4\pi d_L^2$

$$d_L(z) \propto r(z)(1+z)$$

- Angular diameter distance

  Angular diameter = Physical size / d_A

$$d_A(z) \propto \frac{r(z)}{(1+z)}$$

- Comoving number

  $N \propto V^{-1}(z)$

$$\frac{dV(z)}{dz\,d\Omega} \propto \frac{r^2(z)}{H(z)}$$

- Age of the universe

$$t(z) \propto \int_0^z \frac{dz'}{(1+z')H(z')}$$

Take sides!

- Can't hide from the data – ΛCDM too good to ignore
  - SNIa
  - Subtraction: $1 - 0.3 = 0.7$
  - Age
  - Large-scale structure
  - …

  $H(z)$ not given by Einstein–de Sitter $H^2 \neq (8\pi G/3)\rho_{\mathrm{MATTER}}$
- Dark energy (modify right-hand side of Einstein equations)
  - Is it "just" a cosmological constant
  - If not constant, what drives dynamics (scalar field)
  - Interpretation of $w = \rho / p$
- Gravity (modify left-hand side of Einstein equations)
  - Beyond Einstein (non-GR: branes,etc.)
  - (Just) Einstein (Back reaction of inhomogeneities)

Entertaining conjecture

Now entertain conjecture of a time
When creeping murmur and the poring dark
Fills the wide vessel of the universe.

— *Shakespeare, King Henry Vth*

All evidence for creeping murmur (dark energy) is indirect!

- Observed evolution of $H(z)$ does not fit Einstein–de Sitter.
- We *infer* the existence of dark energy!
- Could Friedmann equation be modified, and no dark energy?

Modifying the left-hand side

- Braneworld modifies Friedmann equation — Binetruy, Deffayet, Langlois
- Friedmann equation modified today — Freese & Lewis

$$H^2 = A\rho\left[1+\left(\rho/\rho_{\text{cutoff}}\right)^{n-1}\right]$$

- Gravitational force law modified at large distance — Deffayet, Dvali & Gabadadze

  Five-dimensional at cosmic distances

- Tired gravitons — Gregory, Rubakov & Sibiryakov; Dvali, Gabadadze & Porrati

  Gravitons metastable - leak into bulk

- Gravity repulsive at distance $R \approx$ Gpc — Csaki, Erlich, Hollowood & Terning
- $n=1$ KK graviton mode very light, $m \approx$ (Gpc)$^{-1}$ — Kogan, Mouslopoulos, Papazoglou, Ross & Santiago
- Einstein (Hilbert) got it wrong — Carroll, Duvvuri, Turner, Trodden

$$S = \left(16\pi G\right)^{-1}\int d^4x\sqrt{-g}\left(R-\mu^4/R\right)$$

- Backreaction of inhomogeneities — Räsänen; Kolb, Matarrese, Notari & Riotto; Notari; Kolb, Matarrese & Riotto

Acceleration from inhomogeneities

- Most conservative approach — nothing new
  - no new fields (like $10^{-33}\,\mathrm{eV}$ mass scalars)
  - no modification of GR
  - no extra dimensions, bulks, branes, etc.
  - no faith-based reasoning (no anthropic arguments)
- Magnitude?: calculable from observables related to $\delta\rho/\rho$
- Why now?: acceleration triggered by era of non-linear structure

Inhomogeneities-general

Inhomogeneity / anisotropy → smooth

- Matter smoothing is straightforward (particles → fluid), space-time metric smoothing not straightforward!
- Suppose Einstein equations apply on some small scale where the universe is inhomogeneous and anisotropic
- Smooth on some larger scale
- Smoothing and evolution (going to field eqns.) do not commute
- Einstein tensor computed from smoothed metric is *not* the same as Einstein tensor computed from smoothed stress-energy

Ellis, Barausse, Buchert, Ellis, Kolb, Matarrese, Notari, Räsänen, Riotto, Schwarz

Inhomogeneities-cosmology

- Our Universe is inhomogeneous
- One can define an average density $\langle \rho \rangle$
- The expansion rate of an inhomogeneous universe of average density $\langle \rho \rangle$ is *NOT!* the same as the expansion rate of a homogeneous universe of average density $\langle \rho \rangle$!
- Difference is a new term that enters an effective Friedmann equation — new term need not satisfy energy conditions!
- We deduce dark energy because we are comparing to the wrong model universe (*i.e.,* a homogeneous/isotropic model)
- Acceleration is a pure GR effect

Inhomogeneities-example

- Perturbative example

Kolb, Matarrese, Notari & Riotto

- Perturbed FLRW model:

$$G_{\mu\nu}(\vec{x},t) = G_{\mu\nu}^{\mathrm{FLRW}}(t) + \delta G_{\mu\nu}(\vec{x},t)$$
$$G_{00}^{\mathrm{FLRW}}(t) + \delta G_{00}(\vec{x},t) = \kappa^2 T_{00}(\vec{x},t)$$
$$3(\dot{a}/a)^2 = \kappa^2\left(\langle\rho\rangle - \kappa^{-2}\langle\delta G_{00}\rangle\right)$$

- $(\dot{a}/a)^2$ <u>*is not*</u> $\kappa^2\langle\rho\rangle/3$.

- ($\dot{a}/a$ <u>*is not even the expansion rate*</u>.)

Inhomogeneities-cosmology

- For a general fluid, four velocity $u^\mu = (1,\vec{0})$.
  (Local observer comoving with energy flow)

- For irrotational dust, work in synchronous and comoving gauge

$$ds^2 = -dt^2 + h_{ij}(\vec{x},t)dx^i dx^j$$

- Velocity gradient tensor

$$\Theta^i{}_j = u^i{}_{;j} = \tfrac{1}{2} h^{ik}\dot{h}_{kj} = \Theta\delta^i{}_j + \sigma^i{}_j \qquad (\sigma^i{}_j \text{ is traceless})$$

- Θ is the volume-expansion factor and $\sigma^i{}_j$ is the shear
  ($\Theta = 3H$ and $\sigma^i{}_j = 0$ in the homogeneous case)

Inhomogeneities and q

- Local deceleration parameter positive: Hirata & Seljak; Flanagan; Giovannini; Alnes, Amarzguioui & Gron

$$q = -\frac{\left(3\dot{\Theta} + \Theta^2\right)}{\Theta^2} = 6\left(\sigma^2 + 2\pi G \rho\right) \geq 0$$

- However must course-grain over some finite domain:

$$\left\langle F \right\rangle_D = \frac{\int_D \sqrt{h}\, F\, d^3x}{\int_D \sqrt{h}\, d^3x}$$

- Evolution and smoothing do not commute: Buchert & Ellis; Kolb, Matarrese & Riotto

$$\left\langle F \right\rangle_D^{\bullet} - \left\langle F^{\bullet} \right\rangle_D = \left\langle F\Theta \right\rangle_D - \left\langle \Theta \right\rangle_D \left\langle F \right\rangle_D$$

$$\left\langle \Theta \right\rangle_D^{\bullet} = \left\langle \Theta^{\bullet} \right\rangle_D + \left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2 \geq \left\langle \Theta^{\bullet} \right\rangle_D$$

- $\left\langle \Theta \right\rangle_D^{\bullet} \neq \left\langle \Theta^{\bullet} \right\rangle_D$ although $\left\langle \Theta^{\bullet} \right\rangle_D$ can't accelerate, $\left\langle \Theta \right\rangle_D^{\bullet}$ can!

Inhomogeneities

- Define an course-grained scale factor:

Kolb, Matarrese & Riotto astro-ph/0506534; Buchert & Ellis

$$a_D \equiv \left(V_D / V_{D0}\right)^{1/3} \qquad V_D = \int_D d^3x \sqrt{h}$$

- Course-grained Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D} = \tfrac{1}{3}\langle \Theta \rangle_D$$

- Effective evolution equations:

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}\left(\rho_{\rm eff} + 3p_{\rm eff}\right) \qquad \rho_{\rm eff} = \langle \rho \rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R \rangle_D}{16\pi G}$$

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\rm eff} \qquad p_{\rm eff} = -\frac{Q_D}{16\pi G} + \frac{\langle R \rangle_D}{48\pi G}$$

not described by a simple $p = w\,\rho$

- Kinematical back reaction: $Q_D = \tfrac{2}{3}\left(\langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2\right) - 2\langle \sigma^2 \rangle_D$

Inhomogeneities

- Kinematical back reaction: $Q_D = \frac{2}{3}\left(\left\langle\Theta^2\right\rangle_D - \left\langle\Theta\right\rangle_D^2\right) - 2\left\langle\sigma^2\right\rangle_D$
- For acceleration: $\rho_{\text{eff}} + 3p_{\text{eff}} = \left\langle\rho\right\rangle_D - \frac{Q_D}{4\pi G} < 0$
- Integrability condition (GR): $\left(a_D^6 Q_D\right)^{\bullet} + a_D^4\left(a_D^2\left\langle R\right\rangle_D\right)^{\bullet} = 0$
- Acceleration is a pure GR effect:
  - curvature vanishes in Newtonian limit
  - Q_D will be exactly a pure boundary term, and small
- Particular solution to integrability condition:
  - $3Q_D = -\langle R\rangle_D =$ const. (i.e., $\Lambda_{\text{eff}} = -Q_D$)

Inhomogeneities

- Now specialize: $h_{ij}\left(\vec{x},t\right)=a^2\left(t\right)e^{-2\Psi(\vec{x},t)}\left[\delta_{ij}+\chi_{ij}\left(\vec{x},t\right)\right]$

  $a\sim t^{2/3}$ is the usual FRW scale factor

  Ψ is a scalar perturbation: $\Psi=\Psi_\ell+\Psi_s$ $\ell=$ long, $s=$ short (wrt: D)

  χ_{ij} is a traceless tensor with scalar, vector & tensor d.o.f.

- Absorb Ψ_s into $\tilde{h}_{ij}\left(\vec{x},t\right)$: $h_{ij}\left(\vec{x},t\right)=a^2\left(t\right)e^{-2\Psi_\ell(t)}\tilde{h}_{ij}\left(\vec{x},t\right)$

- In terms of metric functions: $\left\langle R\right\rangle_D=a^{-2}e^{2\Psi_\ell}\left\langle \tilde{R}+4\tilde{\nabla}^2\Psi_\ell-2\tilde{\nabla}^i\Psi_\ell\tilde{\nabla}_i\Psi_\ell\right\rangle$

$$Q_D=\frac{2}{3}\left\langle\tilde{\Theta}^2\right\rangle_D-2\left\langle\tilde{\sigma}^2\right\rangle_D$$

- *Only* super-Hubble modes: Q_D vanishes

  integrability condition $\rightarrow \left\langle R\right\rangle_D\propto a_D^{-2}$

  can have $q\rightarrow 0$, but no acceleration

Gradient expansion

Lifsitz, Khalatnikov, Tomita, Salopek, Stewart, Comer, Deruelle, Langlois, Parry, Nambu, Taruya, Bruni, Sopuerta, Croudace, …

- Local curvature expanded in powers of gradients of perturbations
- Lowest-order solution is "seed" long-wavelength approximation
- Successively add higher-order gradient terms
- "Seed" metric: $\Psi_{IN} = \frac{5}{3}\varphi \qquad \chi_{IN} = 0$
- Up to two gradients:

$$\Psi = \frac{5}{3}\varphi + \frac{1}{18}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[\nabla^2\varphi - \frac{5}{6}(\nabla\varphi)^2\right]$$

$$\chi^i_{\ j} = -\frac{1}{3}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[D^i_{\ j}\varphi + \frac{5}{3}\left(\varphi^{,i}_{\ ,j} - \frac{1}{3}(\nabla\varphi)^2\delta^i_{\ j}\right)\right]$$

- Secular divergence: $a \ \frac{1}{18}\frac{1}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[\nabla^2\varphi - \frac{5}{6}(\nabla\varphi)^2\right]$

RG-improved gradient expansion

Kolb, Matarrese & Riotto

- Absorb secular divergence into constants of integration of the 0th-order solution by a renormalization prescription

Nambu & Yamaguchi

- Solution of renormalization-group equation:

$$\Psi = \frac{5}{3}\varphi + \frac{1}{18}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[\nabla^2\varphi - \frac{5}{6}(\nabla\varphi)^2\right]$$

$$\to \frac{5}{3}\varphi - \frac{1}{2}\ln\left\{1 - \frac{1}{9}\frac{a}{a_0}\left(\frac{2}{H_0}\right)^2 e^{10\varphi/3}\left[\nabla^2\varphi - \frac{5}{6}(\nabla\varphi)^2\right]\right\}$$

- Peculiar expansion $\theta = \Theta - 3H$ is $\theta = -3\dot{\Psi} - \frac{1}{2}\chi^{kl}\dot{\chi}_{kl}$

- At late times $q \to 0$ (generalization of Geshnizjani, Chung, & Ashfordi)

RG-improved gradient expansion

Kolb, Matarrese & Riotto

- At higher orders: $\langle R \rangle_D = \sum_{n \geq 1} e^{2n\Psi_{IN}} c_n a^{n-2}$ $\quad c_n = \mathcal{O}(2n$ derivatives)

$$\rightarrow \Psi_R(a) \sim -\frac{1}{2n} \ln\left(1 + 2nc_n a^n\right) \rightarrow -\frac{1}{2} \ln a \Rightarrow q \rightarrow 0$$

- If *only* long-wavelength perturbations present,
  curvature-dominated universe

- Acceleration must be the result of cross-talk between
  long-wavelength & short-wavelength perturbations —
  small-scale dynamics affect the effective average dynamics

Sub-Hubble instabilities

- Result in 2nd-order perturbation (in ϕ) theory:

Kolb, Notari, Matarrese, & Riotto

$$\frac{\langle\theta = \Theta - 3H\rangle}{3H} = -\frac{20\tau^2}{9}\langle\nabla^2\phi\rangle - \frac{100\tau^2}{9}\langle\phi\rangle\langle\nabla^2\phi\rangle - \frac{23\tau^4}{54}\langle\nabla^2\phi\rangle\langle\nabla^2\phi\rangle$$
$$+\frac{130\tau^2}{27}\langle\phi^{,i}\phi_{,i}\rangle + \frac{20\tau^2}{3}\langle\phi\nabla^2\phi\rangle + \frac{4\tau^4}{27}\left(\langle\nabla^2\phi\nabla^2\phi\rangle - \langle\phi^{,ij}\phi_{,ij}\rangle\right)$$

  - Each derivative accompanied by $\tau = 2/aH$
  - Each factor of τ accompanied by c.
  - Highest derivative is highest power of $\tau \propto c$: "Newtonian"
  - Lower derivative terms $\propto c^{-n}$: "Post-Newtonian"

- Mean:
  - $\langle\nabla^2\phi\rangle = \langle\phi\rangle = 0$
  - $\langle\nabla^2\phi\nabla^2\phi\rangle = \langle\phi^{,ij}\phi_{,ij}\rangle = \mathcal{O}(1)$ Räsänen
  - *But* total Newtonian term vanishes (surface term)
  - Post-Newtonian: $\langle\phi\nabla^2\phi\rangle = \langle\phi^{,i}\phi_{,i}\rangle = \mathcal{O}(10^{-5})$ ***huge!*** (large k^2/a^2H^2)

Sub-Hubble instabilities

- First term in gradient expansion (2 spatial derivatives.):

  $\langle R \rangle_D \propto a_D^{-2} \qquad Q_D = 0 \rightarrow$ no acceleration

- In general, gradient expansion gives

Notari; Kolb, Matarrese, & Riotto

$$\langle R \rangle_D = \sum_{n=1}^{\infty} r_n a^{n-3} \qquad \left(r_n = \sum_{m=n}^{2n} (2n \text{ derivatives}) \phi^m \right)$$

$$Q_D = \sum_{n=2}^{\infty} q_n a^{n-3} \qquad \left(q_n = \sum_{m=n}^{2n} (2n \text{ derivatives}) \phi^m \right)$$

- Newtonian terms, $(\nabla^2 \phi)^n \sim (k/aH)^{2n} \phi^n$, *individually* are large, but only appear as surface terms, hence small in total
- Post-Newtonian terms, $(\nabla \phi)^{2n} \sim (k/aH)^{2n} \phi^{2n}$, individually are small, but do not appear as surface terms
- Dominant term is combination: $(\nabla^2 \phi)^{n-1} (\nabla \phi)^2 \sim (k/aH)^{2n} \phi^{n+1}$

Sub-Hubble instabilities

- Gradient expansion: $\langle R \rangle_D = \sum_{n=1}^{\infty} r_n a^{n-3}$ $\qquad Q_D = \sum_{n=2}^{\infty} q_n a^{n-3}$
- Lowest-order term to make big contribution is $n = 3$ (6 derivatives)
- Disconnected fourth-order moment of ϕ: $\left\langle \frac{\left(\nabla^2 \phi\right)^2}{H_0^4} \right\rangle \left\langle \frac{\left(\nabla \phi\right)^2}{H_0^2} \right\rangle$
- Notice $n = 3$ contributes to Q_D and $\langle R \rangle_D$ terms $\propto a^0$, i.e., expansion as if driven by a cosmological constant !!!
- But why stop at $n = 3$?????

Inhomogeneities

- Does this have anything to do with our universe?

- Have to go to non-perturbative limit

- How to relate observables ($d_L(z)$, $d_A(z)$, $H(z)$, $\ldots$) to Q_D & $\langle R \rangle_D$?

- Can one have large effect and isotropic expansion/acceleration? (*i.e.,* will the shear be small?)

- What about gravitational instability?

- Work to be done!

- Toy model proof of principle: Tolman-Bondi dust model
  (Nambu & Tanimoto (gr-qc/0507057)

Tolman–Bondi–Lemaître

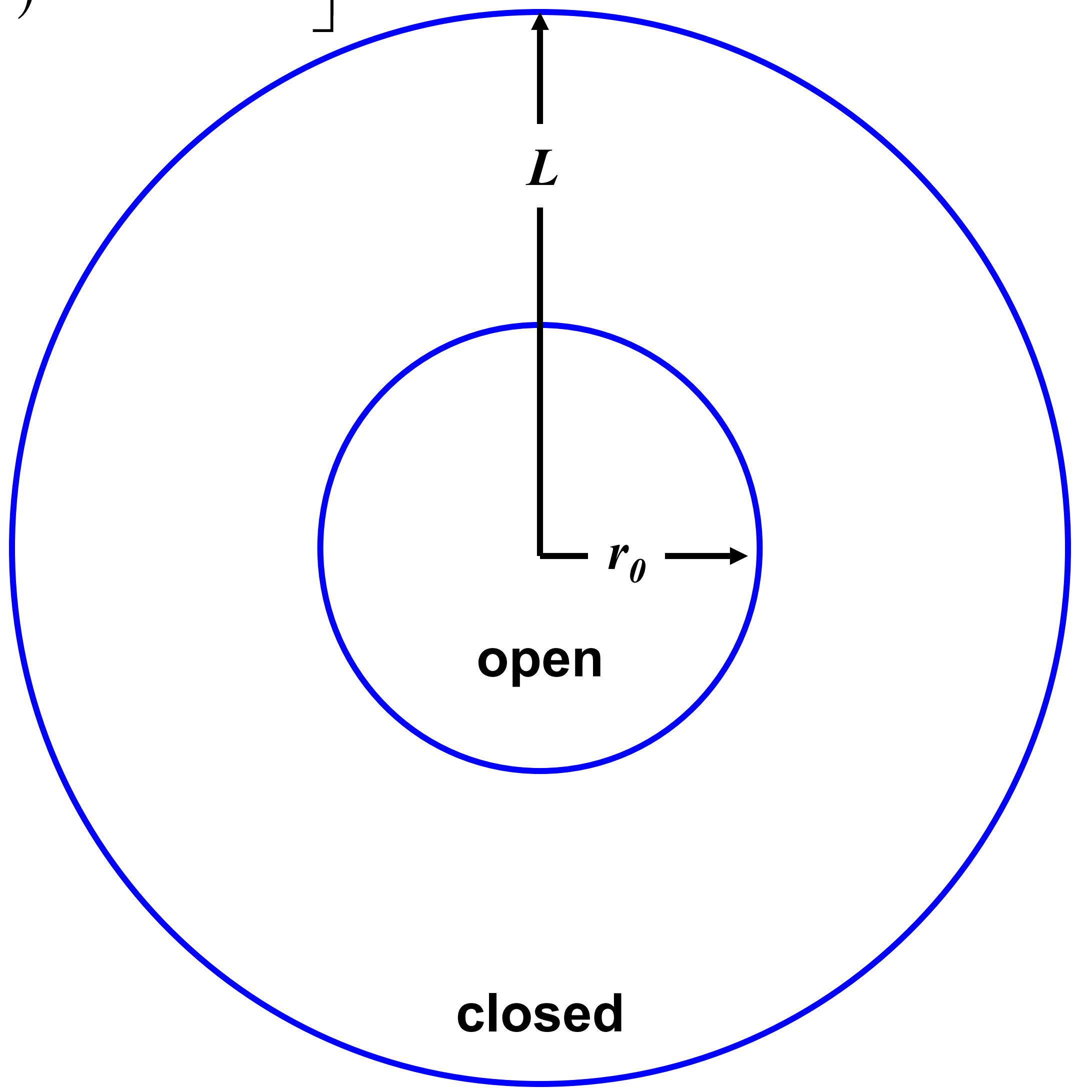
(Nambu & Tanimoto (gr-qc/0507057)

$$ds^2 = -dt^2 + a^2\left[\left(1+\frac{a_{,r}r}{a}\right)^2 \frac{dr^2}{1-k(r)r^2} + r^2 d\Omega_2^2\right]$$

- dust model: $\rho = \rho_0 / a^3$
- spatial curvature:
  $k = -1$ for $0 \le r \le r_0$
  $k = +1$ for $r_0 \le r \le L$
- "Friedmann" equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_0(r)}{a^3} - \frac{k(r)}{a^2}$$

- Not to be regarded as a realistic model



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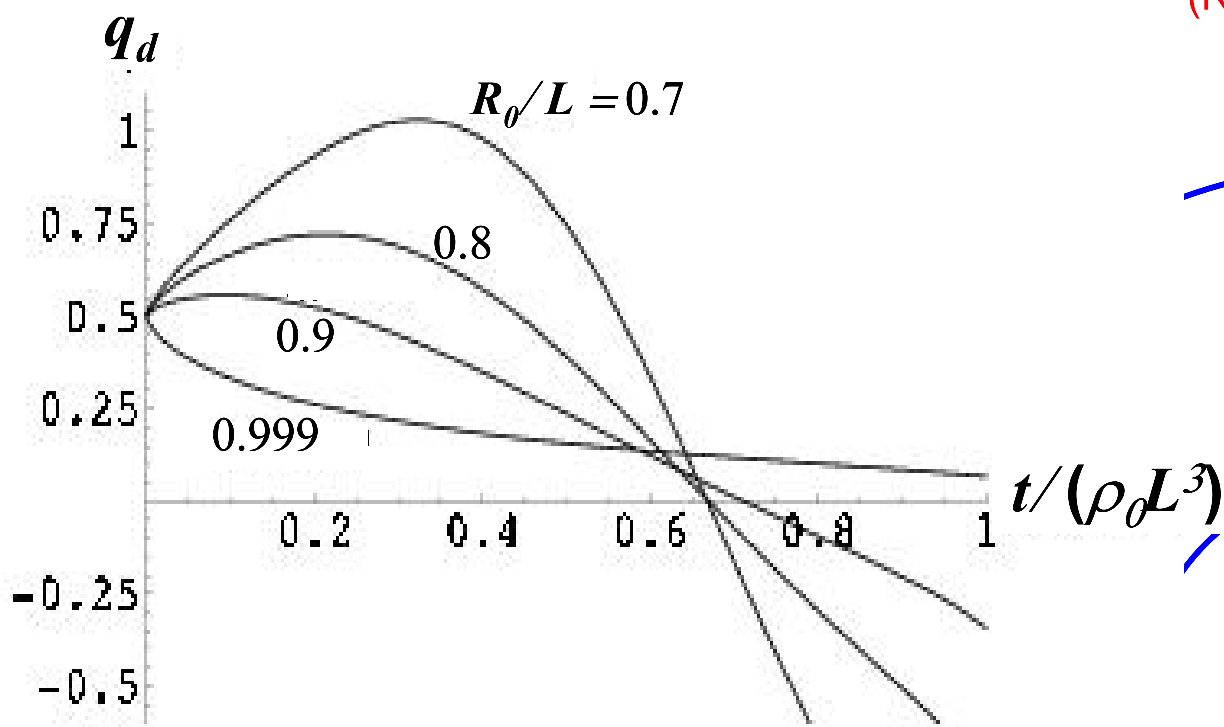
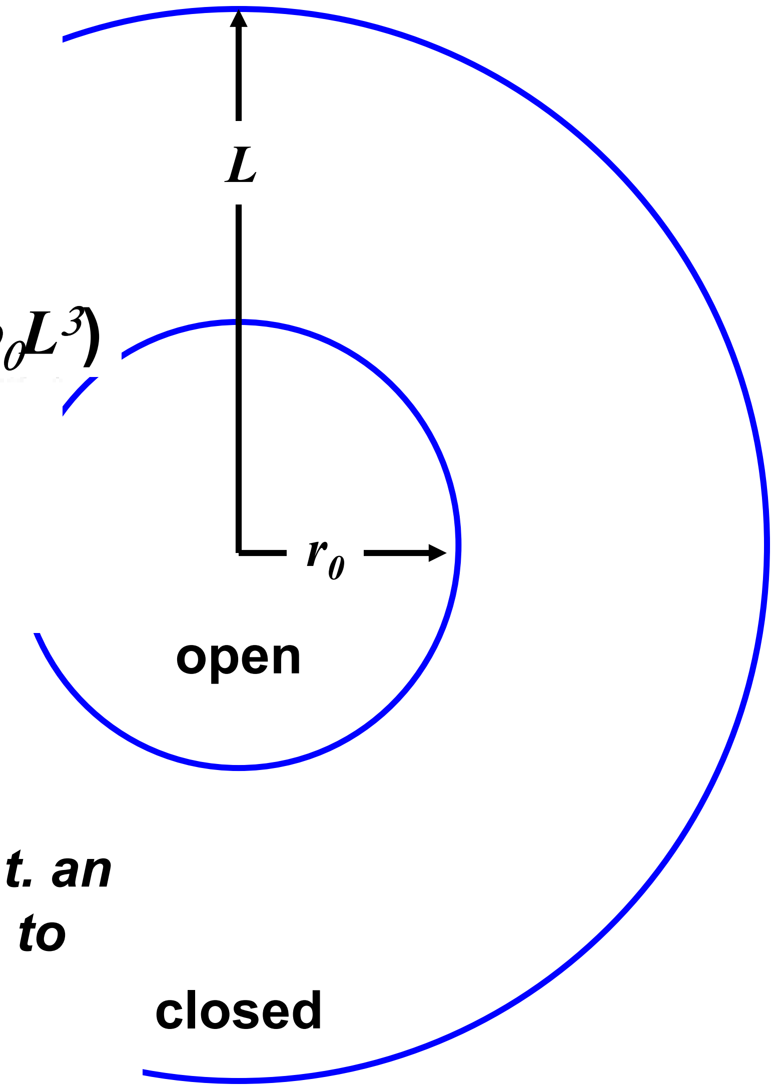




Acceleration in our local Hubble patch is possible if the <u>mean rarefaction factor</u> (w.r.t. an underlying FRW model) grows fast enough to overshoot the FRW background evolution.

Kolb, Matarrese, Riotto

Tolman–Bondi–Lemaître

(Nambu & Tanimoto (gr-qc/0507057)

If have exact solution

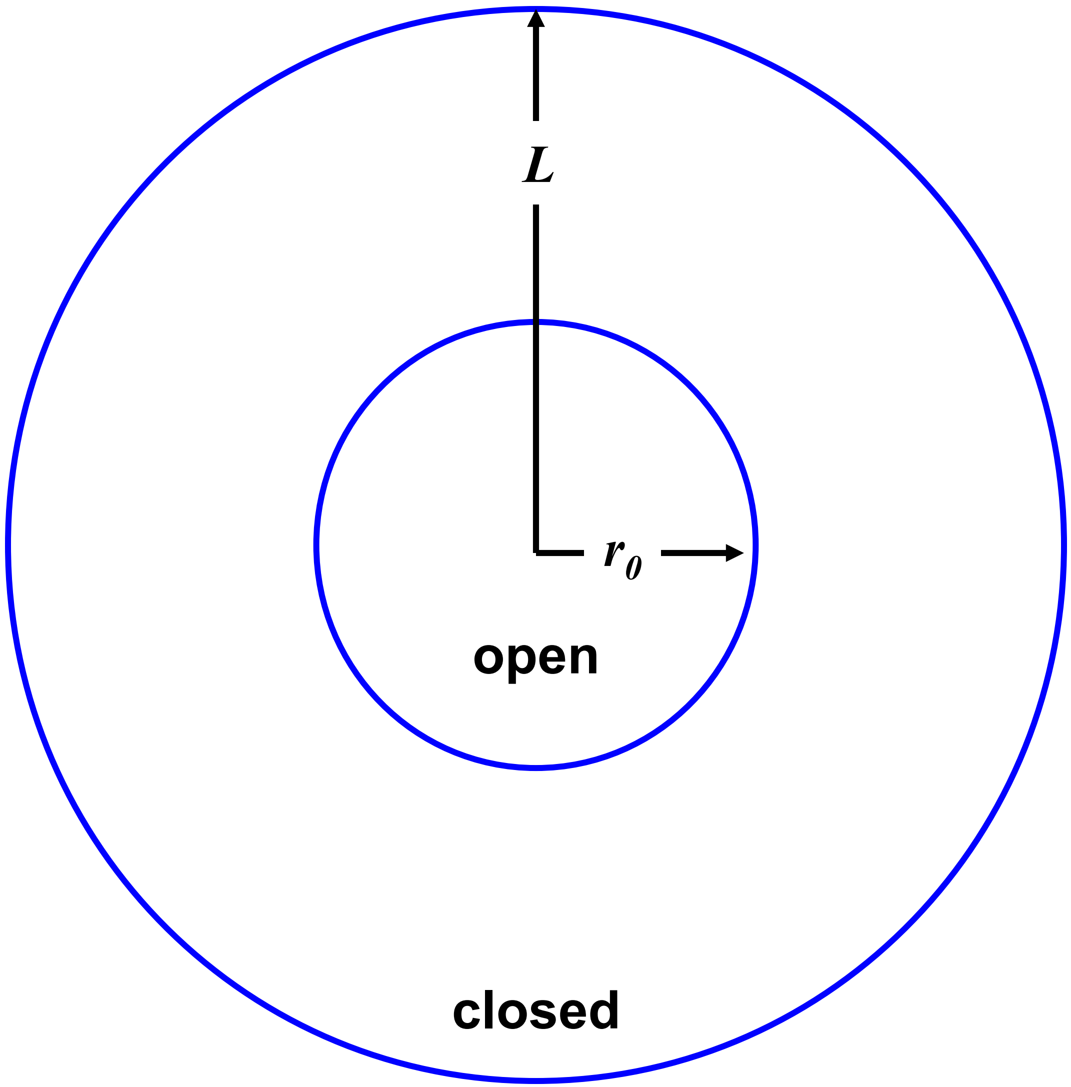
can investigate how observables related to Q_D and $\langle R \rangle_D$

Not quite there

metric has discontinuity at $r = r_0$

Fix

wall at $r = r_0$?
make it smooth?



Conclusions

- Must properly smooth inhomogeneous Universe
- In principle, acceleration possible even if "locally" $\rho + 3p > 0$
- Super-Hubble modes, of and by themselves, cannot accelerate
- Sub-Hubble modes have large terms in gradient expansion
  - Newtonian terms can be large but combine as surface terms
  - Post-Newtonian terms are not surface terms, but small
  - Mixed Newtonian $\times$ Post-Newtonian can be large
  - Effect from "mildly" non-linear scales
- The first large term yields effective cosmological constant
- No reason to stop at first large term
- Can have $w < -1$?
- Advantages to scenario:
  - No new physics
  - "Why now" due to onset of non-linear era

Acceleration without Dark Energy

COSMO–05 Bonn, August 2005

Rocky Kolb, Fermilab & Chicago