MSSM Electroweak Baryogenesis and Flavour Mixing in Transport Equations

Thomas Konstandin

in collaboration with T. Prokopec, M.G. Schmidt and M.Seco

hep-ph/0410135, hep-ph/0505103

Outline

1. Introduction
   - Electroweak Baryogenesis
2. Former Approaches
   - Summary
3. First Principle Approach
   - The Kadanoff-Baym Equations
   - Results
4. Conclusions

Introduction

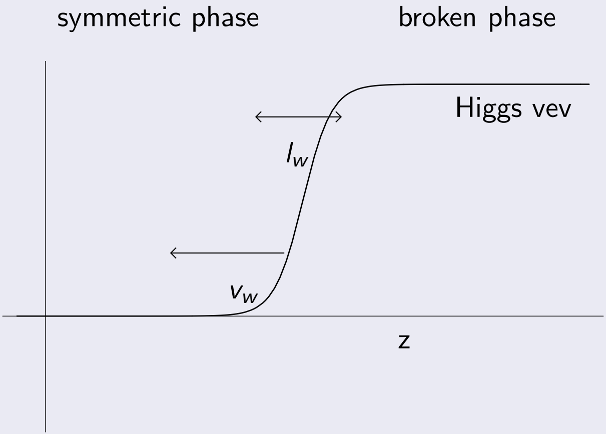
Statement I

Electroweak Baryogenesis

= Transport + CP Violation

Picture of Electroweak Baryogenesis

SHAPOSHNIKOV ('87)



CP-violation: Chargino masses in the MSSM

In the MSSM case the higgsino/wino mass matrix is:

$$m = \begin{pmatrix} M_2 & g\, h_2(z) \\ g\, h_1(z) & \mu_c \end{pmatrix}$$

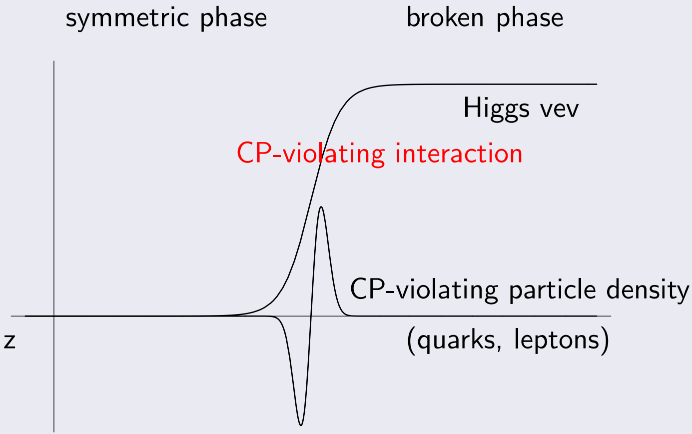
with M_2 and μ_c containing a CP-violating complex phase.

Qualitative difference in CP-violation

- $\left(\partial_z m m^\dagger - m \partial_z m^\dagger\right) \sim \mathrm{Im}(M_2 \mu_c^*) \neq 0$ and CP-violation is already present on the tree level.
- In the SM: $\partial_z m \sim m \to \left(\partial_z m m^\dagger - m \partial_z m^\dagger\right) = 0$ and CP-violation depends on loop calculations.

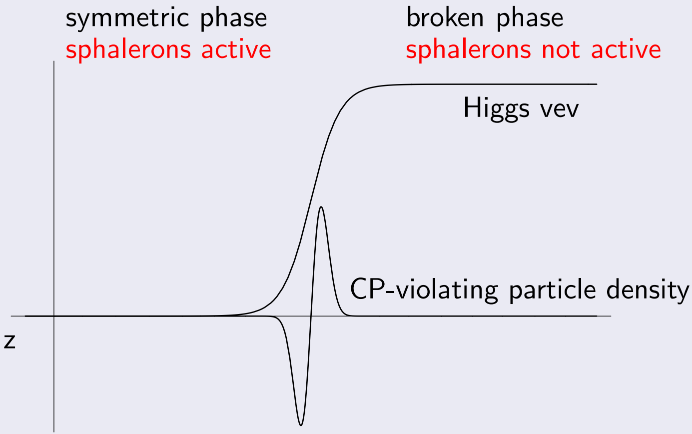
Picture of Electroweak Baryogenesis

SHAPOSHNIKOV ('87)



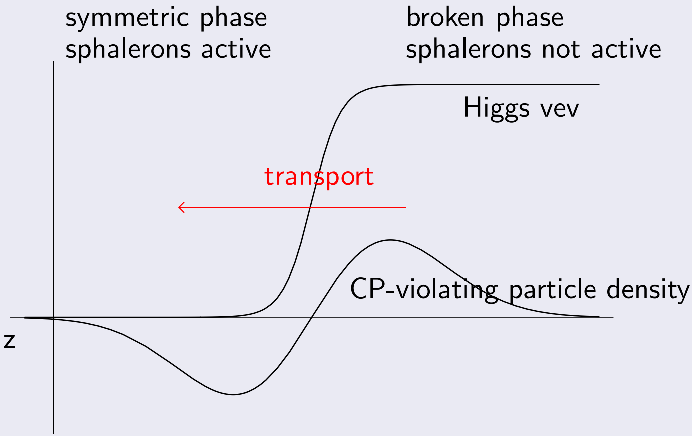
Picture of Electroweak Baryogenesis

Shaposhnikov ('87)



Picture of Electroweak Baryogenesis

SHAPOSHNIKOV ('87)



Former Approaches

Statement II

Weaknesses of Former Approaches
based on Classical Transport

A) Flavour Basis Invariance

B) Ambiguities

Particle distribution function of species j: $f_j(\vec{p}, \vec{x}, t)$
Boltzmann-Equation:

$$\left(\partial_t + \frac{\vec{p}}{m} \cdot \nabla + \frac{\vec{F}_j}{m} \cdot \partial_{\vec{p}}\right) f_j = \text{gain- and loss-terms}$$

Summary of former Approaches

Approach	Cline et al.	Carena et al.
CP-violation	dispersion relation	local source term
	WKB	perturbation theory
basis	mass eigenbasis	flavour eigenbasis
quasiparticles	charginos	higgsinos/winos
transport	classical	classical
	Boltzmann type	diffusion
mixing	not included	in the source
		not in the diffusion
$\hbar$ **order**	second order	first order

To decide which approach is correct one has to derive semi-classical transport equations from first principles.

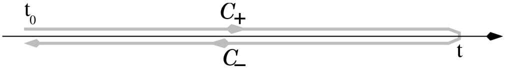
Quantum Transport

Statement III

Quantum Transport Equation
= Kadanoff-Baym-Equation of
the Wightman Function
in Wigner Space

The Wightman Function

In statistical QFT all two-point functions obtain an additional 2×2 structure from the *in-in*-(closed-time-path)-formalism



$$\Sigma = \begin{pmatrix} \Sigma^t & \Sigma^> \\ \Sigma^< & \Sigma^{\bar{t}} \end{pmatrix}, S = \begin{pmatrix} S^t & S^> \\ S^< & S^{\bar{t}} \end{pmatrix}.$$

In Fourier space the Greens function entry $S^<$ is called Wightman function and can interpreted as particle density, e.g. in thermal equilibrium

$$iS^<_{eq} = 2\pi\,\mathrm{sign}(k_0)\,\delta(k_\mu^2 - m^2)\; n_{BE}(k_0)$$

such that

$$\int_{k_0>0} \frac{dk_0}{2\pi}\, 2ik_0\, S^<_{eq} \sim n_{BE}(\sqrt{\vec{k}^2 + m^2})$$

Dynamics of the Wightman Function

The dynamics of the Wightman function is described by the Kadanoff-Baym equations that are the statistical analogue to the Schwinger-Dyson equations:

$$\int d^4z \left(S_0^{-1}(x_\mu, z_\mu) - \Sigma(x_\mu, z_\mu)\right) S(z_\mu, y_\mu) \quad = \quad \mathbb{1}\, \delta(x_\mu - y_\mu)$$

Since the Greens-function depends not only on the relative coordinate, but also on the average coordinate $X_\mu = (x_\mu + y_\mu)/2$, Fourier-transformation leads to the Moyal star product in Wigner space:

$$e^{-i\Diamond}\{S_0^{-1}(X_\mu, k_\mu) - \Sigma(X_\mu, k_\mu)\}\{S(X_\mu, k_\mu)\} \quad = \quad \mathbb{1}$$

with the diamond operator

$$2\Diamond\{A, B\} := \partial_{X^\mu} A\, \partial_{k_\mu} B - \partial_{k_\mu} A\, \partial_{X^\mu} B$$

Gradient Expansion

Statement IV

Semi-Classical Expansion

$= $ Gradient Expansion in $\Diamond$

Gradient Expansion

Since the background in the MSSM is weakly varying ($l_w \approx 20/T_c$) the Moyal star product can be simplified by the semi-classical approximation

$$\partial_k \partial_X \approx \frac{1}{T_c l_w} \approx \frac{1}{20} \ll 1 \quad \rightarrow \quad e^{-i\Diamond} \approx 1 - i\Diamond$$

Neglecting selfenergies, the simplest example for an transport equation in a varying background is for one bosonic flavour with real z-dependent mass

$$\begin{aligned} (k^2 - m^2(z))S^< &= 0 \\ (k^\mu \partial_\mu - \frac{1}{2}(\partial_z m^2(z))\,\partial_{k_z})S^< &= 0. \end{aligned}$$

Fermionic Systems

After spin projection the fermionic system of equations reads

$$
\begin{aligned}
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right)S_0^s - (2isk_z + s\partial_z)\,S_3^s - 2im_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_1^s - 2im_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_2^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right)S_1^s - (2sk_z - is\partial_z)\,S_2^s - 2im_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_0^s + 2m_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_3^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right)S_2^s + (2sk_z - is\partial_z)\,S_1^s - 2m_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_3^s - 2im_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_0^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right)S_3^s - (2isk_z + s\partial_z)\,S_0^s + 2m_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_2^s - 2m_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}}\,S_1^s &= 0\,,
\end{aligned}
$$

where $S_0 \ldots S_3$ are 2×2 matrices in flavour space and s denotes the spin.

Transport in the Chargino sector

The chargino transport equations for the left/right handed CP violating deviations from equilibrium ($\delta S^< = S^< - S^<_{eq}$) are of the form

$$k_0 \partial_t \delta S^< + k_z \partial_z \delta S^< + \frac{i}{2}\left[m^2, \delta S^<\right] + \mathrm{Force}(\delta S^<) = \mathrm{Source}(S^<_{eq})$$

Comments

- The term $\frac{i}{2}\left[m^2, \delta S^<\right]$ will lead to an oscillatory behaviour of the off-diagonal particle densities, similar to neutrino oscillations with frequency $\sim (m_1^2 - m_2^2)/k_z$.
- Without this oscillation term the first order mixing terms will not lead to CP-violation

Summary

Approach	Heidelberg
technique	Kadanoff-Baym equations
basis	basis independent
quasiparticles	no quasi-particles / no dispersion relation
transport	semi-classical
	diffusion
mixing	included
$\hbar$ **order**	first order and second order
comment	accounts for oscillations
	unambiguous

Oscillation Effects

Statement V

The Flavour Oscillation Leads to

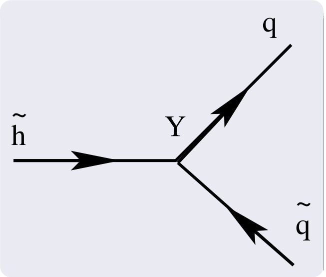
A) Suppression of Mixing far from Degeneracy

B) BAU of maximally $4 \times \eta_{exp}$

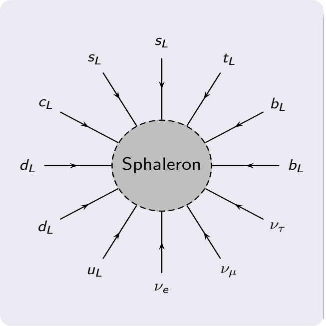
Determination of the BAU

Huet, Nelson ('95)

The missing parts to determine the baryon asymmetry of the universe are:

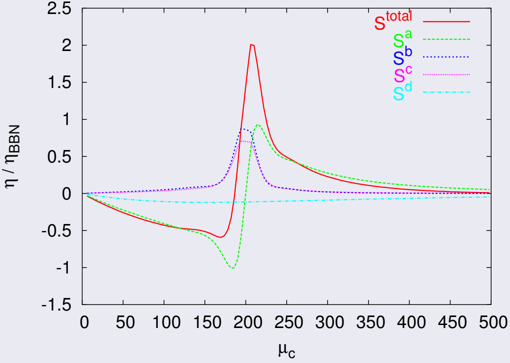


and



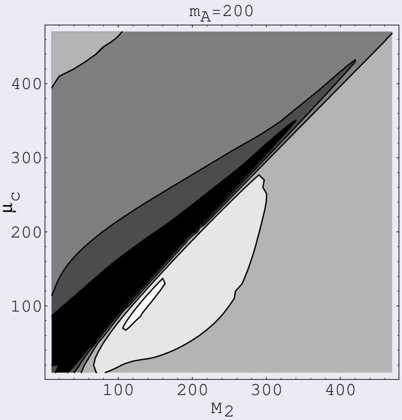
Numerical Results I

Parameters chosen: $M_2 = 200$ GeV, $v_w = 0.05$, $l_w = 20/T_c$, CP-phase maximal.



Numerical Results II

Parameters chosen: $v_w = 0.05$, $l_w = 20/T_c$, CP-phase maximal.



Pessimistic Conclusion

MSSM electroweak baryogenesis is a rather unlikely scenario based on

- A light stop to acquire a strong first order phase transition
- The condition $\mu_c \approx M_2 \lesssim 400$ GeV of the *a priori* unrelated parameters M_2 and μ_c
- A large CP-violating phase that hardly satisfies experimental EDM bounds

Optimistic Conclusion

We achieved to attain a formalism that

- Describes semi-classical transport
- Is basis independent
- Is derived from first principles
- Unambiguous

and it unveiled an oscillation in the dynamics that is significant for the first order mixing effects.

Based on our analysis, MSSM electroweak baryogenesis is a scenario that

- Can explain the BAU
- Predicts $\mu_c \approx M_2 \lesssim 400$ GeV
- Will be testable soon

Conclusion

Statements I-V

I) Electroweak Baryogenesis = Transport + CP Violation

II) Weaknesses of Former Approaches based on Classical Transport

- Basis Invariance
- Ambiguities

III) Quantum Transport Equation = Kadanoff-Baym-Equation of the Wightman Function in Wigner Space

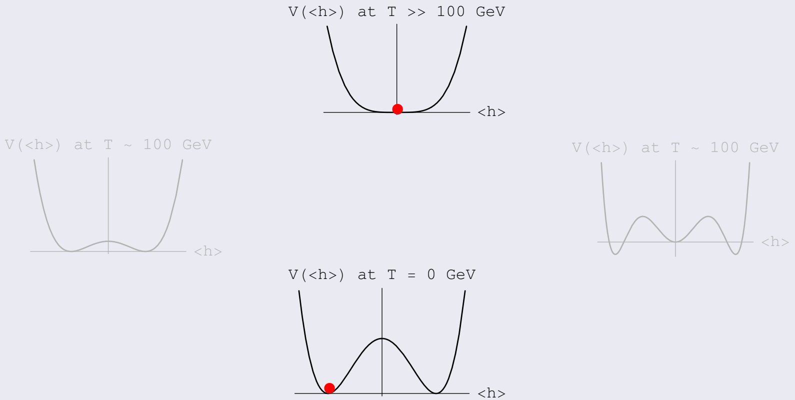
IV) Semi-Classical Expansion = Gradient Expansion

V) The Flavour Oscillation Leads to

- Suppression of Mixing far from Degeneracy
- BAU of maximally $4 \times \eta_{exp}$

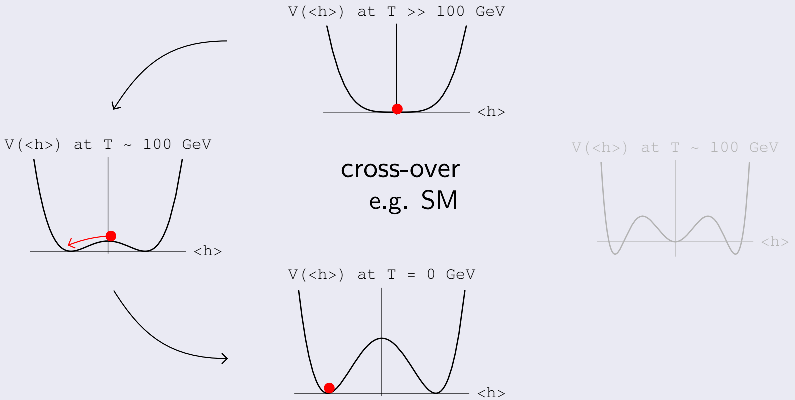
Non-equilibrium: The electroweak phase transition

Cross-over *versus* first order electroweak phase transition:



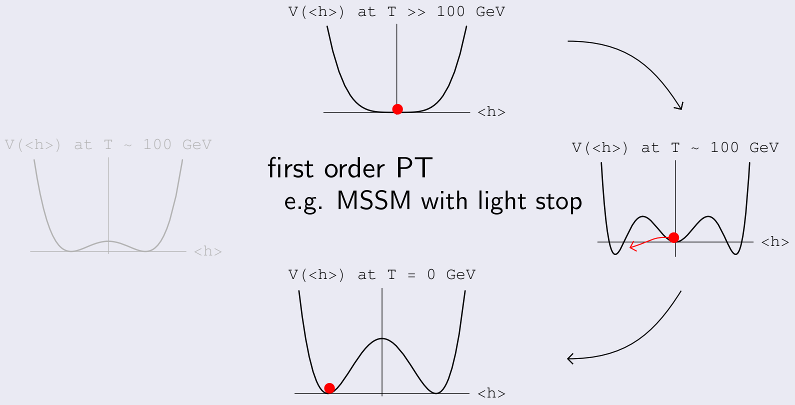
Non-equilibrium: The electroweak phase transition

Cross-over *versus* first order electroweak phase transition:



Non-equilibrium: The electroweak phase transition

Cross-over *versus* first order electroweak phase transition:



Sakharov conditions

Baryogenesis is one of the cornerstones of the Cosmological Standard Model.

The celebrated Sakharov conditions state the necessary ingredients for baryogenesis:

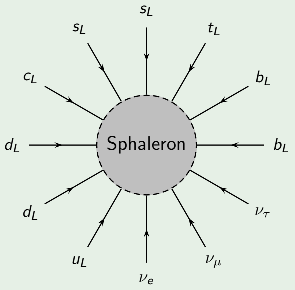
Sakharov conditions

- C and CP violation
- B-violation
- non-equilibrium

C- and B-violation: The sphaleron process

In the hot universe B- and C-violation is present in the SM and its extensions due to sphaleron precesses.

The effective sphaleron vertex



- $\Delta B = 3$, $\Delta L = 3$, $\Delta N_{CS} = 1$
- $B - L$ conserving
- $B + L$ violating
- Exponentially suppressed by the W mass
- Topological effect of the SU(2) gauge sector

Former Approach using Dispersion Relations

Cline, Joyce, Kainulainen ('97, '00)

The CP violating dispersion relation $E(p, z)$ for a varying mass (e.g. $m(z) = |m(z)|e^{i\theta(z)}$) is determined by the WKB method

$$E^2 = \vec{p}^2 + m^2 \pm \frac{m^2 \partial_z \theta}{2 p_z}$$

and put into the classical Boltzmann equation for the particle density f:

$$(\partial_t + \partial_{p_z} E \, \partial_z - \partial_z E \, \partial_{p_z}) f \quad = \quad \text{Coll.}$$

This procedure is performed in mass eigenbasis and does not contain mixing effects.

Former Approach using a Local Source

Carena, Moreno, Quiros, Seco, Wagner ('00, '02)

The additional term in the Schwinger-Dyson equation is interpreted as an interaction term

The CP violating source is calculated from

$$j^\mu(X_\mu) = \frac{1}{\tau} \int \frac{d^4 p}{(2\pi)^4} \; p^\mu \delta S^<(p_\mu, X_\mu)$$

and put 'by hand' into diffusion equations

$$(D\partial_z^2 - v_w \partial_z - \Gamma) f = j^0.$$

The source j^μ is determined in flavour eigenbasis and contains mixing.

Status of Electroweak Baryogenesis

EWB in the SM excluded

- CP violation too small
- No strong first order phase transition

EWB in the MSSM not excluded/disproved

- CP violation in the neutralino/chargino sector
- Strong first order phase transition in case of a light stop