Coupling Quintessence to Inflation in SUGRA

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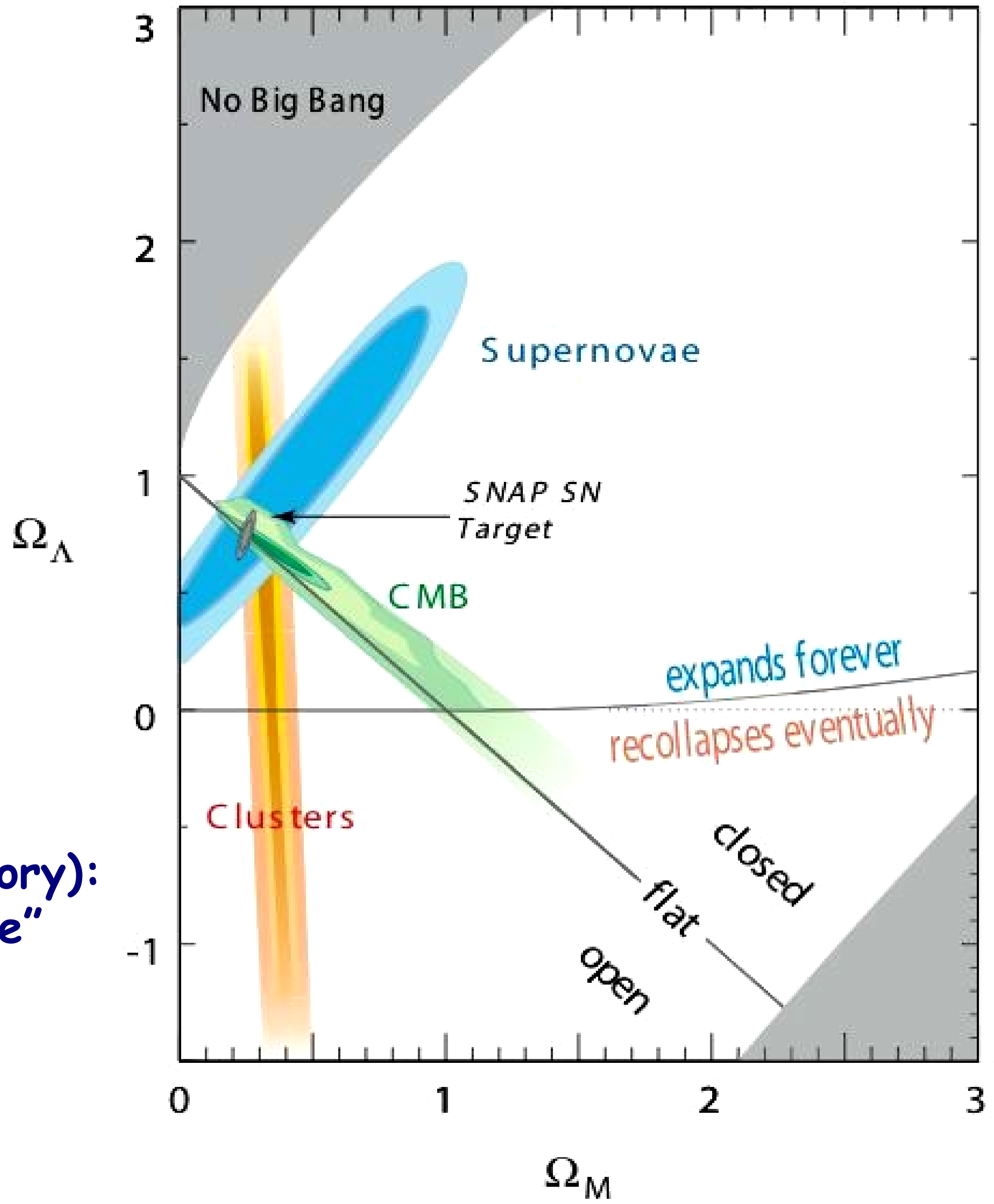
P. Brax § J. Martin, PRD 71 063530 (2005), astro-ph/0502069

- Observations seem to indicate that our universe is presently in a phase of accelerated expansion

- Dark energy accounts for 70% of the energy density present in our universe

$$\Omega_{\text{Dark energy}} \simeq 0.7$$

- Various explanations (although none of them is fully satisfactory): here, we consider "quintessence"

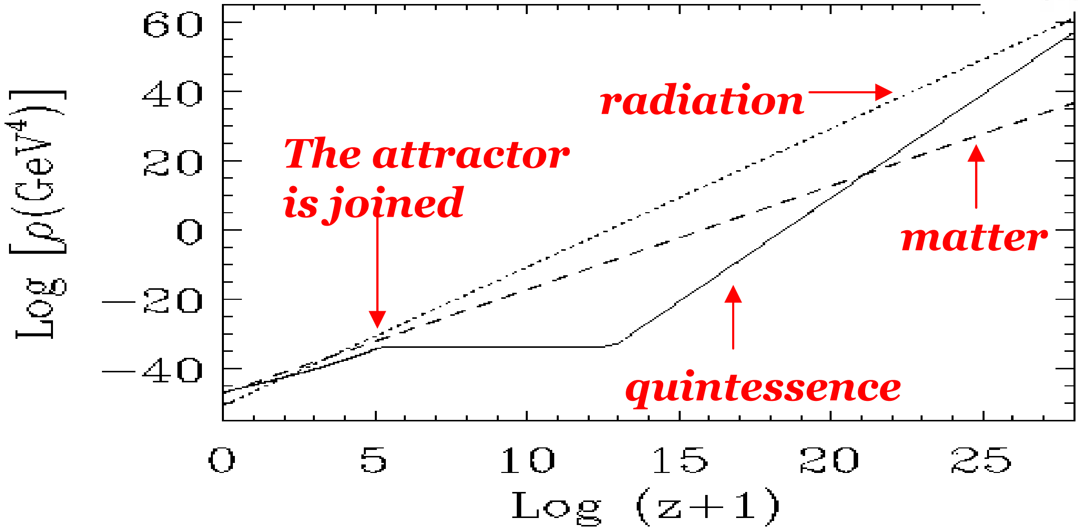


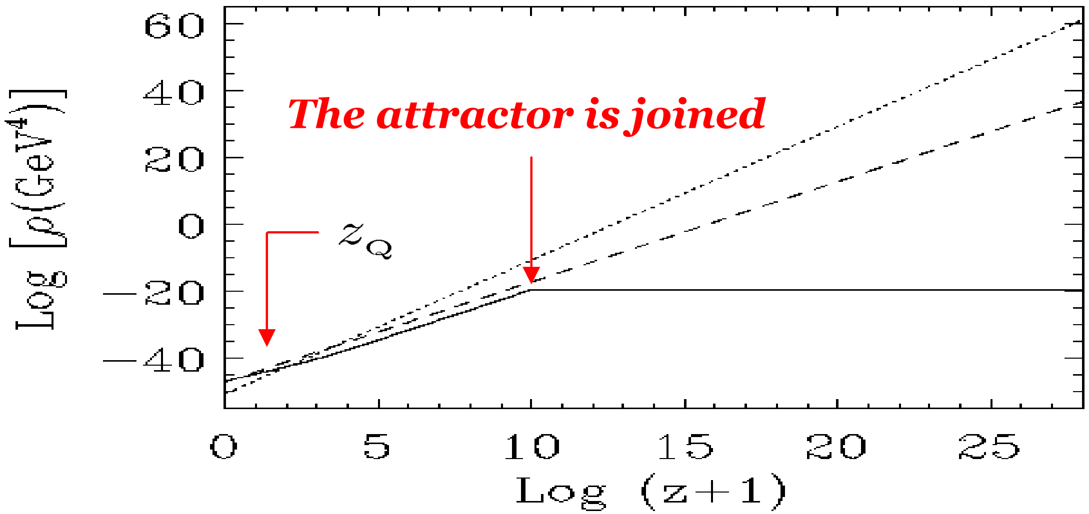
Shortcomings:

- Does not say anything about the cosmological constant problem
- Another scalar field …
- The mass scale has to be tuned by hand

Advantages:

- Gives a model where the equation of state is redshift-dependent and where the perturbations can be consistently computed
- Model building: SUGRA potential …
- The mass of the field is small but the free parameter is of the order of natural high energy scales
- There is an attractor for a huge range of initial conditions. It is not a simple mapping of the equation of state.





$$10^{-37}\mathrm{GeV}^4 < \rho_Q < 10^{61}\mathrm{GeV}^4$$

Outline

- It is important to study how the quintessence field interacts with the other sectors of the theory

- What are the form of the interaction terms with the standard model fields, with the inflaton etc …?

- The main issues addressed here are:

  -We consider the evolution of the quintessence field during a phase of (chaotic) inflation

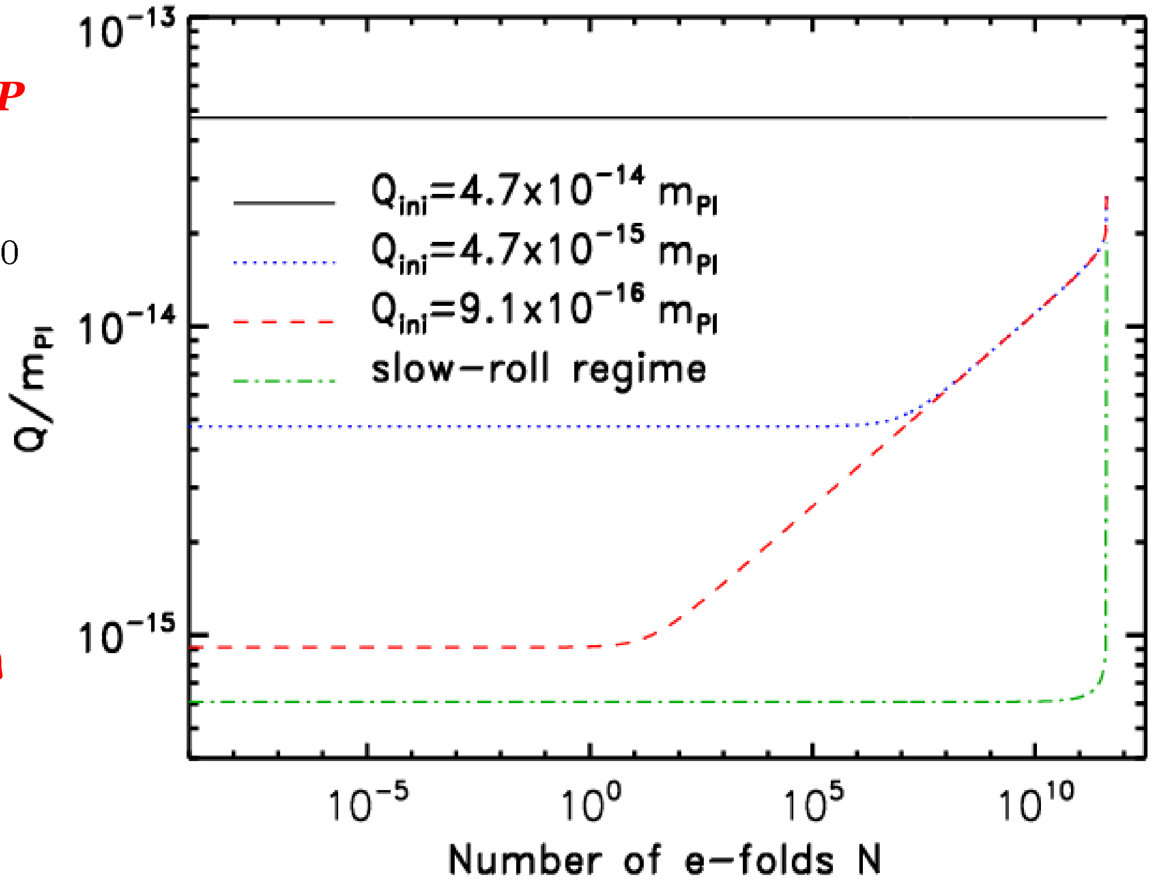
  -Is the interaction of the quintessence field with the inflaton and its evolution during inflation compatible with the allowed range of initial conditions? In other words, is the evolution of quintessence during inflation going to drive the field away from the allowed range of initial (final …) conditions after inflation?

Coupling the inflaton to quintessence

- The basic assumption is that Q is a test field in a background the evolution of which is controlled by the inflaton ϕ with $V(\phi) = m^2\phi^2/2$

$$\frac{Q}{m_{\rm Pl}} = \left[\left(\frac{Q_{\rm ini}}{m_{\rm Pl}}\right)^{\alpha+2} + \alpha(\alpha+2)\frac{m_{\rm Pl}^2}{m^2}\left(\frac{M}{m_{\rm Pl}}\right)^{4+\alpha} \ln\left(\frac{\phi_{\rm ini}}{\phi_{\rm cl}}\right)\right]^{1/(\alpha+2)}$$

$$\begin{cases} m \simeq 10^{-5} \times m_{\rm Pl} & \textit{COBE \& WMAP} \\ \left(\frac{M}{m_{\rm Pl}}\right)^{4+\alpha} = \frac{\rho_{\rm cri}}{m_{\rm Pl}^4} = \frac{H_0^2}{m_{\rm Pl}^2} \simeq 10^{-120} & \end{cases}$$



Typically, the quintessence field is frozen during inflation

Coupling the inflaton to quintessence

- **Another basic assumption is that Q and the inflaton belongs to different sectors of the theory. This means that**

$$K = K_{\rm Quint}(X,Y,Q) + K_{\rm Inf}(\rho,\phi)\,, \qquad W = W_{\rm Quint}(X,Y,Q) + W_{\rm Inf}(\rho,\phi)$$

$$G_{A\bar{B}} = \begin{array}{c} \overbrace{\qquad\qquad}^{\text{Inflation}} \quad \overbrace{\qquad\qquad\qquad}^{\text{Quintessence}} \\ \left[\begin{array}{cc|ccc} G_{\rho\rho^\dagger} & G_{\rho\phi^\dagger} & 0 & 0 & 0 \\ G_{\phi\rho^\dagger} & G_{\phi\phi^\dagger} & 0 & 0 & 0 \\ \hline 0 & 0 & \kappa & 0 & 0 \\ 0 & 0 & 0 & \kappa\dfrac{(QQ^\dagger)^q}{m_{\rm C}^{2q}} & 0 \\ 0 & 0 & 0 & 0 & \kappa \end{array}\right] \end{array}$$

$$V_{\rm Inf} = \frac{1}{\kappa^2}{\rm e}^{G_{\rm Inf}}\left[G_{\rm Inf}^{\bar{A}B}(G_{\rm Inf})_{\bar{A}}(G_{\rm Inf})_B - 3\right]$$

$$V_{\rm Quint}(Q) = {\rm e}^{\kappa Q^2/2}\frac{M^{4+\alpha}}{Q^\alpha}$$

$$\boxed{V = {\rm e}^{\kappa\xi^2}\left[{\rm e}^{\kappa Q^2/2}V_{\rm Inf}(\rho,\phi) + {\rm e}^{\kappa K_{\rm Inf}}V_{\rm Quint}(Q) + \kappa^2\left(\xi^2 + \frac{Q^2}{2}\right)|W_{\rm Inf}|^2\,{\rm e}^{\kappa\left(K_{\rm Inf}+Q^2/2\right)}\right]}$$

- To go further, a model for (chaotic) inflation is needed. One takes

$$\left.\begin{aligned} K_{\rm Inf} &= -\frac{3}{\kappa}\ln\left[\kappa^{1/2}\left(\rho+\rho^\dagger\right)+\frac{\kappa}{2}\left(\phi-\phi^\dagger\right)\right] \\ &+ \frac{1}{2}\left(\phi-\phi^\dagger\right)\,, \\ W_{\rm Inf} &= \frac{m}{\sqrt{2}}\phi^2 \end{aligned}\right\}$$

$$\boxed{V_{\rm Inf} = \frac{2m^2}{\Delta^2\left(3-\Delta\right)}\phi^2 = \frac{1}{2}m^2\phi^2}$$

N.B.: $\Delta \equiv \kappa^{1/2}\left(\rho+\rho^\dagger\right) = 2$

$$\boxed{V(\phi,Q) = \underbrace{\frac{1}{2}m^2\phi^2 e^{\kappa\xi^2+\kappa Q^2/2}}+\underbrace{\frac{1}{8}e^{\kappa\xi^2+\kappa Q^2/2}\frac{M^{4+2p}}{Q^{2p}}}\underbrace{+4\pi^2\frac{m^2}{m_{\rm Pl}^4}\left(\xi^2+\frac{1}{2}Q^2\right)\phi^4 e^{\kappa\xi^2+\kappa Q^2/2}}}$$

$Q/m_{\rm Pl} \ll 1$

$$\sim \frac{1}{2}m^2\phi^2$$

Ratra-Peebles/SUGRA

$$\sim \frac{M^{4+\alpha}}{Q^\alpha}$$

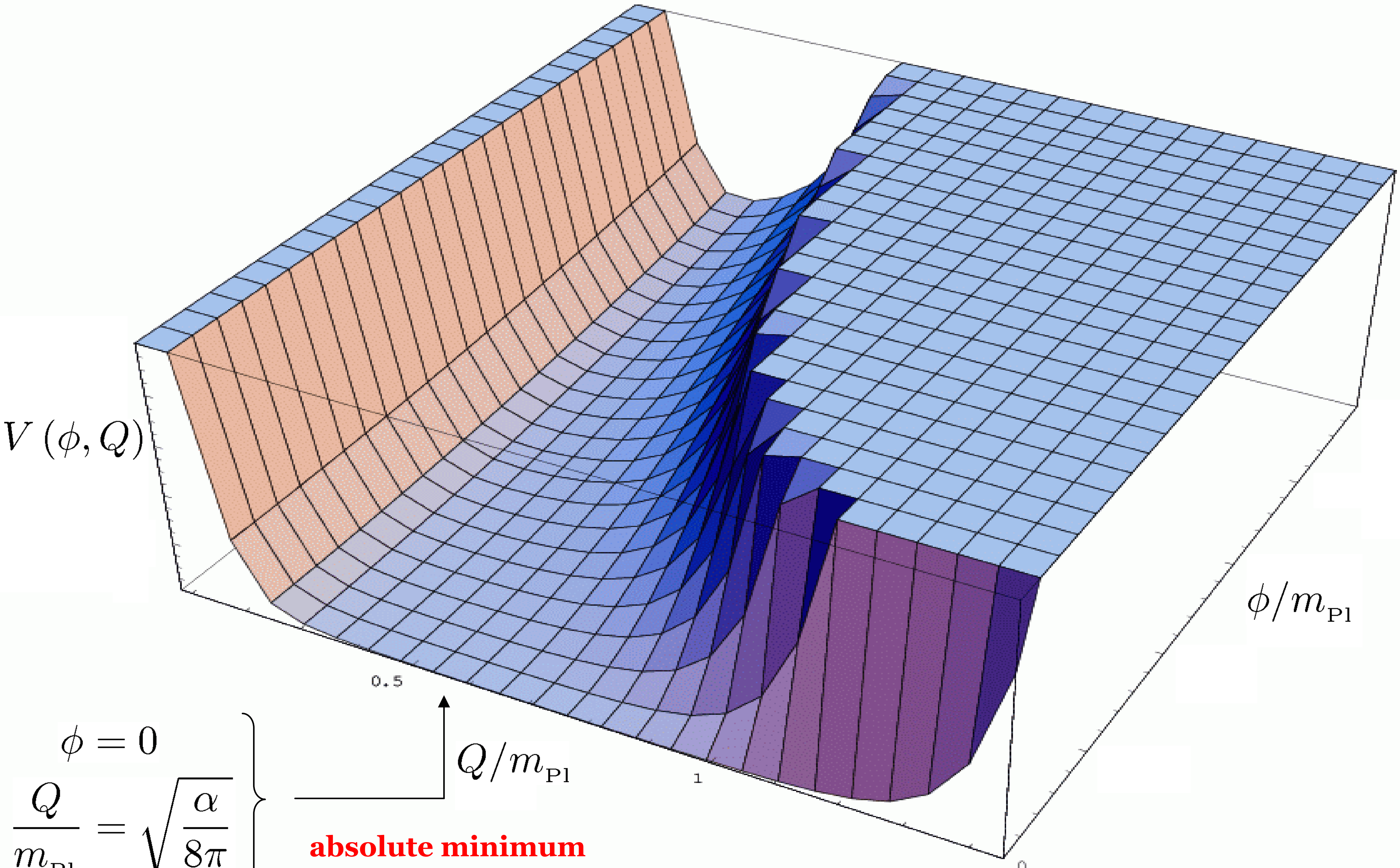
$$\sim m^2\phi^4 Q^2/m_{\rm Pl}^4$$

N.B.:

$$10^{-18}m_{\rm Pl} < Q_{\rm ini} < 10^{-2}m_{\rm Pl}\,, \alpha = 6$$

$$Q \gg \xi$$

Coupling the inflaton to quintessence



$$\left.\begin{aligned} \phi &= 0 \\ \frac{Q}{m_{\rm Pl}} &= \sqrt{\frac{\alpha}{8\pi}} \end{aligned}\right\}$$

absolute minimum

Coupling the inflaton to quintessence

- If the quintessence field is a test field, then Q evolves in an effective time-dependent potential given by

$$\frac{V_{\rm eff}(Q)}{m_{\rm Pl}^4} \simeq \frac{1}{8}\left(\frac{M}{m_{\rm Pl}}\right)^{4+2p}\left(\frac{Q}{m_{\rm Pl}}\right)^{-2p} + 2\pi^2\left(\frac{m}{m_{\rm Pl}}\right)^2\left(\frac{Q}{m_{\rm Pl}}\right)^2\left(\frac{\phi}{m_{\rm Pl}}\right)^4$$

Slow-rolling inflaton field

$$\frac{\phi}{m_{\rm Pl}} = \sqrt{\left(\frac{\phi_{\rm ini}}{m_{\rm Pl}}\right)^2 - \frac{N}{2\pi}}$$

- The effective potential possesses a time-dependent minimum

$$\boxed{Q_{\rm Min}(N) = m_{\rm Pl} \times \left\{\frac{p}{16\pi^2}\left(\frac{H_0}{m_{\rm Pl}}\right)\left(\frac{m}{m_{\rm Pl}}\right)^{-2}\left[\frac{\phi(N)}{m_{\rm Pl}}\right]^{-4}\right\}^{1/[2(p+1)]}}$$

N.B.: at the minimum, Q is not light

$$\frac{m_{\rm eff}^2}{H^2} = 6\pi(p+1)\left[\frac{\phi(N)}{m_{\rm Pl}}\right]^2 \gg 1$$

Coupling the inflaton to quintessence

- **The evolution of the minimum is " adiabatic"**

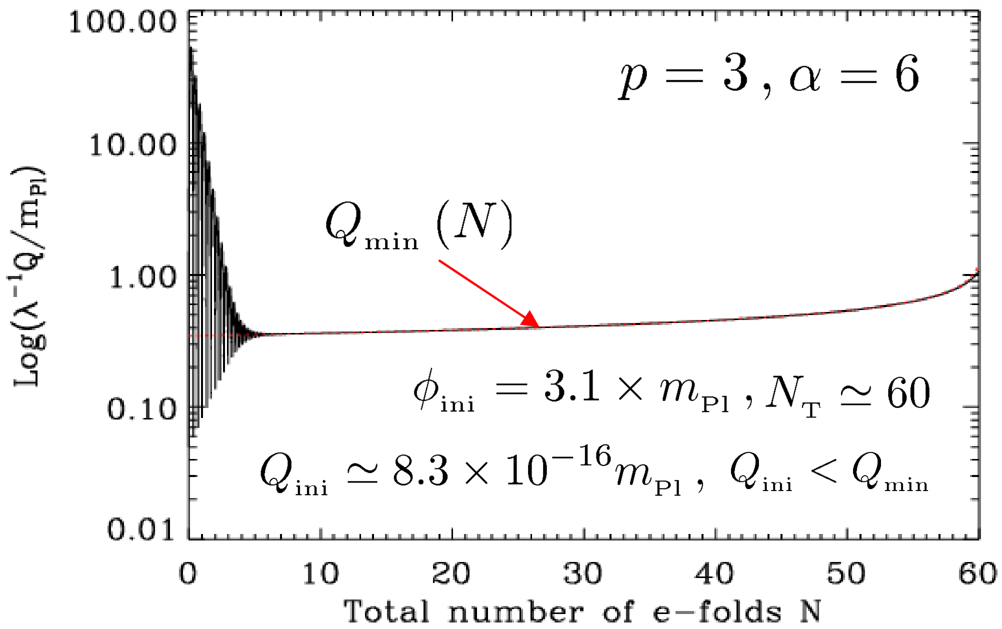
$$\frac{\Delta N_{\rm min}}{\Delta N_{\rm osci}} = \mathcal{O}(1)\frac{N_{\rm T}^{1/2}}{\epsilon} \gg 1$$

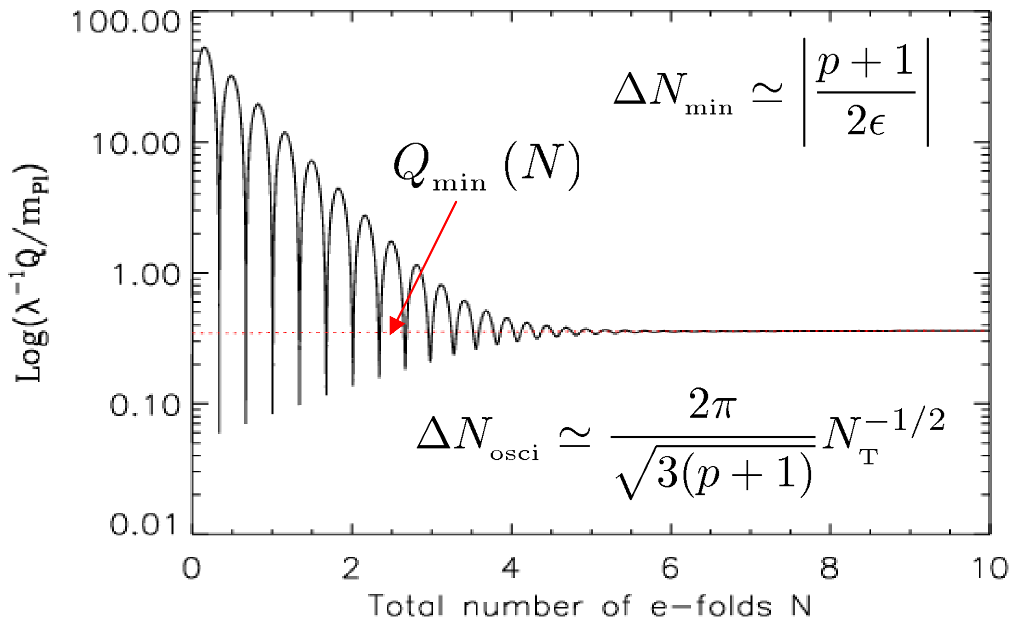
- **The minimum is an attractor**

$$Q_{\rm min}(N_{\rm T}) \sim 10^{-55/(p+1)} \times m_{\rm Pl}$$

- **The effect of the interaction term is important and keeps Q small during inflation**

$$\boxed{\left.\frac{Q_{\rm no\ inter}}{Q_{\rm inter}}\right|_{\rm end} \sim 10^{55/(p+1)} \times \frac{Q_{\rm ini}}{m_{\rm Pl}}}$$





Inflation +25

A conference to celebrate
the 25th anniversary of inflationary cosmology

Institut d'Astrophysique de Paris

June 26 to 30, 2006

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… registration opens early 2006 …