Cosmological Model Selection with Nested Sampling

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Introduction

- There is an abundance of cosmological data, and so many more models are testable than ever before.
- A distinction must be made between *parameter fitting* and *model selection.*
- Model selection statistics are necessary to choose between models, and determine the need for new paremeters.

Bayesian Evidence

- Bayes' theorem gives the posterior probability of the parameters (θ) of a model (H) in light of the data (D)

$$P(\theta \mid D,H) = \frac{P(D \mid \theta,H)P(\theta \mid H)}{P(D \mid H)}$$

- By marginalizing over the parameters, we compute the evidence.

$$E = P(D \mid H) = \int d\theta P(D \mid \theta,H)P(\theta \mid H)$$

Nested Sampling

- Created by John Skilling, Nested Sampling computes the evidence directly using an Monte-Carlo routine

$$E = \int L(\theta)\, p(\theta)\, d\theta$$

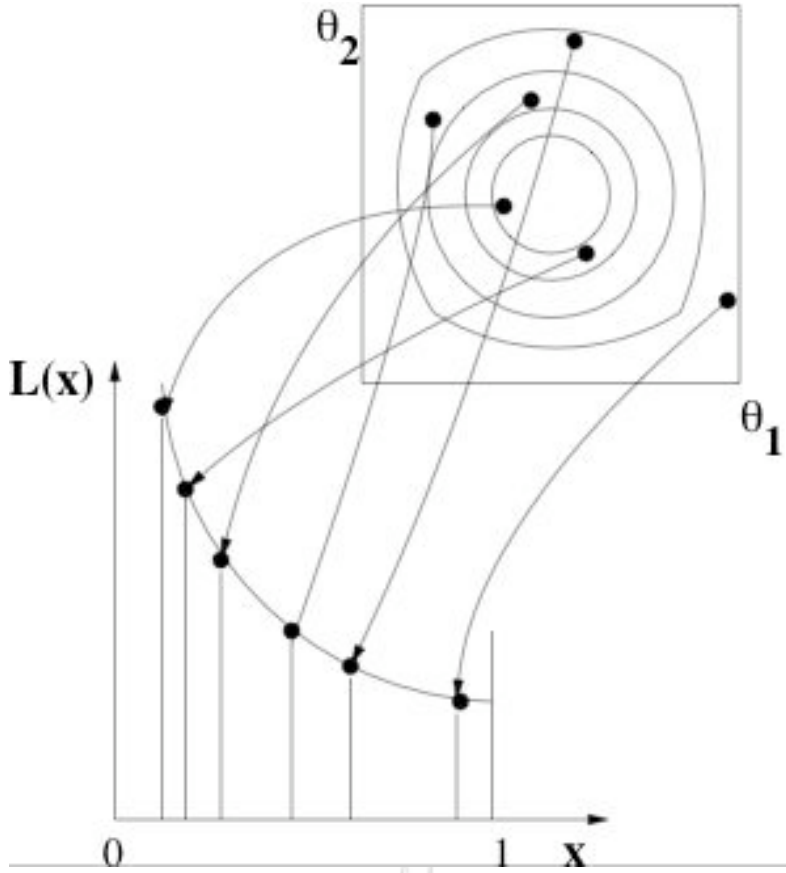
- We re-parameterize the problem so we integrate over the prior mass (*X*) directly

$$E = \int L(X)\, dX$$

- Here *X* is normalised to unity.

Integration

- We sample the prior mass uniformly with some large number of points, and sort the points by likelihood.
- This creates a series of nested iso-likelihood surfaces.
- We then increment the evidence by the prior mass volume weighted by the likelihood.



Uniform Sampling

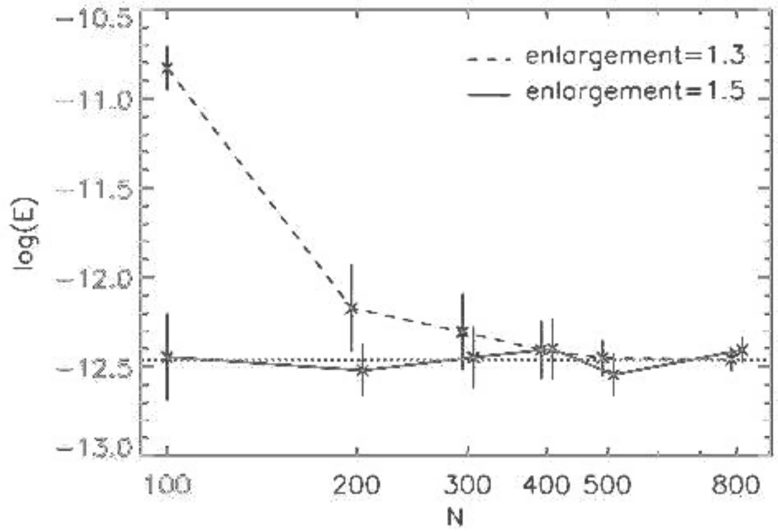
- We cannot go from the co-ordinates of the points to the value of X, or the reverse.
- Instead we approximate X by its statistics. After i replacements $X \sim (N/N+1)^i$
- This assumption means that the points and their replacements must be drawn uniformly from the prior mass remaining.

Method

1. Sample N points randomly from within the prior. Initially we will have the full prior range available, i.e. $(0,X_0=1)$.
2. Select the point with the lowest likelihood L_j. The volume corresponding to this point can be estimated probabilistically as being $(N/N+1)^j$
3. Increment the evidence by $E_j=L_j*(X_{j-1}-X_{j+1})/2$
4. Replace the lowest likelihood point with a new point with higher likelihood, which is uniformly distributed within the remaining prior volume $(0,X_j)$.
5. Repeat the previous steps 2-4, until such time as the evidence has been estimated to some accuracy.

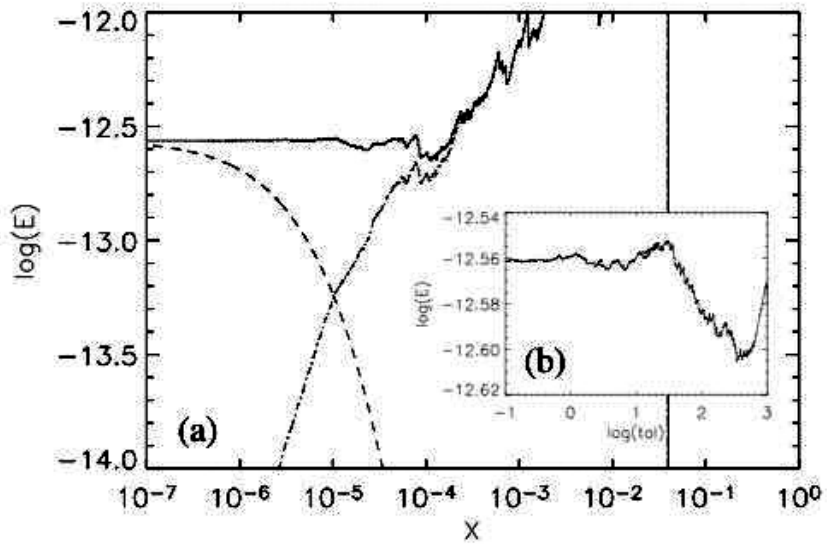
Replacement points

- Find ellipsoid bounded by all points, in space rotated by the covariance matrix (so parameter are uncorrelated).
- Enlarge the ellipsoid by a certain enlargement factor
- Generate new point randomly in ellipsoid.



Stopping Criteria

- We can make an maximal estimate of the remaining evidence, $L_{max}X_j$
- Termination of the process occurs when this maximal estimate is some fraction of the accumulated evidence.



Prior ranges (ΛCDM model)

Parameter	Minimum	Maximum
$\Omega_b h^2$	0.018	0.032
$\Omega_{cdm} h^2$	0.04	0.16
Θ	0.98	1.1
τ	0	0.5
$\text{Log}(A_s x 10^{10})$	2.6	4.2

Evidences

Model	LCDM+ HZ	LCDM +n_s	LCDM+ n_s (wide)	HZ+w	w+n_s
n_s	1	0.8-1.2	0.6-1.4	1	0.8-1.2
w	-1	-1	-1	-0.33 - -1	-0.33- -1
e.f.	1.5	1.7	1.7	1.7	1.8
N_{like} (10^4)	8.4	17.4	16.7	10.6	18.0
Log(E)	0.0 ± 0.08	-0.6 ±0.09	-1.2 ± 0.08	-0.5 ±0.08	-1.5 ± 0.08

Conclusions

- Nested Sampling is accurate, generally applicable and computationally feasible.
- It requires only $O(10^5)$ likelihood evaluations, so is efficient.
- Current data offer no indication of the need to add either w or n_s as extra parameters to the standard ΛCDM+HZ cosmological model.