Black hole production in preheating

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1, Primordial Black Holes(PBHs)

Primordial black holes are formed by the gravitational collapse of the overdense region in the early universe.

Jeans length

$$\lambda_J \sim \sqrt{\frac{w}{G\rho}} \qquad P = w\rho$$

In the radiation dominated universe

$$\lambda_J \sim 1/H$$

H :Hubble parameter

Numerical simulations (Shibata&Sasaki '99, Musco et al. '05) suggest BHs are formed if

$$0.3 < \delta_c < 0.5$$

δ_c :Density perturbation on comoving hypersurface

Depends on the initial conditions of the fluctuations.

Formed BH mass is equal to the horizon mass.

Abundance of the produced BHs at the formation time.

Assuming that the probability distribution of the density perturbation smoothed over the horizon scale is Gaussian,

Mass fraction of PBHs

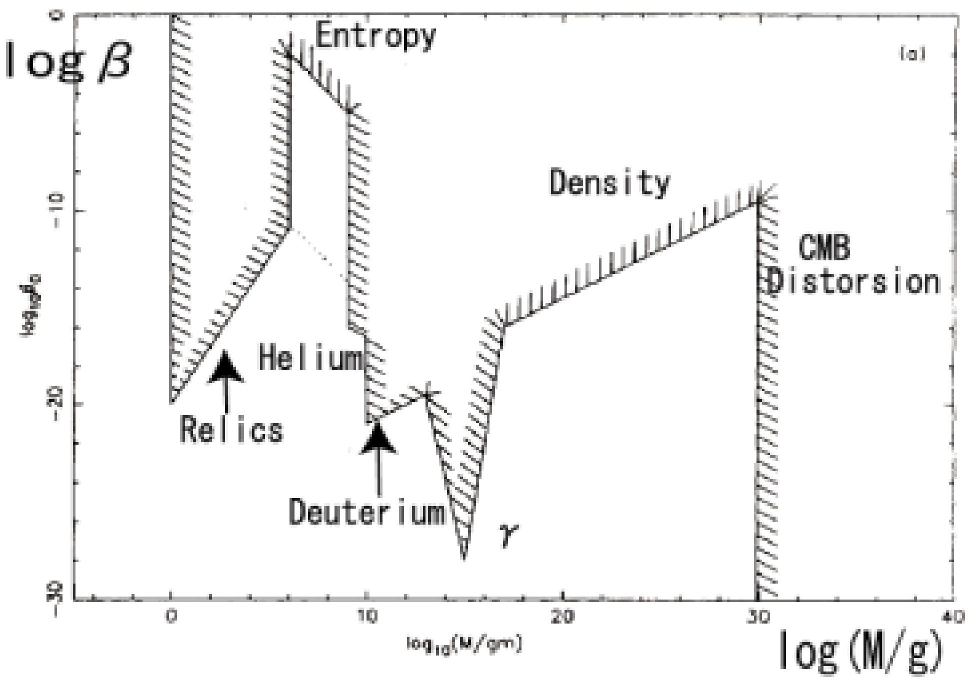
$$\beta \sim \int_{\delta_c} d\delta \exp\left(-\frac{\delta^2}{2\sigma^2}\right) \sim \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$$

In the radiation dominated universe,

$$\rho_{PBH} \propto a^{-3}$$
$$\rho_{rad} \propto a^{-4}$$
$$\Longrightarrow \quad \Omega_{PBH} \propto a$$

Even if β is very small, it conflicts with the standard Big Bang cosmology.

Constraints on β for various mass scales



$$\sigma < 0.03$$

dependence on σ is weak.

Carr, Gilbert,Lidsey '94

PBHs can constrain the inflationary models.
Scales are much smaller than the CMB-relevant scales.

PBHs important even if never formed.(Carr)

We show that PBH production generically does not exceed astrophysical bounds during preheating phase after inflation.

2, Preheating

- Preheating occurs at the first stage of reheating
- Preheating significantly differs from the usual reheating scenario.
- Energy transfer from inflaton to created particles lasts only during the several oscillations of the inflaton.

Simple model (e.g. Kofman et al. '94)

$$V_{eff} = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

Homogeneous part of the inflaton

$$\phi(t) = \frac{m_p}{\sqrt{3\pi m t}}\sin mt$$

Effective mass of a χ field

$$m^2_{\chi,eff} = g^2\phi^2(t)$$

Time dependent

2, Preheating

Evolution equation for χ field

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \underline{(k^2/a^2 + g^2\phi^2(t))}\chi_k = 0$$

Causes non-adiabatic excitation of the field fluctuations by parametric resonance

This is quantified by the dimensionless parameter

$$R = \frac{\dot{\omega}_k}{\omega_k^2} \qquad \omega_k^2 = k^2/a^2 + g^2\phi^2(t)$$

$R \gg 1$ Particle production

Exponential growth of the particle number

Since density perturbation is a linear combination of field fluctuations, density perturbation also grows exponentially.

If density perturbation becomes too large by parametric resonance, it can lead to copious over-production of PBHs.

Mathieu equation

$$X_k = a^{3/2}\chi_k \qquad A_k = 2q + \frac{k^2}{a^2m^2}$$

$$\frac{d^2X_k}{d^2z} + (A_k - 2q\cos 2z)X_k = 0 \qquad q = \frac{g^2\bar{\phi}^2(t)}{4m^2}$$

Broad resonance $\quad q \gg 1$

From the adiabatic condition, the maximum wave number in the resonance band is

$$k_{max} \sim mq^{1/4}$$

backreaction

Rapid draining of energy due to the particle production soon affects the dynamics of the background inflaton itself (backreaction).

Growth of field fluctuations and hence growth of density perturbation stops.

This stage is very complicated because non-linear interactions become non-negligible.

Final amplitude of density perturbations crucially depends on the time backreaction becomes important.

All non-linear effects such as rescattering must be taken into account in order to evaluate the final amplitude of the density perturbation.

Numerical simulation

3, Preheating and BH production

We have used modified LATTICEEASY.

LATTICEEASY developed by Gary Felder and Igor Tkachev. (hep-ph/0011159)

C++ program for doing lattice simulations of the evolution of interacting scalar fields in an expanding universe.

http://www.science.smith.edu/departments/Physics/fstaff/gfelder/latticeeasy/index.html

LATTICEEASY

1, solves the classical equations of motion.

This is valid in preheating stage because occupation number soon becomes much larger than unity.

2, include all the effects of non-linear interactions between scalar field fluctuations such as rescattering and backreaction.

Equations solved in LATTICEEASY.

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\triangle\phi + V'(\phi) = 0$$

$$\ddot{\chi} + 3H\dot{\chi} - a^{-2}\triangle\chi + V'(\chi) = 0$$

$$H^2 = \frac{8\pi G}{3}\langle\rho\rangle$$

Initial conditions

Given by vacuum fluctuations

Take the variance of these vacuum fluctuations as if it were statistical variance.

boundary condition

Periodic boundary condition

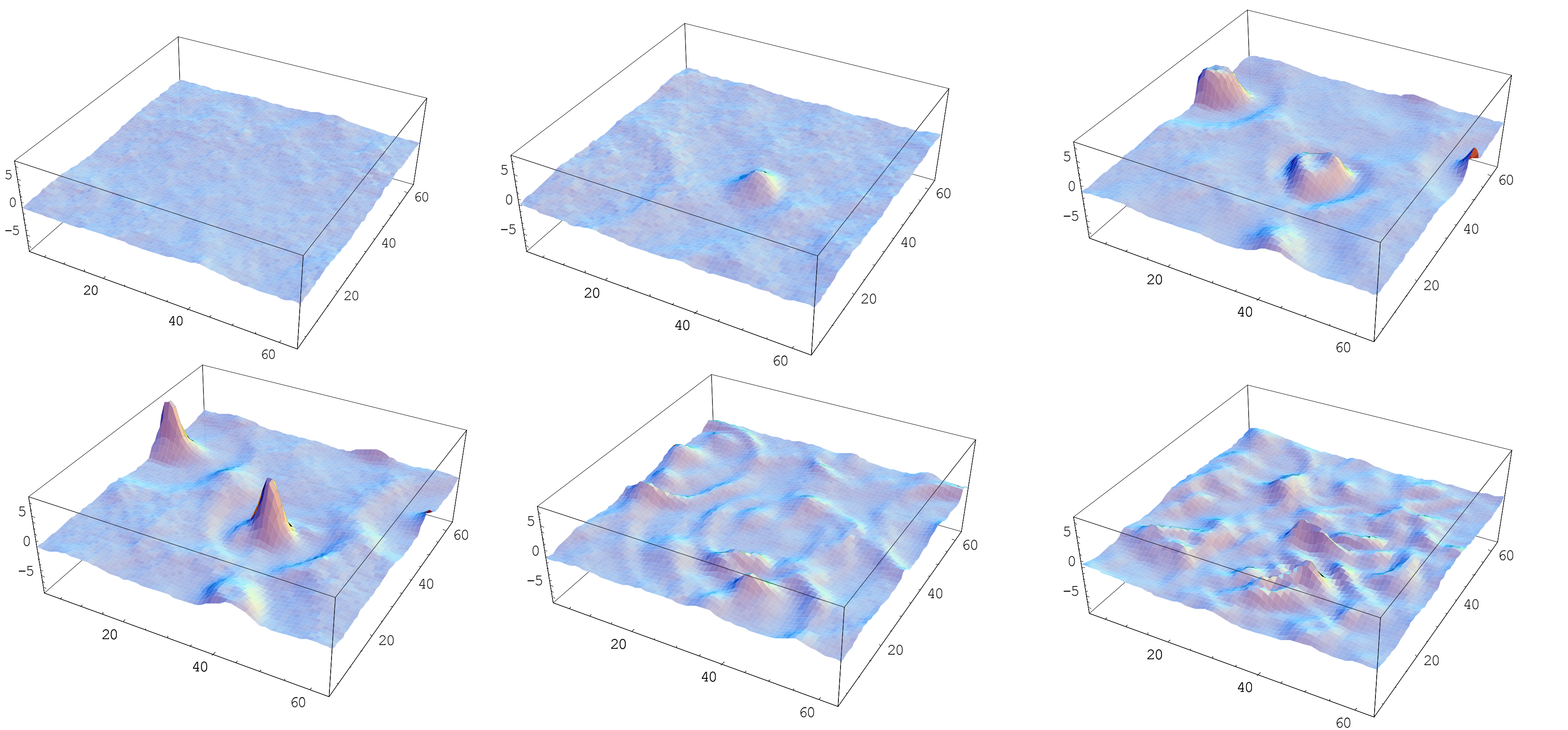
3, Preheating and BH production

We have solved equations of motion for two typical preheating models.

$$V_{eff} = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

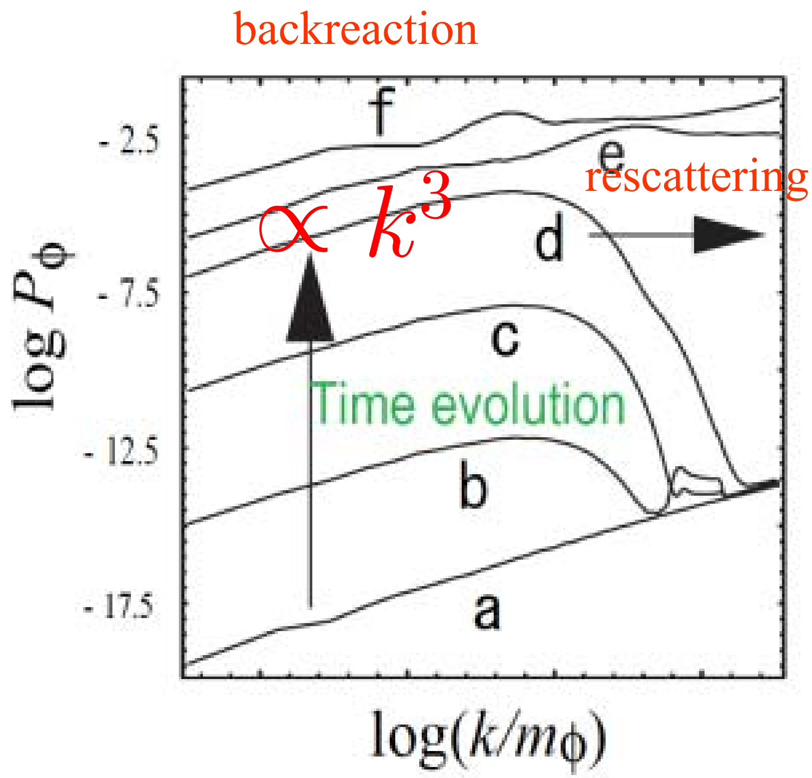
conformally invariant

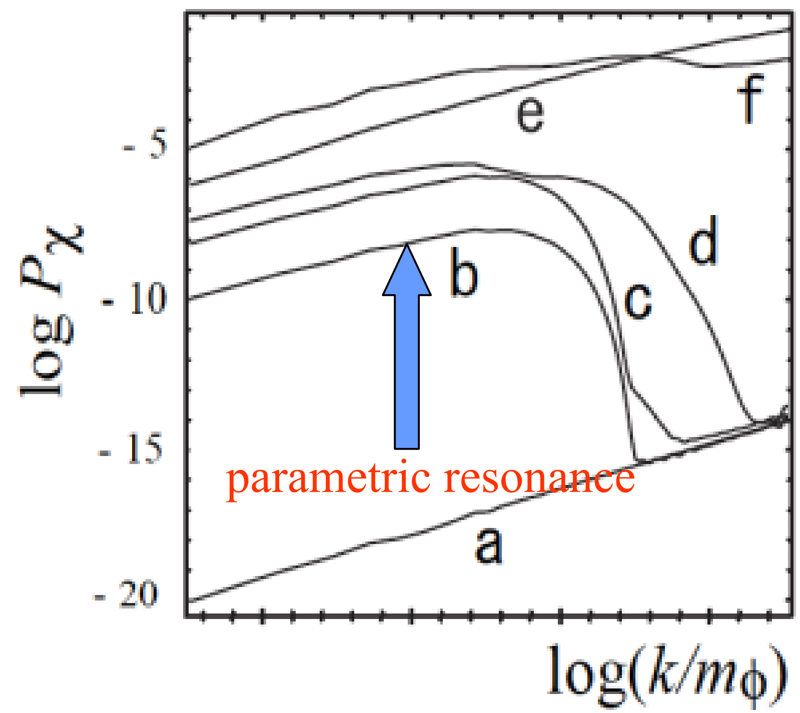
$$V_{eff} = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$



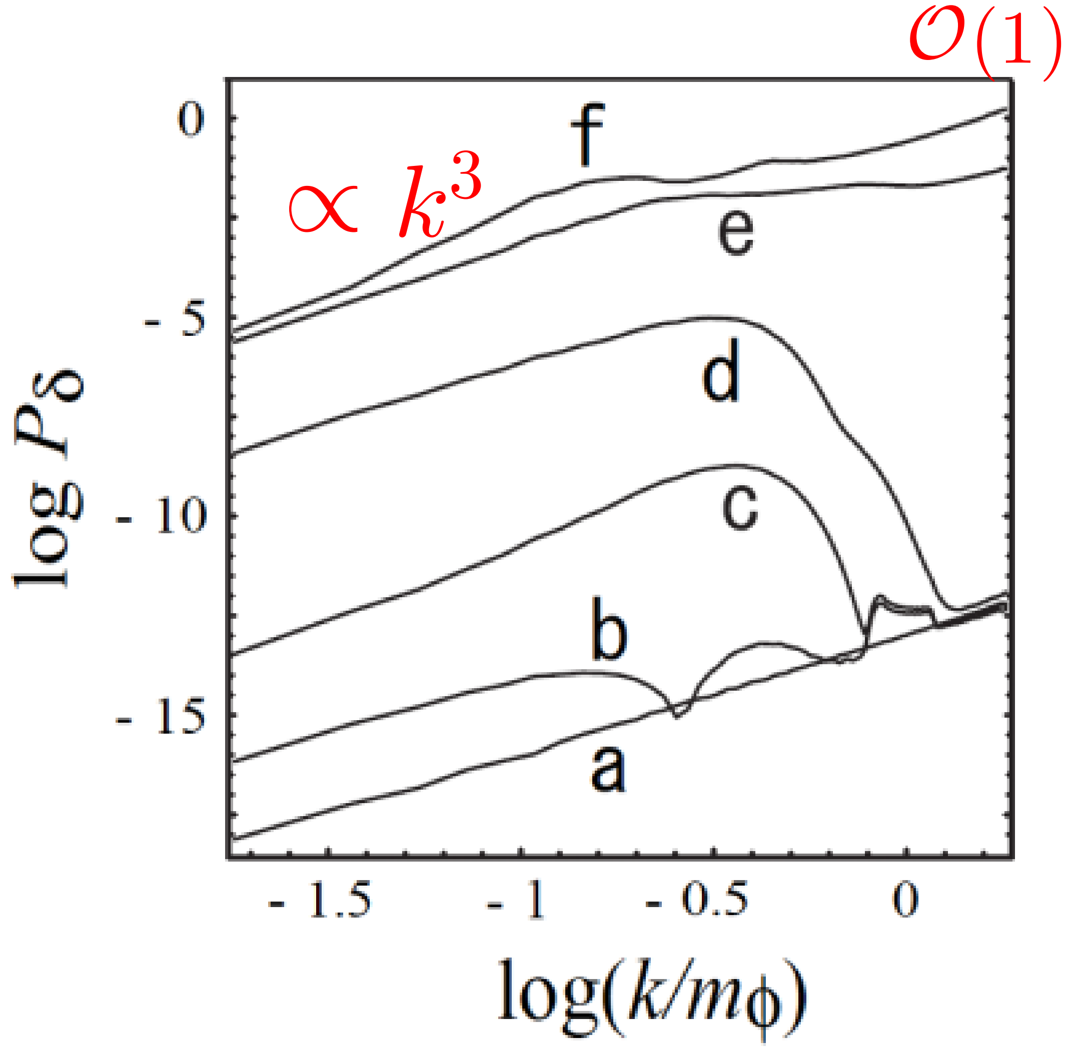
3, Preheating and BH production

First case



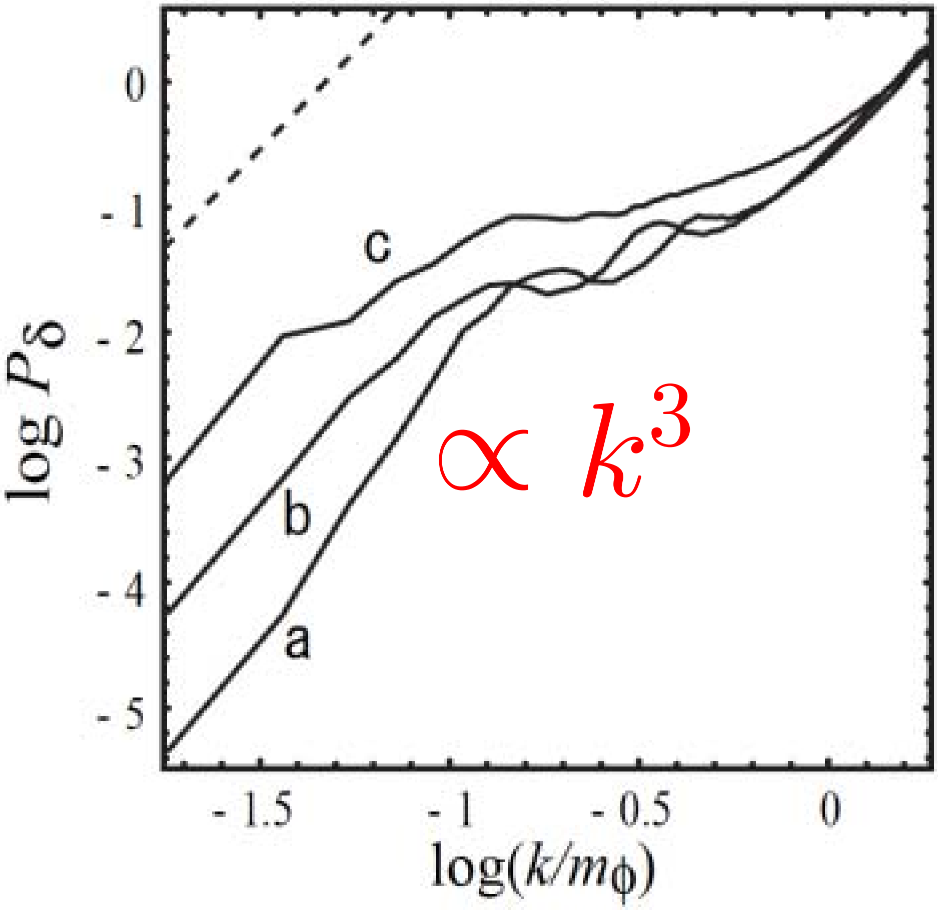


Evolution of density perturbations



density perturbations after preheating

Threshold of BH overproduction



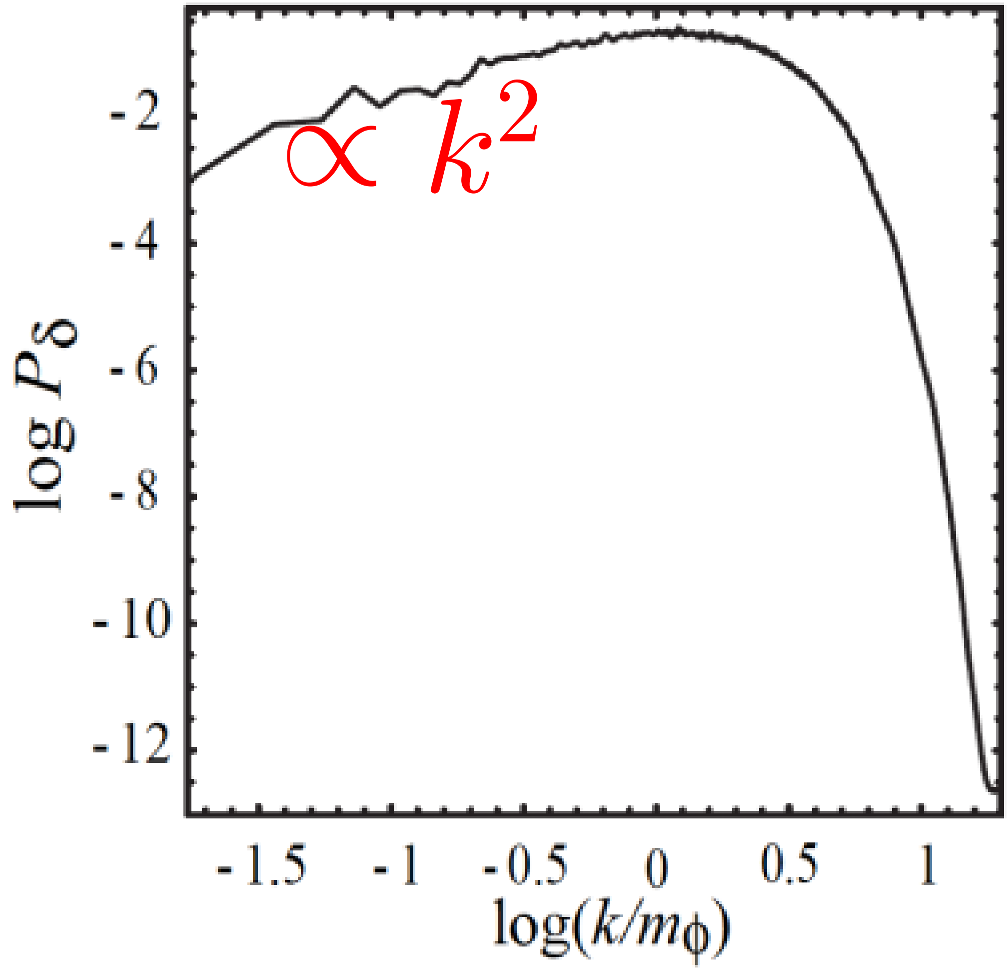
Slope of the power spectra for long wavelength is *universal* with a value of 3.

All three lines lie significantly under the dashed line.

PBHs are not overproduced in these cases.

Two-dimensional case

Density perturbations at the end of preheating



Interpretation of the results.

Assume that the resultant density perturbations have a typical scale r, and Correlations on larger length scales are strongly suppressed.

$$\frac{1}{k^D}\mathcal{P}_\delta(k) \propto \int d^D x \langle \delta(0)\delta(\vec{x})\rangle e^{i\vec{k}\vec{x}}$$

D :Number of spatial dimensions

is dominated by a small region with $|\vec{x}| < r$.

For long wavelength modes with $k \ll r^{-1}$, $e^{i\vec{k}\vec{x}}$ can be approximated by unity.

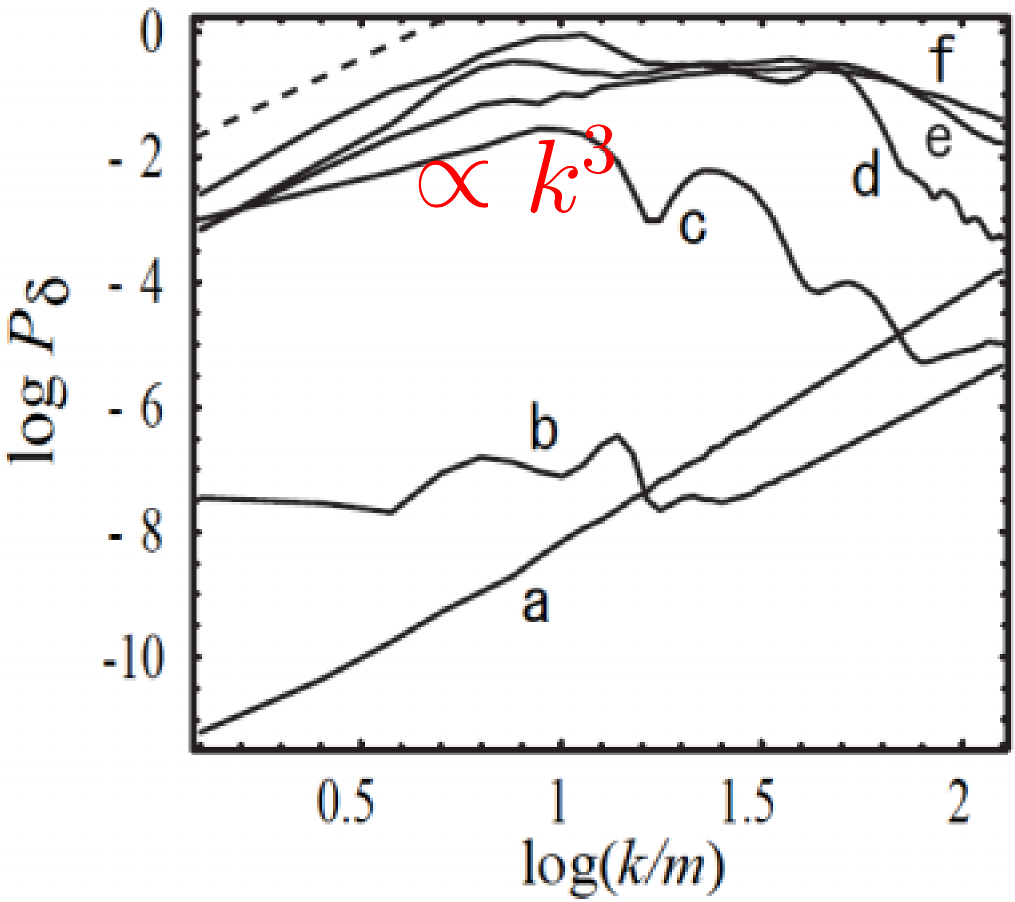
$$\mathcal{P}_\delta(k) \propto k^D$$

In general parametric resonance causes loss of correlation between density fluctuations beyond a typical length scale.

Second model

Evolution of the density perturbations

Threshold of BH overproduction



BHs are not overproduced in this model

In general,

Peak of the power spectrum

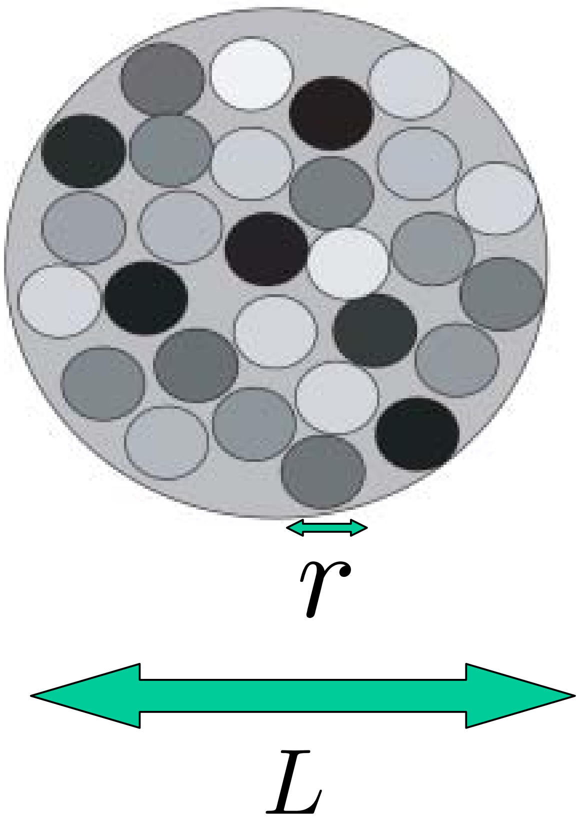
$$\frac{k_{max}}{H} \sim \frac{m_p}{\phi} q^{1/4}$$

is always at sub-Hubble wavelengths

q :resonance parameter

3, Preheating and BH production

Gaussianity

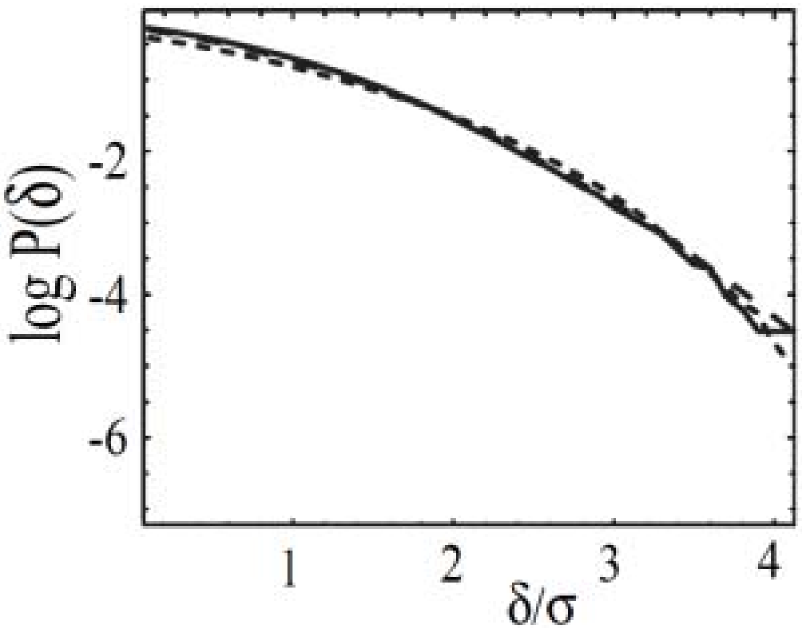


Probability distribution of the amplitude of the density perturbations at the end of preheating.

Large number of independent variables of $\mathcal{O}((L/r)^3)$

Probability is very close to Gaussian by the central limit theorem

Gaussian distribution



4, Summary

1, BH production generically does not exceed astrophysical bounds during the resonant phase after inflation.

2, After preheating, power spectra are universal, with no memory of the power spectrum at the end of inflation.

3, Probability distribution of density perturbations is Gaussian when smoothed over the Hubble scale.