Anomaly-Induced Inflation

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It would be interesting to have a natural mechanism for inflation based on the vacuum **quantum effects of matter fields.**

Gravity is not quantized!

Modified Starobinsky model.

We can describe two phases of inflation:

1) Initial period of inflation: **matter fields are approximately massless;**
2) Final phase: **masses become relevant.**

I. Free (or AF) Massless Fields.

N_0 *scalars*, $N_{1/2}$ *fermions*, N_1 *vectors*

Notice: Vacuum quantum effects come from **virtual** particles. $N_{0,\,1/2,\,1}$ have **no relation to the real matter** in the Universe.

Classical vacuum action of conformal theory

$$S_{vac} = \int d^4x\sqrt{-g}\,\Big\{l_1C^2 + l_2E + l_3\Box R\Big\}\,.$$

$C = C_{\mu\nu\alpha\beta}$ is Weyl tensor,
$E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ is Gauss-Bonnet term.
S_{vac} does not affect cosmological solution.

Quantum correction: **Conformal Anomaly**

$$T = \langle T^\mu_\mu \rangle = -\,(wC^2 + bE + c\,\Box R)\,,$$

$w,\,b,\,c$ are β-functions for $l_1,\,l_2,\,l_3$

$$\begin{pmatrix} w \\ -\,b \\ c \end{pmatrix} = \frac{1}{360(4\pi)^2}\begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

Remark: Alternating sign for c.

Recent investigation of $\Box R$-type ambiguity:
M. Asorey, E. Gorbar & I.Sh. Clas.Q.Gr.(2003).

Anomaly-Induced Effective Action (EA)

$$-\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} = T\,.$$

(Reigert, Fradkin & Tseytlin, 84)

The EA **is exact** solution for FRW metric

$$\bar{\Gamma}_{ind} = S_c[\bar{g}_{\mu\nu}] + \int d^4x\sqrt{\bar{g}}\,\{w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R})$$

$$+2b\sigma\bar{\Delta}\sigma\,\} - \frac{3c-2b}{36}\int d^4x\sqrt{g}\,R^2,$$

where $g_{\mu\nu} = a^2(x)\bar{g}_{\mu\nu}$, $a^2(x) = e^{2\sigma(x)}$,
$S_c[g_{\mu\nu}]$ an arbitrary conformal functional,
$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$.

Local covariant solution via auxiliary fields (A.Jacksenaev & I.Sh., Phys.Lett.B, 1994)

$$\Gamma_{ind} = S_c - \frac{3c-2b}{36}\int_x R^2 + \frac{1}{2}\int_x \{\varphi\Delta\varphi - \psi\Delta\psi$$

$$+\varphi\left[\sqrt{-b}(E - \frac{2}{3}\Box R) - \frac{w}{\sqrt{-b}}C^2\right] + \frac{w}{\sqrt{-b}}\psi C^2\}\,.$$

The most useful form of the vacuum EA for the conformal matter fields.

Cosmological Model based on the action

$$S_{total} = -\frac{M_P^2}{16\pi}\int d^4x\sqrt{-g}\,(R+2\Lambda)$$

$$+S_{matter}+S_{vac}+\bar{\Gamma}_{ind}\,.$$

Equation of motion in phys. time $dt = a(\eta)d\eta$

$$\frac{\ddddot{a}}{a}+\frac{3\dot{a}\dddot{a}}{a^2}+\frac{\ddot{a}^2}{a^2}-\left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}-2k\left(1+\frac{2b}{c}\right)\frac{\ddot{a}}{a^3}$$

$$-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}-\frac{2\Lambda}{3}\right) = 0\,,$$

where $k=0,\pm1$, $\Lambda-$ cosmological constant.

Particular solutions (Starobinsky, Ph.L.B., 1980)

$$a(t) = a_0\begin{pmatrix} \exp[Ht]\,, & k=0 \\ \cosh[Ht]\,, & k=1 \\ \sinh[Ht]\,, & k=-1 \end{pmatrix},$$

the Hubble parameter $H=\dot{a}/a$ is

$$H = \frac{M_P}{\sqrt{-32\pi b}}\left(1\pm\sqrt{1+\frac{64\pi b}{3}\frac{\Lambda}{M_P^2}}\right)^{1/2}$$

For $0 < \Lambda \ll M_P^2$ there are two solutions:

$$H \approx \sqrt{\Lambda/3} \qquad\qquad (IR)$$

$$H \approx \sqrt{-\frac{M_P^2}{16\pi b} - \frac{\Lambda}{3}} \approx \frac{M_P}{\sqrt{-16\pi b}} \qquad\qquad (UV)$$

Perturbations of the conformal factor:

$$\sigma(t) \to \sigma(t) + y(t).$$

The criterion for a stable inflation

$$c > 0 \quad \Longleftrightarrow \quad N_1 < \frac{1}{3} N_{1/2} + \frac{1}{18} N_0\,,$$

in agreement with Starobinsky.

Remarks. The value of Λ and the choice of $k = 0, \pm 1$ do not influence the stability condition.

Unstable case: one can fine-tune the initial conditions such that the universe performs sufficient inflation and then graceful exit to the FRW behaviour.

Stable case: no fine-tuning is needed. The Universe starts inflation from an arbitrary position at the phase plane.

The original Starobinsky model

M.V. Fischetti, J.B. Hartle & B.L. Hu, Phys.Rev. **D20** (1979) 1757;

A.A. Starobinski, Phys.Lett. **91B** (1980) 99;

V.F. Mukhanov & G.V. Chibisov, JETP Lett. 33 (1981) 532; JETP (1982) 258;

A.A.Starobinski, Let.Astr.Journ. 9 (1983) 579;

A. Vilenkin, Phys. Rev. **D32** (1985) 2511;

P. Anderson, Phys. Rev. **D28** (1983) 271; **D29** (1984) 615; **D29** (1986) 1567.

is based on the unstable case
and involves **heavy fine-tunings.**

Our purpose is to avoid fine-tunings at all.

This seems to be possible if we use the notions of Effective Quantum Fields theory, that is take care on separating the light and heavy degrees of freedom.

Simple test of the model. Late Universe, $k = \Lambda = 0$. Only photon is active

$$N_0 = 0\,, \qquad N_{1/2} = 0\,, \qquad N_1 = 1\,.$$

The typical energy is $H_o \approx 10^{-42}\,GeV$, therefore all massive particles (even neutrino) $m_\nu \geq 10^{-12}\,GeV$ **decouple** from gravity.

Recent study of decoupling in gravity:
E.Gorbar & I.Sh, JHEP (2003, 2004).

$c < 0 \quad \Longrightarrow \quad$ **today inflation is unstable.**

Consider $a(t) \sim t^{2/3}$ in

$$\frac{\ddddot{a}}{a} + \frac{3\,\dot{a}\dddot{a}}{a^2} + \frac{\ddot{a}^2}{a^2} - \left(5 + \frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}$$

$$- \frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = \frac{\rho_m\,a^{-3}}{c}\,.$$

$$t \to \infty \qquad \begin{cases} \text{classical terms} & \sim 1/t^2 \\ \text{quantum corrections} & \sim 1/t^4 \end{cases}$$

Late universe: quantum effects irrelevant.

Second test. $H_o \approx \sqrt{\frac{\Lambda}{3}}$ without matter.

A.Pelinson,I.Sh.,F.Takakura, N.Ph.B(2003).

Recent SN data indicate $\rho_{vac} \approx 10^{-47} GeV^4$.

$$H \to H_o + const \cdot e^{\lambda t} \implies$$

$$\lambda^3 + 7H_o\lambda^2 + \left[\frac{(3c-b)4{H_o}^2}{c} - \frac{M_P^2}{8\pi c}\right]\lambda$$

$$- \frac{32\pi b{H_o}^3 + M_P^2 H_o}{2\pi c} = 0\,.$$

The solution for $\Lambda = H_o = 0$ is

$$\lambda_1^{(0)} = 0\,, \quad \lambda_{2/3}^{(0)} = \pm \frac{M_P}{\sqrt{8\pi|c|}}\, i\,.$$

Expanding in $H_o/M_P \approx 10^{-61}$:

$$\lambda_1 = -4H_o\,, \qquad \lambda_{2/3} = -\frac{3}{2}H_o \pm \frac{M_P}{\sqrt{8\pi|c|}}\, i\,.$$

$\Lambda > 0$ **protects our world from quantum corrections!**

Coming back to the Early Universe.

Stable inflation does not depend on initial data. Fine! **But how does it end?**

Suppose at **UV** $(H \gg M_F)$ there is **SUSY**, e.g. **MSSM** with a particle content

$$N_1 = 12\,, \qquad N_{1/2} = 32\,, \qquad N_0 = 104\,.$$

This provides **stable inflation**, because $c > 0$

$$N_1 \,<\, \frac{1}{3}\,N_{1/2} + \frac{1}{18}\,N_0\,.$$

Similar for any realistic supersymmetric model.

Already for MSM inflation is unstable $c < 0$.

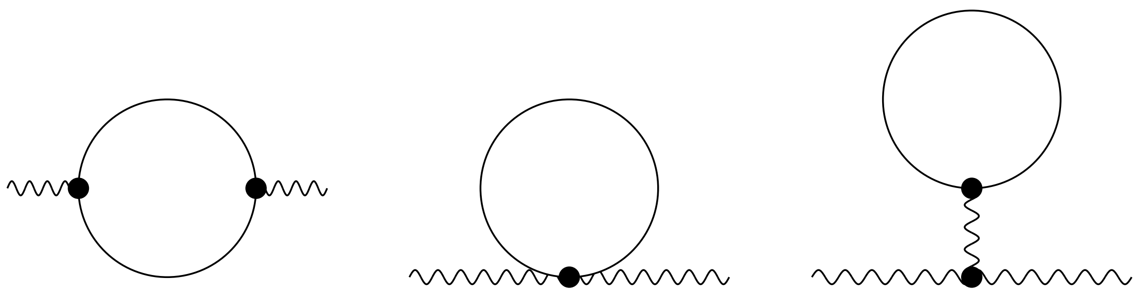
Why inflation ends?
Because all **sparticles** are heavy $\Rightarrow$ **decouple** at low energies, when H becomes smaller than the mass of the quantum field.

Relevant Feynman diagrams include loops of matter field & external lines of σ.

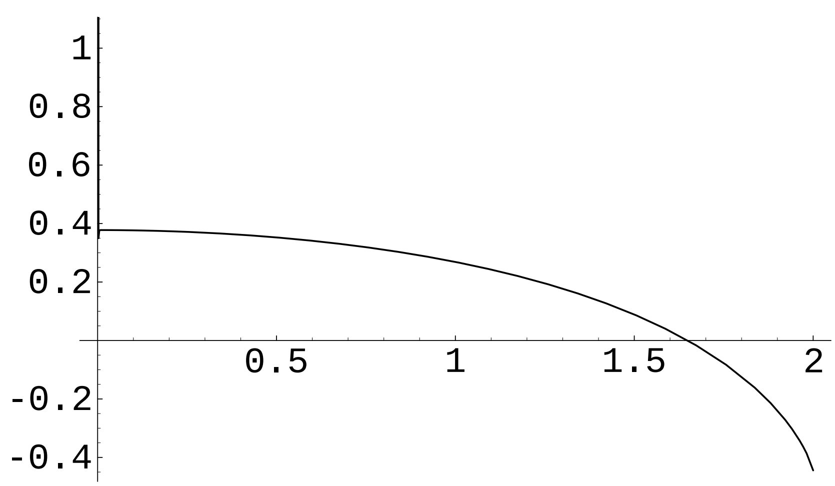
Calculations for the linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,.$$

Corrections to the graviton propagator:



One of the results is the plot for c smoothly changing sign between **UV** and **IR** due to the s-particles decoupling in a models with **broken SUSY.** (Gorbar & Sh., 2003).



This property may provide interface between stable inflation in UV and the FRW-like evolution in IR.

Is the semiclassical approximation reliable?

For MSSM $$H \approx \frac{M_P}{\sqrt{-16\pi b}} \propto 1$$

while for SUSY GUT $H \ll M_P$.

When inflation ends?

At $H = M_* \geq M_F$, because for MSM

$N_{1,1/2,0} = (12, 24, 4) \quad \Longrightarrow \quad c < 0.$

Temperature after the end of inflation

$$M_* \propto M_F \quad \Longrightarrow \quad T \sim \sqrt{M_* \, M_P} = 10^{11} \, GeV \,,$$

a standard estimate for the inflaton models.

If SUSY is broken at the GUT scale

$$M_* \propto 10^{14} \, GeV \qquad \Longrightarrow \quad T \sim 10^{16} \, GeV \,.$$

Inflation does not solve monopole problem.

Inflation favors low-energy SUSY !

II. Massive fields.

Next question. **Why the energy scale H decreases during inflation?**
In the exponential phase Hubble parameter $H(t) = const$ and does not decrease.

Other unclear point.
Using anomaly-induced EA for massive fields is not a correct approximation.

Maybe all difficulties can be solved if **taking masses of the fields into account?**

We developed (I.Sh. & J.Solà, Ph.L.B.,2002) reliable **Ansatz for the EA of massive fields,** based on the Cosmon Model

R.D.Peccei, J.Solà, C.Wetterich, Ph.Lett. B 195(1987)183
and on the "conformization" of GR
S. Deser, Ann. Phys. **59** *(1970) 248*.

The idea is to construct the **conformal formulation** of the SM and use it to derive EA for massive fields.

Conformal formulation of gauge theory.

The conformally non-invariant terms:

$$\sqrt{-g}\; m_s^2\, \varphi^2\,, \qquad\qquad \sqrt{-g}\; m_f\, \bar{\psi}\psi$$

and

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda)\,.$$

Replacing dim. parameters by new scalar χ:

$$m_{s,f} \to \frac{m_{s,f}}{M}\,\chi\,, \quad M_P^2 \to \frac{M_P^2}{M^2}\,\chi^2\,, \quad \Lambda \to \frac{\Lambda}{M^2}\,\chi^2\,.$$

M is related to a scale of conformal symmetry breaking. In the IR $\chi \sim M$.

Massive terms get replaced by Yukawa and $(scalar)^4$ type interactions with χ.

In the gravity sector

$$\mathcal{L}^*_{EH} = -\frac{M_P^2}{16\pi\, M^2}\left\{[R\chi^2 + 6\,(\partial\chi)^2] + \frac{2\Lambda\,\chi^4}{M^2}\right\}$$

in order to provide local conformal invariance.

The new theory is conformal invariant

$$\sigma = \sigma(x), \qquad \begin{cases} \chi \to \chi\, e^{-\sigma}\,, & g_{\mu\nu} \to g_{\mu\nu}\, e^{2\sigma} \\ \varphi \to \varphi\, e^{-\sigma}\,, & \psi \to \psi\, e^{-3/2\,\sigma} \end{cases}$$

The conformal symmetry comes together with a new scalar χ, absorbing conformal degree of freedom. Fixing $\chi \to M$ we come back to original formulation.

The **conformal anomaly** becomes

$$< T > = -\{wC^2 + bE + c\Box R +$$

$$+ \frac{f}{M^2}\,[R\chi^2 + 6(\partial\chi)^2] + \frac{g}{M^4}\,\chi^4\},$$

f, g are β-functions for $1/16\pi G$ and $\Lambda/8\pi G$.

$$f = \sum_i \frac{N_f}{3\,(4\pi)^2}\, m_f^2\,, \qquad \tilde{f} = \frac{16\pi f}{M_P^2}\,,$$

$$g = \frac{1}{2(4\pi)^2} \sum_s N_s\, m_s^4 - \frac{2}{(4\pi)^2} \sum_f N_f\, m_f^4\,,$$

N_f and N_s are multiplicities of the fields.

Anomaly-induced EA

in terms of $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma}$ and $\chi = \bar{\chi} \cdot e^{-\sigma}$

$$\bar{\Gamma} = S_c[\bar{g}_{\mu\nu}, \bar{\chi}] - \frac{3c+2b}{36} \int d^4x\sqrt{-g}R^2$$

$$+ \int d^4x\sqrt{-\bar{g}}\{w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}) + 2b\sigma\bar{\Delta}\sigma$$

$$+ \frac{f}{M^4}\sigma[\bar{R}\bar{\chi}^2 + 6(\partial\bar{\chi})^2] + \frac{g}{M^4}\bar{\chi}^4\sigma\}.$$

This may be seen as a generalization of Renorm. Group. In curved space-time RG corresponds to the scaling $g_{\mu\nu} \to g_{\mu\nu} \cdot e^{-2\tau}$:

$$\Gamma[e^{-2\tau}g_{\alpha\beta}, \Phi_i, P, \mu] = \Gamma[g_{\alpha\beta}, \Phi_i(\tau), P(\tau), \mu]\,,$$

In the leading-log approximation we meet the RG improved classical action of vacuum

$S_{vac}[g_{\alpha\beta}, P(\tau), \mu]\,,$ where $P(\tau) = P_0 + \beta_P\,\tau\,.$

We observe 100% **equivalence** in all terms which do not vanish for $\sigma = \tau = const.$

Cosmological implications of the quantum effects of massive particles

$$S_t = S_{matter} + S^*_{EH} + S_{vac} + \bar{\Gamma}\,.$$

The equation of motion for $\Lambda = 0, \;\; g = 0$

$$a^2\,\ddot{\ddot{a}} + 3\,a\,\dot{a}\,\dddot{a} - \left(5 + \frac{4b}{c}\right)\dot{a}^2\,\ddot{a} + a\,\ddot{a}^2$$

$$-\frac{M_P^2}{8\pi c}\left(a^2\ddot{a} + a\dot{a}^2\right)\,[1 - \tilde{f}\cdot \ln a] \;=\; 0\,,$$

Let us solve $M_P^2 \to M_P^2\,[1 - \tilde{f}\cdot \ln a]$.

$$\dot{\sigma} = H = H_o\sqrt{1 - \tilde{f}\sigma(t)}\,, \qquad H_o = \frac{M_P}{\sqrt{-16b}}\,.$$

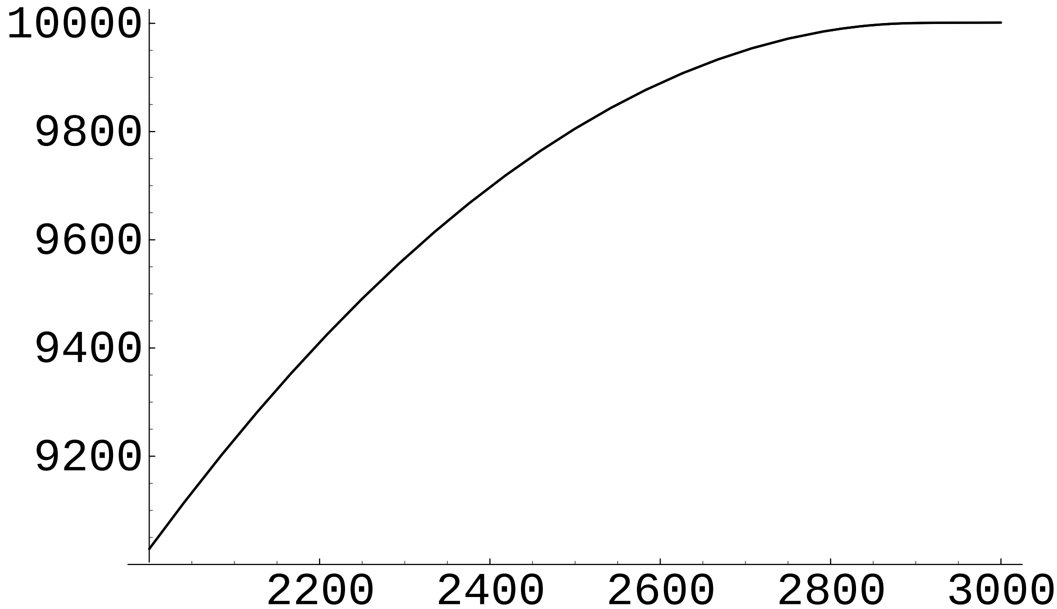
leads to the simple solution

$$\sigma(t) \;=\; H_o\,t \;-\; \frac{H_0^2}{4}\,\tilde{f}\;t^2\,.$$

This formula fits with the numerical solutions with a wonderful 10^{-6} precision!

$\tilde{f} > 0 \;\;\Rightarrow\;\;$ we arrive at **tempered inflation.**

Stability under $\;\sigma \to \sigma + y(t)\;$ holds until **SUSY breaking** point $\;H_f \,=\, M_*\,.$



The relation

$$\sigma(t) = H_o\, t - \frac{H_0^2}{4}\, \tilde{f}\; t^2\,.$$

can be used to evaluate a total number of the inflationary e-folds for different models.

MSSM with SUSY breaking at $M_* \sim 1\, TeV$.

$$\tilde{f} \sim (M_*/M_P)^2 = 10^{-32} \qquad \Longrightarrow$$

the total amount of the e-folds $n_e \sim 10^{32}$.

Indeed, only the last (at most 65) e-folds are relevant for observations.

The main difficulty of the original model:

(Starobinsky, 1980; Vilenkin, 1985)

non-stability under metric perturbations.

Let us see how it looks now.

The consistency with CMB requires
$H(t) \leq 10^{-5} M_P$
in the **last** 65 e-folds of inflation.

GW's emitted **before** are **unobservable**.

I. Creation of gravitational waves.

$H_f = H(t_f) = M_* \;\sim\;$ to $\sigma_f = \sigma(t_f)$.

$H_i = H(t_i) \;\sim\; \sigma_i = \sigma_f - 65$.
Simple calculus gives

$$H_i^2 \,=\, H_f^2 + \frac{65}{48\pi^2} \sum N_i m_i^2 \,.$$

H_i and H_f have the same order of magnitude.

The amplitude of the emitted waves can be consistent with CMBR without fine-tuning of any parameter.

II. Amplification of the gravitational waves.

Approximations for the amplitude $h = h(t, \vec{x})$:
(i) $\quad H(t) \approx const$;
(ii) $\quad \sigma(t) = \ln a(t)$ is **huge** (10^{32} in MSSM).

a) Independent on the class. action $a_1 \int C^2$;
b) **Doesn't depend on** $S_c[g_{\mu\nu}]$
c) Coefficients are constant to $\mathcal{O}(1/\sigma_f)$.

$$b_0 \ddddot{h} + b_1 \dddot{h} + b_2 \ddot{h} + b_3 \dot{h} + b_4 h$$

$$+ n_1 e^{-2\sigma} \nabla^2 \dot{h} + n_2 e^{-2\sigma} \nabla^2 \ddot{h} + n_3 e^{-4\sigma} \nabla^4 h = 0\,,$$

$$b_{0,1,2,3,4} = \Big(w,\, 6Hw,\, 11H^2 w,\, 6H^3 w, -\frac{12H^4 b}{\sigma_f}\Big)$$

This equation leads to a flat spectrum below the trans-Planckian scales.
An elementary analysis indicates
the absence of growing modes.
Again, no special fine-tuning is required.

Similar results:

A.A.Starobinski, Let.Astr.Journ. 9(1983)579.
S.W. Hawking, T. Hertog and H.S. Real, Phys.Rev. D63 (2001) 083504.

Conclusions

1. Modified Starobinsky model is promising candidate to naturally describe inflation. The main advantage is no need for fine-tuning for initial data or for the inflaton potential. Actually, there is no inflaton, inflation is caused by quantum effects.

2. We have a nice link between UV and IR asymptotic states, with a stable $\Lambda > 0$ universe in the last and stable tempered inflation in the former. The gravitational waves can be controlled, also without fine-tuning.

3. Small information is available about the most interesting intermediate state. In order to obtain this information one needs further development of the semiclassical approach. This represents a strong motivation for the future work.