Winding knots and Chern-Simons number in a tachyonic electroweak transition

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1. Introduction:

   cold electroweak baryogenesis

2. Winding knots and Chern-Simons number

Cold ElectroWeak Baryogenesis

Cold:
EW transition due to 'inflaton'(σ)–Higgs(φ) coupling, e.g.

$$\mathcal{L}_{\sigma\varphi} = -\lambda_{\sigma\phi}\sigma^2\varphi^\dagger\varphi$$

time-dependent effective φ-mass

$$\mu^2_{\text{eff}} = \mu^2_\varphi + \lambda_{\sigma\phi}\sigma^2$$

becomes negative at $t = t_c$, $\sigma^2(t_c) = -\mu^2_\varphi/\lambda_{\sigma\phi}$
$\Rightarrow$ tachyonic transition at $\approx$ zero temperature

anomaly in divergence of baryon current

$$\partial_\mu j_B^\mu = 3\,q, \qquad\qquad q = \frac{1}{16\pi^2}\,\mathrm{tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

topological-charge density

tachyonic EW transition with CP violation $\Rightarrow$ baryon asymmetry

$$B(t) = 3\int_0^t dt' \int d^3x\,\langle q(\mathbf{x},t')\rangle$$

García-Bellido, Grigoriev, Kusenko, Shaposhnikov, PRD 60 (1999) 123504; Krauss, Trodden, PRL 83 (1999) 1502 rewrite

$q = \partial_\mu j^\mu_{\mathsf{CS}}$, Chern–Simons current

$N_{\mathsf{CS}} = \int d^3x \, j^0_{\mathsf{CS}}$, Chern–Simons number

$$B(t) = 3\langle N_{\mathsf{CS}}(t) - N_{\mathsf{CS}}(0)\rangle$$

models of ElectroWeak-scale inflation*

numerical studies of tachyonic EW transition**

*Copeland, Lyth, Rajantie, Trodden, PRD; Van Tent, JS, Tranberg, JCAP

**García-Bellido, García-Pérez, González-Arroyo, PRD 67 (2003) 103501; Skullerud, JS, Tranberg, JHEP 0308 (2003) 045 with effective CP violation

$$\mathcal{L}_{\Delta CP} = -\kappa\,\varphi^\dagger\varphi\,\mathrm{tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- estimates of baryon asymmetry using effective chemical potential for N_{CS} and effective sphaleron rate*

- estimates using $\mathcal{L}_{\Delta CP}$ directly in e.o.m.**

*García-Bellido, García-Pérez, González-Arroyo, PRD 69 (2004) 023504

**Tranberg, JS, JHEP 0311 (2003) 016

latter work: SU(2)-Higgs model

$$-\mathcal{L}_{\mathrm{SU(2)H}} = \frac{1}{2g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^\dagger D^\mu\varphi + \mu^2_{\mathrm{eff}}\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2$$

with ΔCP

$$\mathcal{L} = \mathcal{L}_{\mathrm{SU(2)H}} + \mathcal{L}_{\Delta CP}$$

initial conditions: quench

$$\mu^2_{\mathrm{eff}} = +\mu^2,\ t < t_c, \quad = -\mu^2,\ t > t_c$$

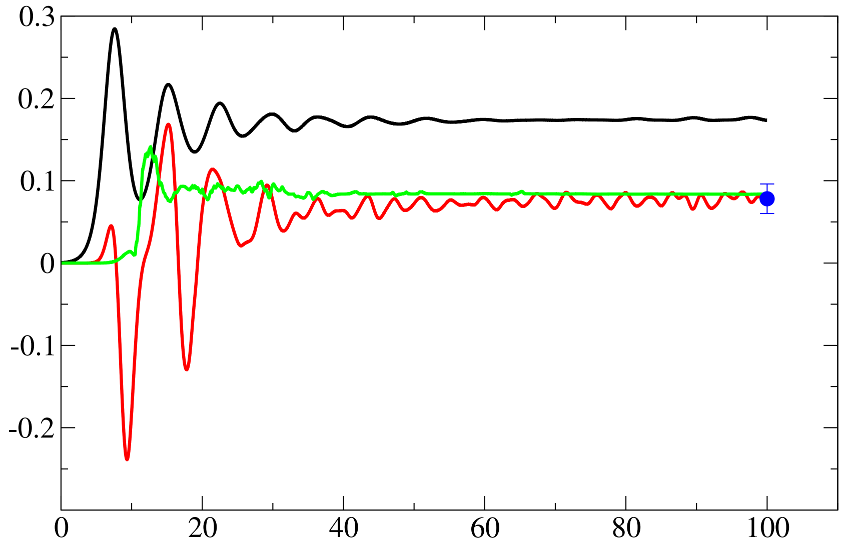
exponential growth of Fourier modes $\varphi_{\mathbf{k}}$ with imaginary frequencies $\omega_k = \sqrt{k^2 - \mu^2}$, i.e. with $\mathbf{k}^2 < \mu^2$

classical approximation*

1. draw initial condition from ensemble at $t < t_c$, $|\mathbf{k}| < \mu$
2. classical evolution for $t > t_c$
3. average over ensemble of initial conditions

(initial ensemble approximated by free vacuum)

JS, Cosmo-01; García-Bellido, García-Pérez, González-Arroyo, PRD 67 (2003) 103501; Tranberg, JS, JHEP 12 (2002) 020; Arrizabalaga, JS, Tranberg, JHEP 0410 (2004) 017 improved results, e.g.



$\langle\varphi^\dagger\varphi\rangle$(black), $\langle N_{\text{CS}}\rangle$(red) and Higgs winding number $\langle N_{\text{w}}\rangle$(green) versus time; $\kappa = 3/16\pi^2 m_W^2$, $m_{\text{H}} = \sqrt{2}\, m_W$

*Tranberg, Lattice '05

$$\begin{aligned}\frac{n_B}{n_\gamma} &= (4 \pm 1)10^{-5}\, k,\; m_{\mathsf{H}} = \sqrt{2}\, m_W \\ &= -(4 \pm 1)10^{-5}\, k,\; m_{\mathsf{H}} = 2\, m_W\end{aligned}$$

$k = 16\pi^2 \kappa\, m_W^2 (= 3$ 'δ_{CP}')

Winding knots and Chern-Simons number

- $\mathcal{L}_{\Delta CP}$ from beyond the SM? Expect more terms in effective lagrangian

- CKM-type CP violation, how to deal with?

- semi-analytic approach?*

study properties of transition (no $\mathcal{L}_{\Delta CP}$):
Chern-Simons densities, Higgs-winding densities, profiles of defects, . . .

interesting in its own right

*previous modelling by
Turok, Zadrozny, PRL 65 (1990) 2331; NPB 358 (1990) 471
Lue, Rajagopal, Trodden, PRD 56 (1997) 1250

winding number

$$\Omega = \frac{1}{\sqrt{\varphi^\dagger\varphi}} \begin{pmatrix} \varphi_d^* & \varphi_u \\ -\varphi_u^* & \varphi_d \end{pmatrix} \in SU(2)$$

$$n_{\text{w}} = -\frac{1}{24\pi^2}\,\epsilon_{jkl}\,\text{tr}\,\partial_j\Omega\Omega^\dagger\,\partial_k\Omega\Omega^\dagger\,\partial_l\Omega\Omega^\dagger$$

$$N_{\text{w}} = \int d^3x\,n_{\text{w}}$$

$N_{\text{w}} =$ integer, topological invariant defined for $\rho \neq 0$,

$$\rho^2 \equiv 2\varphi^\dagger\varphi \quad (= \mu^2/\lambda \equiv v^2 \text{ in classical vacuum})$$

$N_{\text{CS}} \neq$ integer in general

At low energy $N_{\text{CS}} \approx N_{\text{w}}$:

$D_\mu\varphi = (\partial_\mu - iA_\mu)\varphi \approx 0 \Rightarrow A_\mu \approx -i\partial_\mu\Omega\Omega^\dagger$, $\varphi \approx \Omega\varphi_0$

in classical (gauge-equivalent) vacua: $N_{\text{CS}} = N_{\text{w}}$

usual suspects

sphaleron:
unstable static solution of the e.o.m. with localized energy; minimum of energy barrier between two (gauge-equivalent) classical vacua; $N_{\mathrm{CS}} = 1/2$, N_{w} not defined because of zero in ρ; in an ideal sphaleron-transition N_{CS} increases from 0 to 1, N_{w} jumps from 0 to 1 at the sphaleron*

texture:
without gauge field: configuration with $\rho = v$, $N_{\mathrm{w}} = 1$, localized gradient energy; shrinks and decays under e.o.m. into outgoing waves with $N_{\mathrm{w}} \to 0$
with gauge gauge field: $N_{\mathrm{w}} - N_{\mathrm{CS}} = 1$; small specimens decay by shrinking with $\Delta N_{\mathrm{w}} = -1$, $\Delta N_{\mathrm{CS}} = 0$; large specimens decay by spreading with $\Delta N_{\mathrm{w}} = 0$, $\Delta N_{\mathrm{CS}} = 1$

* numbers modulo 1

new(?): half-knots

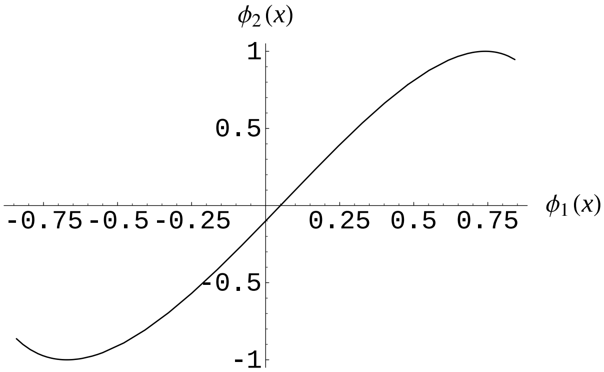
near-zeros in ρ give peaks in n_{w} with $\int_{\text{peak}} n_{\text{w}} \approx \pm 1/2$

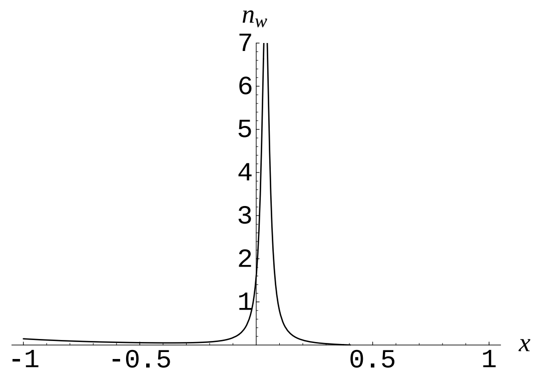
1+1 D, one-component φ:

$$\Omega = \frac{\varphi}{\sqrt{\varphi^*\varphi}} = e^{i\omega} \in U(1)$$

$$n_{\mathsf{W}} = -\frac{1}{2\pi}\, i\partial_x\Omega\,\Omega^* = \frac{1}{2\pi}\,\partial_x\omega, \qquad N_{\mathsf{W}} = \int dx\, n_{\mathsf{W}}$$

example: $\varphi = \phi_1 + i\phi_2$, $\phi_1 = \sin(x)$, $\phi_2 = \sin(2x - 0.1)$





parametric plot for $-1 < x < 1$; $\int_{\mathsf{peak}} n_{\mathsf{W}} \approx 0.5$

similar in 3+1 D

near-zeros of Higgs doublet (all its four real components small) gives peaks in $n_{\rm w}$ with $\int_{\rm peak} n_{\rm w} \approx \pm 1/2$

linear approximation

$$\varphi_\alpha = c_\alpha + d_{\alpha k} x^k \to \int d^3x\, n_{\rm w} = \pm 1/2 \quad \text{exactly}$$

flip of sign possible when ρ goes through zero

with gauge field: $\int_{\rm peak} n_{\rm w} - \int_{\rm peak} n_{\rm CS} \approx \pm 1/2$
expect decays via $\Delta \int_{\rm peak} n_{\rm w} = \pm 1/2$, $\Delta \int_{\rm peak} n_{\rm CS} = 0$, or *vice versa*

some results of simulation

$m_{\mathsf{H}} = \sqrt{2}\, m_W$ ($m_{\mathsf{H}} = \sqrt{2}\, \mu = \sqrt{2\lambda}\, v$, $m_W = \frac{1}{2}\, gv$)

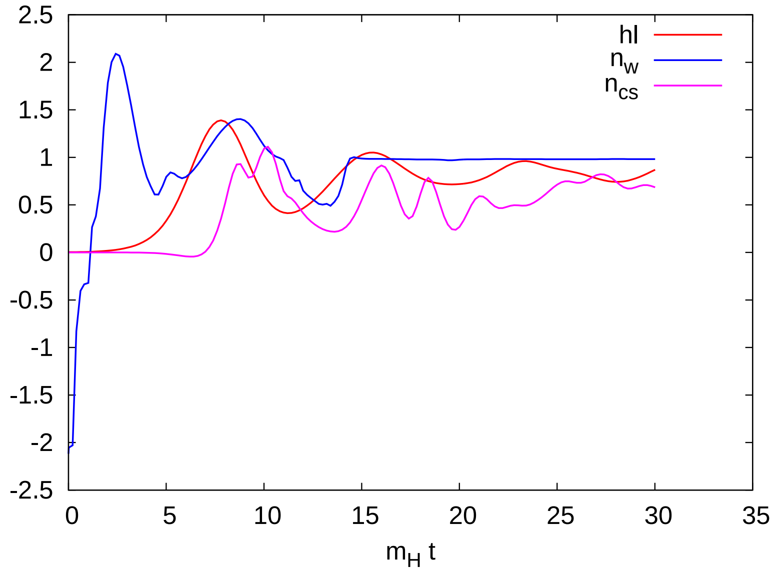
volume: $L^3 = (21\, m_{\mathsf{H}}^{-1})^3$

boundary conditions: periodic

initial conditions: quench

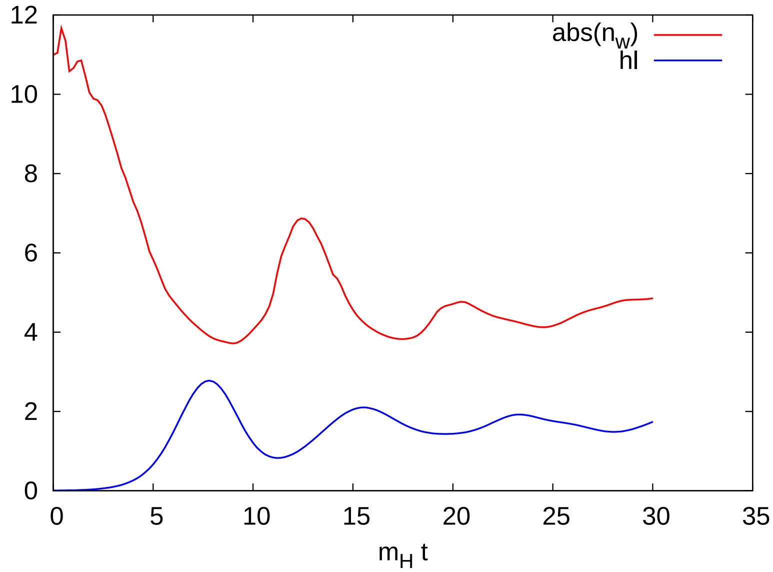
60^3 lattice, spacing: $a = 0.35\, m_{\mathsf{H}}^{-1}$

results for typical i.c. 'nr. 30'



time-dependence of hl $\equiv \overline{\rho^2/v^2}^{\mathrm{vol}}$, N_{w} and N_{CS}

A_μ (sourced by φ) substantial only after $m_{\mathrm{H}}t \approx 7$

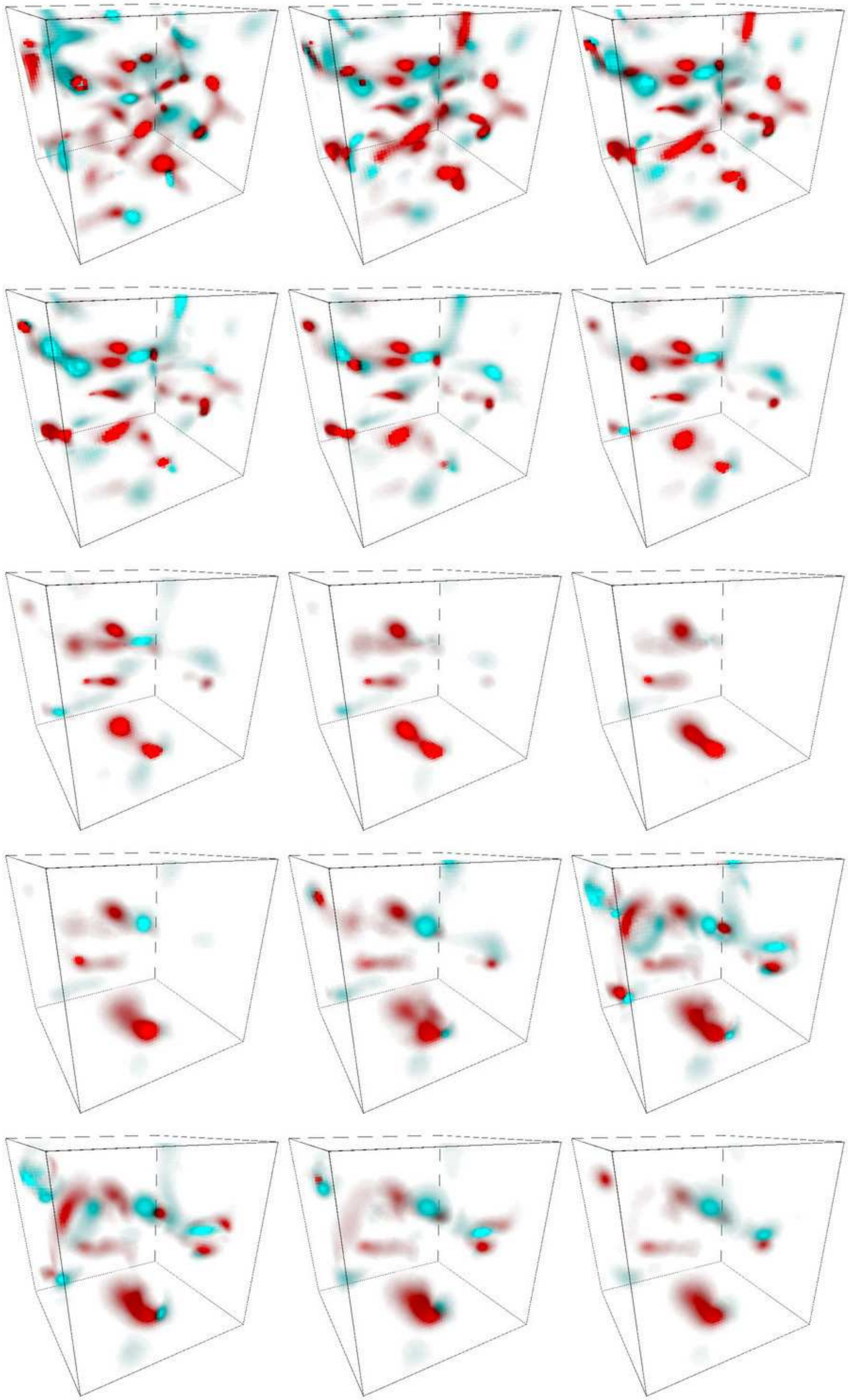


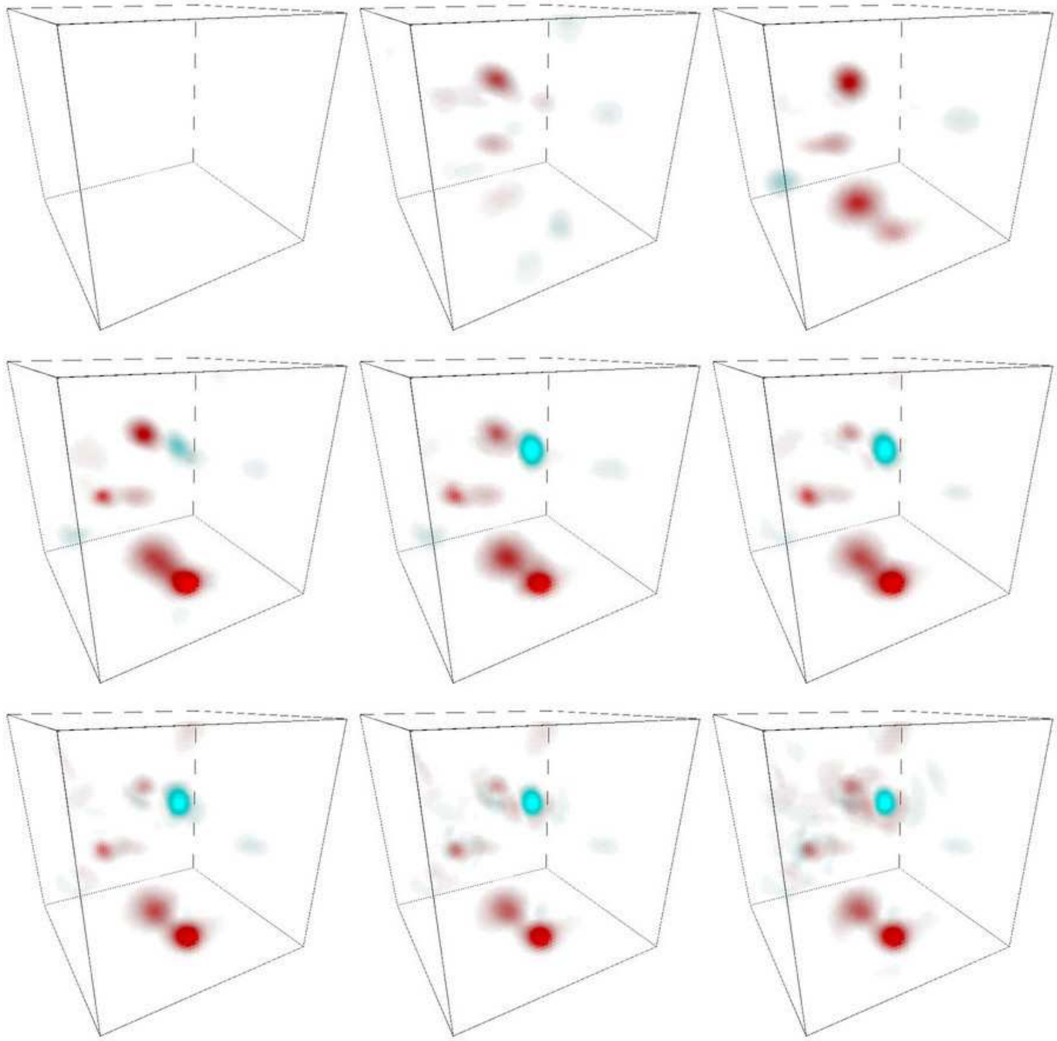
$\int d^3x\, |n_\mathrm{W}|$ and hl

large winding density n_W when hl is small

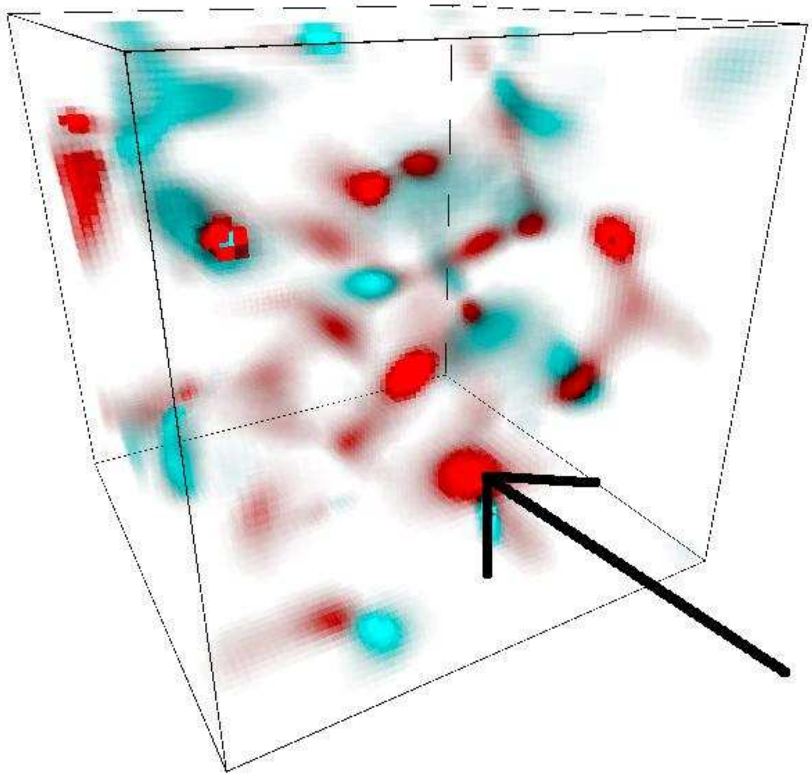
1st, 2nd, 3rd, ... generation of winding blobs next: 3D plots of n_{W} at times $tm_{\mathsf{H}} = 1, 2, \ldots, 15$

followed by n_{CS} at $tm_{\mathsf{H}} = 7, \ldots, 15$



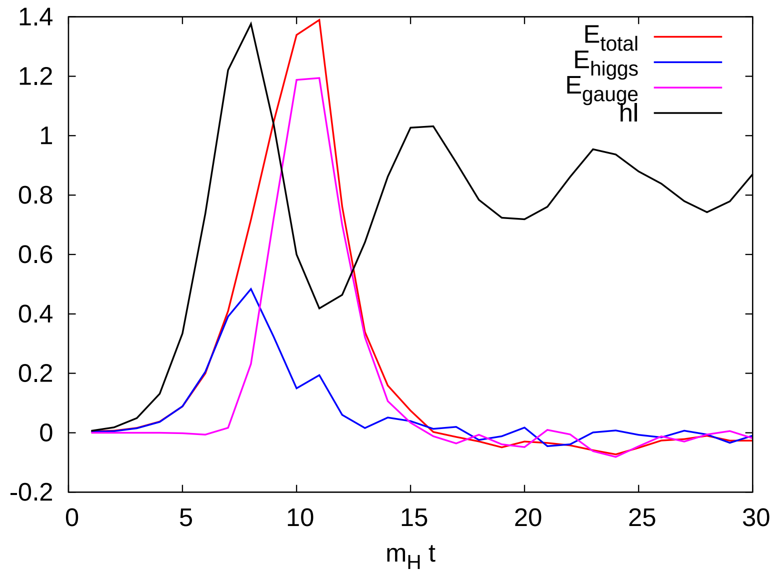


Chern-Simons density correlated with winding density

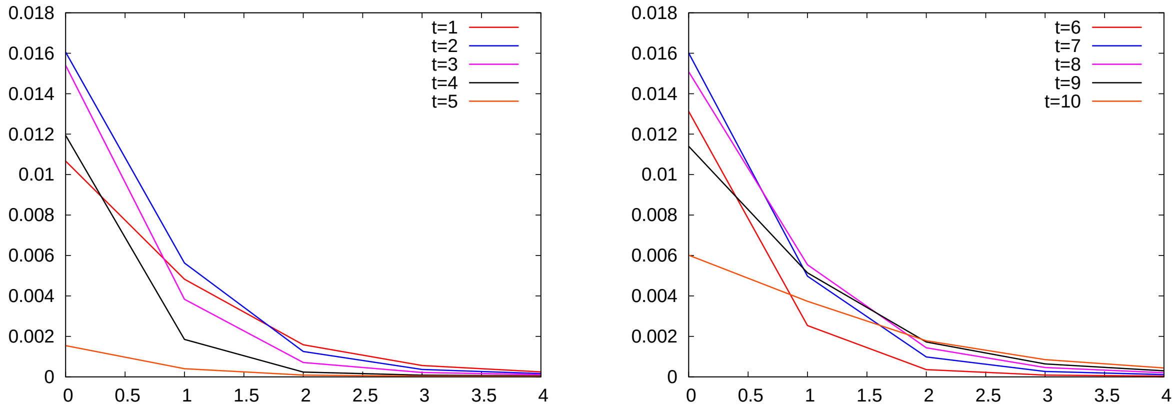


select an early blob (here $tm_{\mathsf{H}} = 1$)

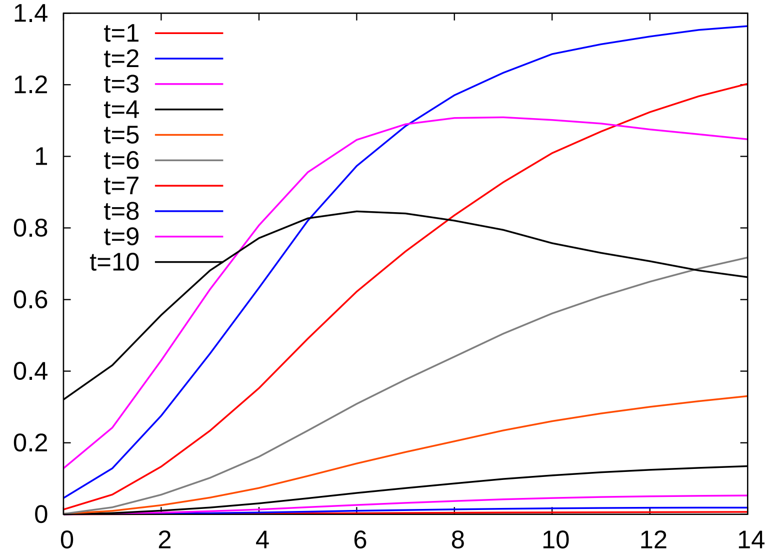
ball of radius $2.1\, m_{\rm H}^{-1} = 6a$ measured from maximum winding density



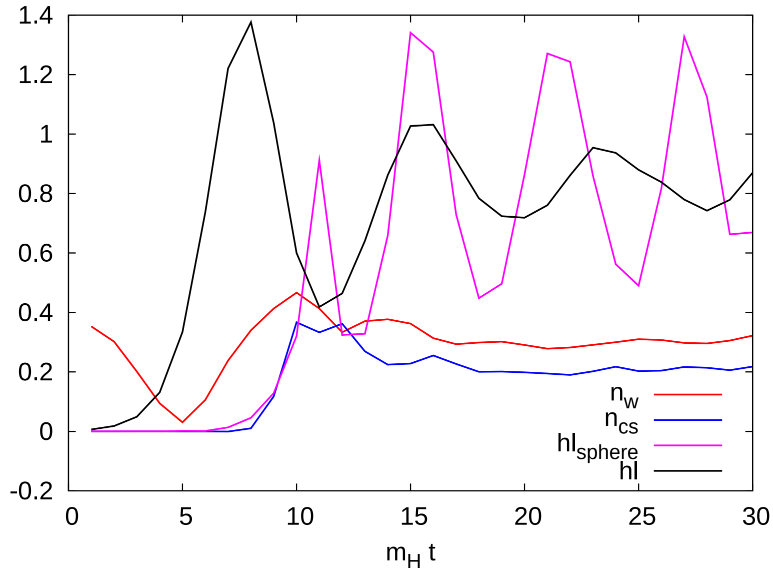
energy in the ball: $E_{\rm total} = E_{\rm gauge} + E_{\rm Higgs}$ with average energy subtracted, in units of the sphaleron energy, and hl



n_{W} versus r for $tm_{\mathrm{H}} = 1, \ldots, 10$, ($r$ in lattice units, $3a \approx m_{\mathrm{H}}^{-1}$); continued spreading after $tm_{\mathrm{H}} = 10$



ρ^2/v^2 versus r at various times; continued approach to non-zero const. after time 10



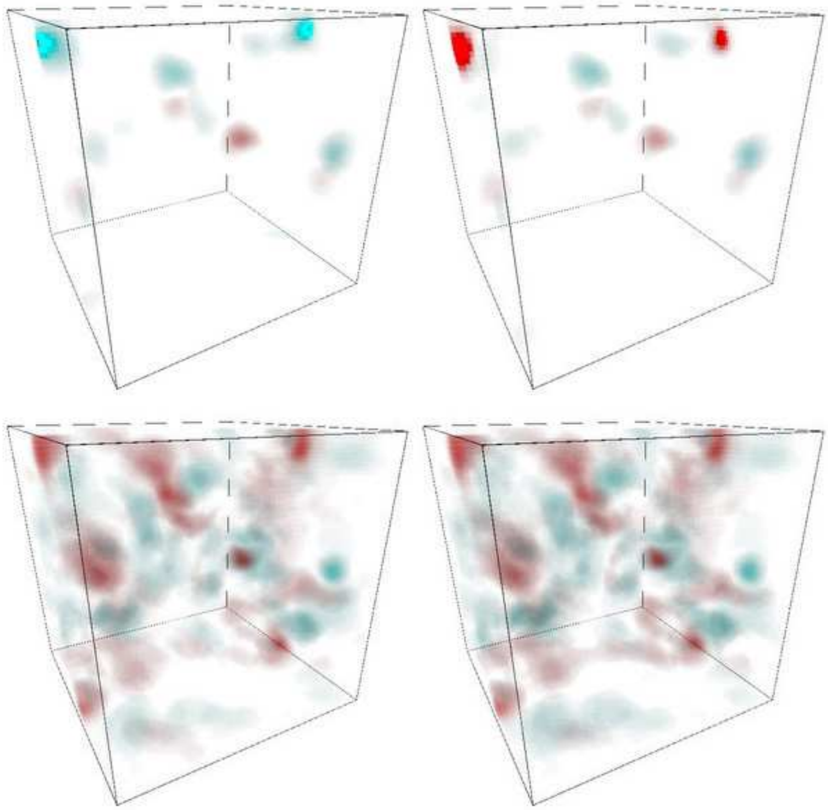
$\int_{\text{ball}} n_{\text{W}}$, $\int_{\text{ball}} n_{\text{CS}}$, $\overline{\rho^2/v^2}^{\text{ball}}$ and hl half-knot forms, nearly changes into an anti-half-knot at $tm_{\mathsf{H}} = 5$, then decays by spreading to an equilibrium gauge-half-knot, by adjustment of the gauge field's Chern-Simons number

local contribution $\approx +1/2$ to ΔN_{CS}

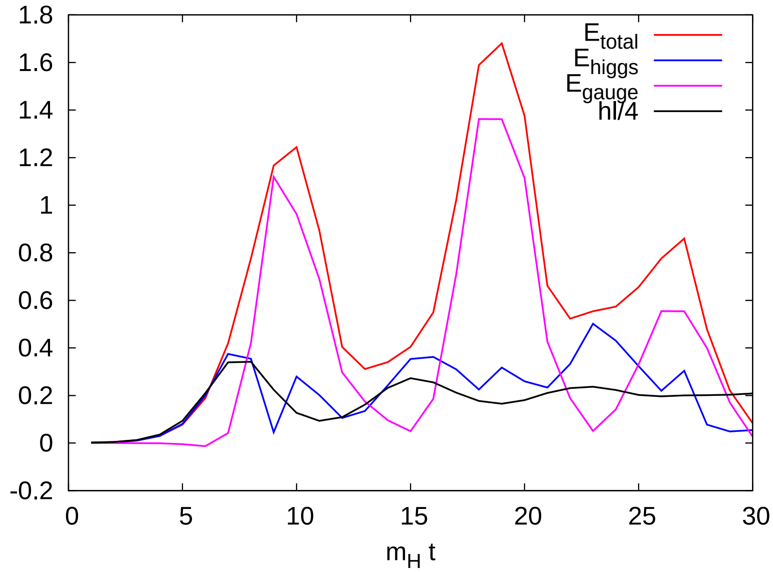
i.c. ‘nr. 31’

late transition between $tm_{\mathsf{H}} = 23$ and 24

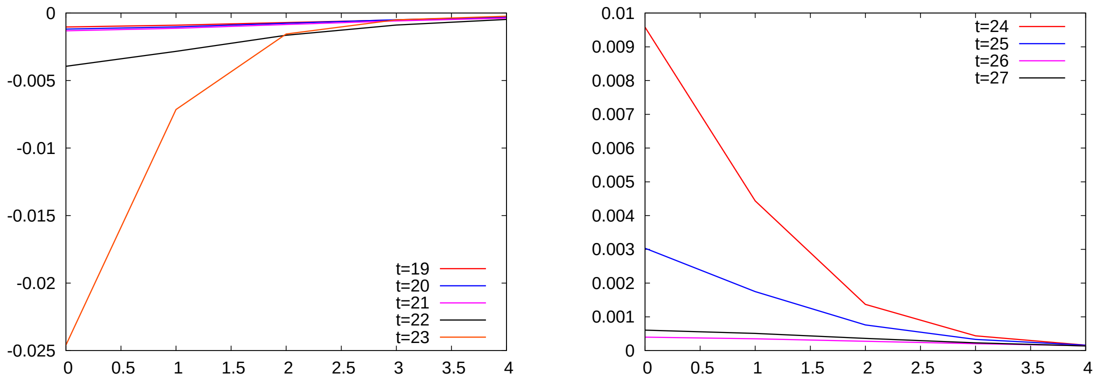
next: 3D plot shows change of sign in n_{W} but not in n_{CS} in blob near ‘ceiling’ of box



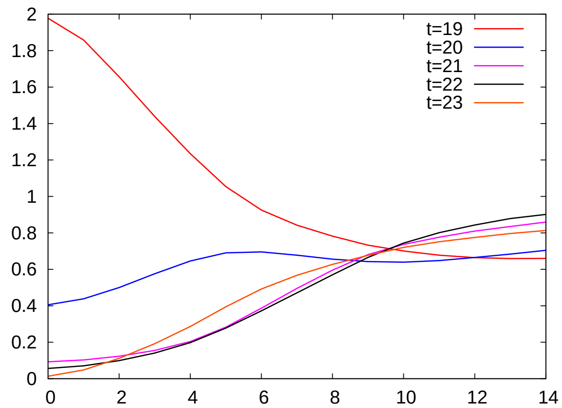
n_{W} (upper) and n_{CS} (lower) at time 23 (L) and 24 (R)

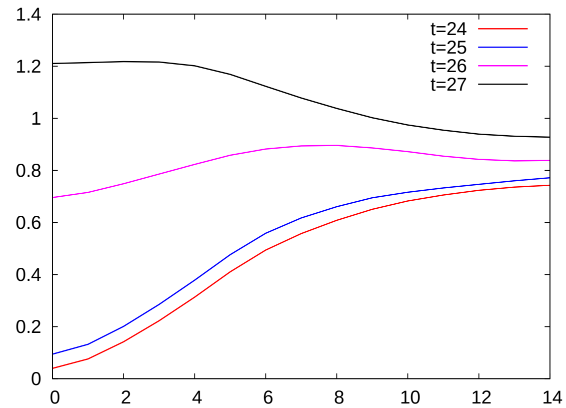


energy in ball with radius $2.1\, m_{\mathrm{H}}^{-1}$

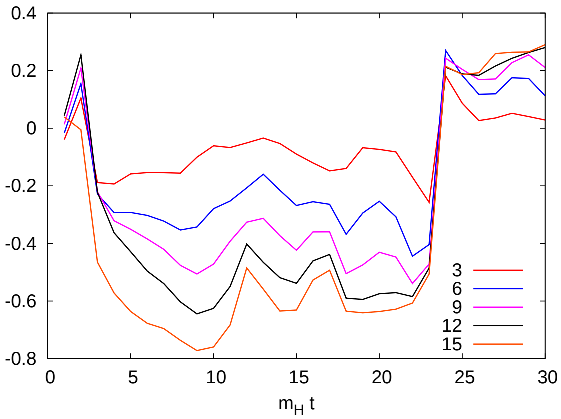


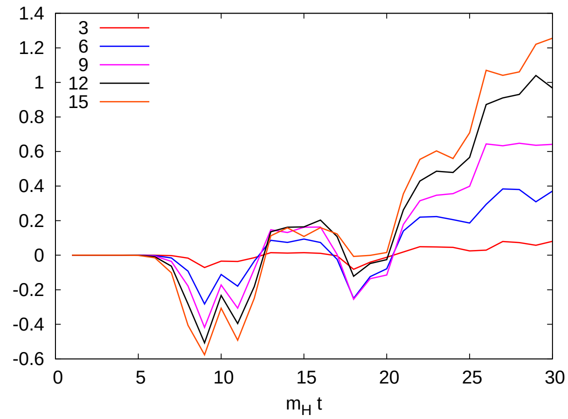
n_{W} versus r at various times





ρ^2/v^2 versus r at various times





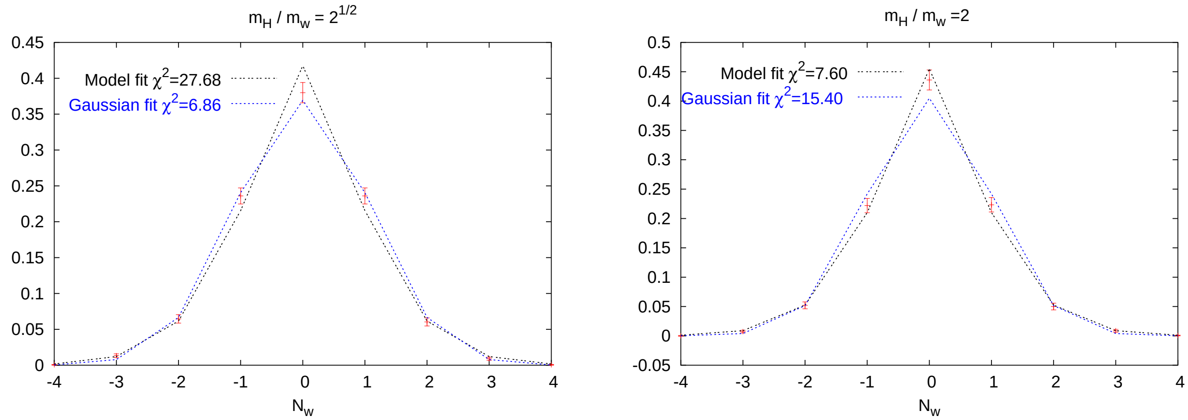
$\int_{\mathrm{ball}} n_{\mathrm{W}}$ (L) and $\int_{\mathrm{ball}} n_{\mathrm{CS}}$ (R), balls of radius $3a, \ldots, 15a$

it's a sphaleron transition

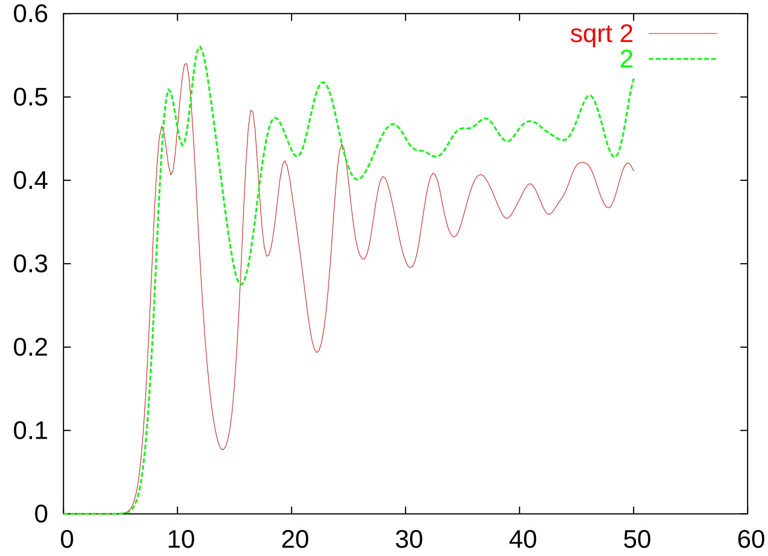
locally $\Delta N_{\mathsf{W}} \approx +1$, $\Delta N_{\mathsf{CS}} \approx +1$

some more results:

N_{w} distribution at time $tm_{\text{H}} = 50$ is nearly gaussian*



'Model' is the distribution $I_{N_{\text{w}}}(\sigma)e^{-\sigma}$, derived from a Poisson-like generation of local winding centers contributing ± 1 to N_{w}; analogous model with local $\pm 1/2$ contributions is closer to 'Gaussian'



$\langle N_{\mathrm{CS}}(t)^2 \rangle$ versus tm_{H} for $m_{\mathrm{H}}/m_W = \sqrt{2}$ and 2

2nd . . . generation of N_{w} is smaller & effective sphaleron rate larger for the case $m_{\mathrm{H}}/m_W = 2$

Conclusion

individual trajectories ('histories') look messy

generation of Chern-Simons number (baryon number) appears to go through localized blobs

could recognize half-knots, sphaleron transitions and texture-like decays

appearence of half-knots when $\overline{\rho^2}^{\rm vol}$ is small, 1st, 2nd, . . . generations

1st generation Higgs-field half-knots seem to be important in a rapid tachyonic transition (quench), gauge field important later in the transition

- many ?? raised/left