``Welcome to the dark side of the world.”

A unified approach to scaling solutions and its applications to dark energy

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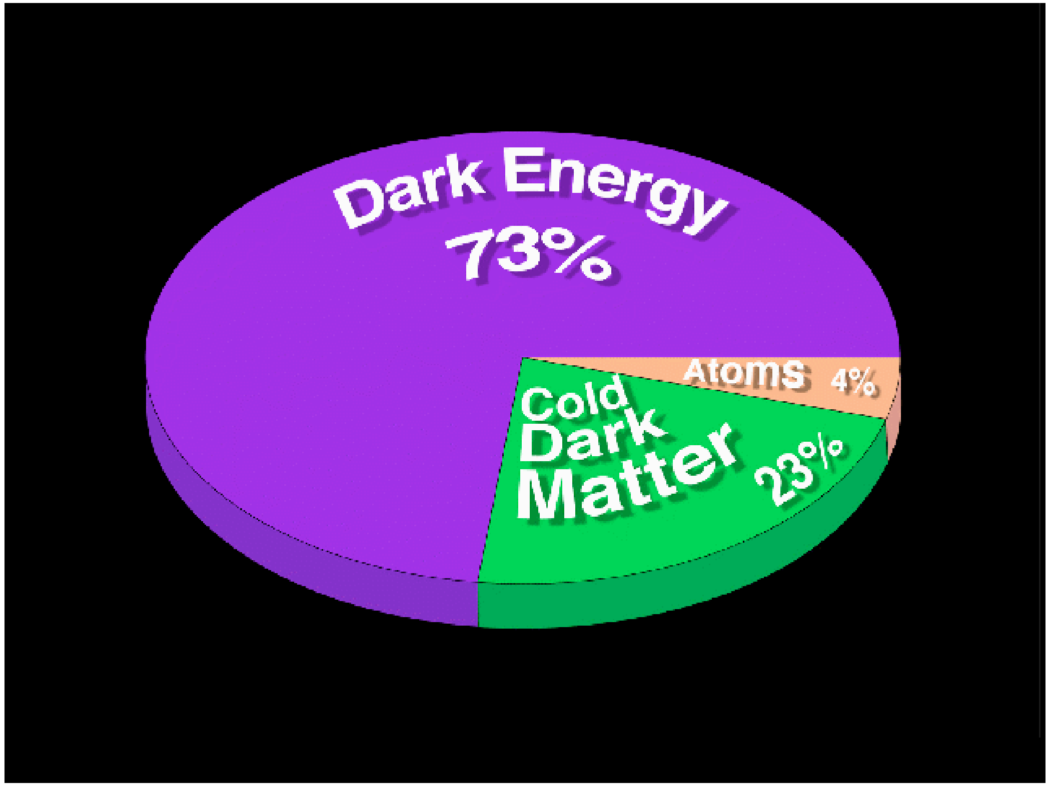
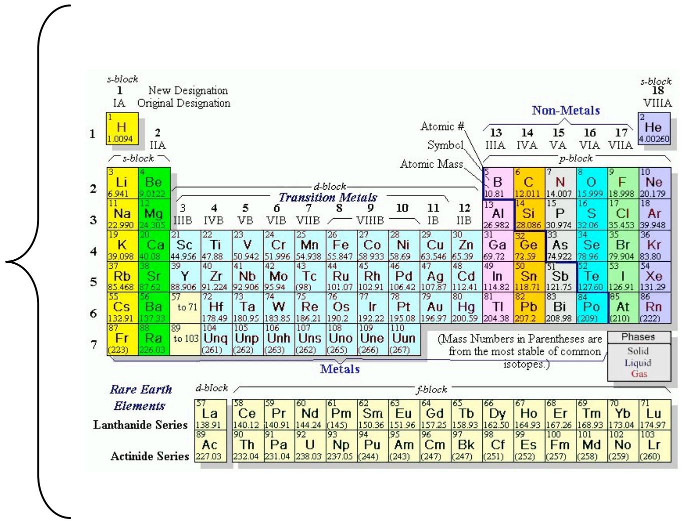
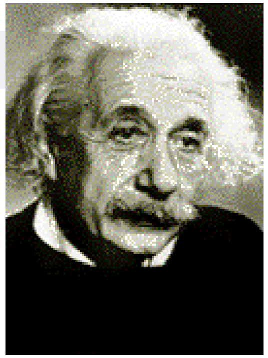
Dark energy

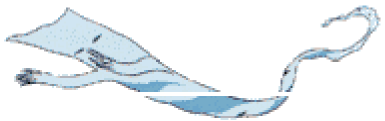
Observations suggest that more than 70% of the energy density of the current universe is dark energy that gives an accelerated expansion.





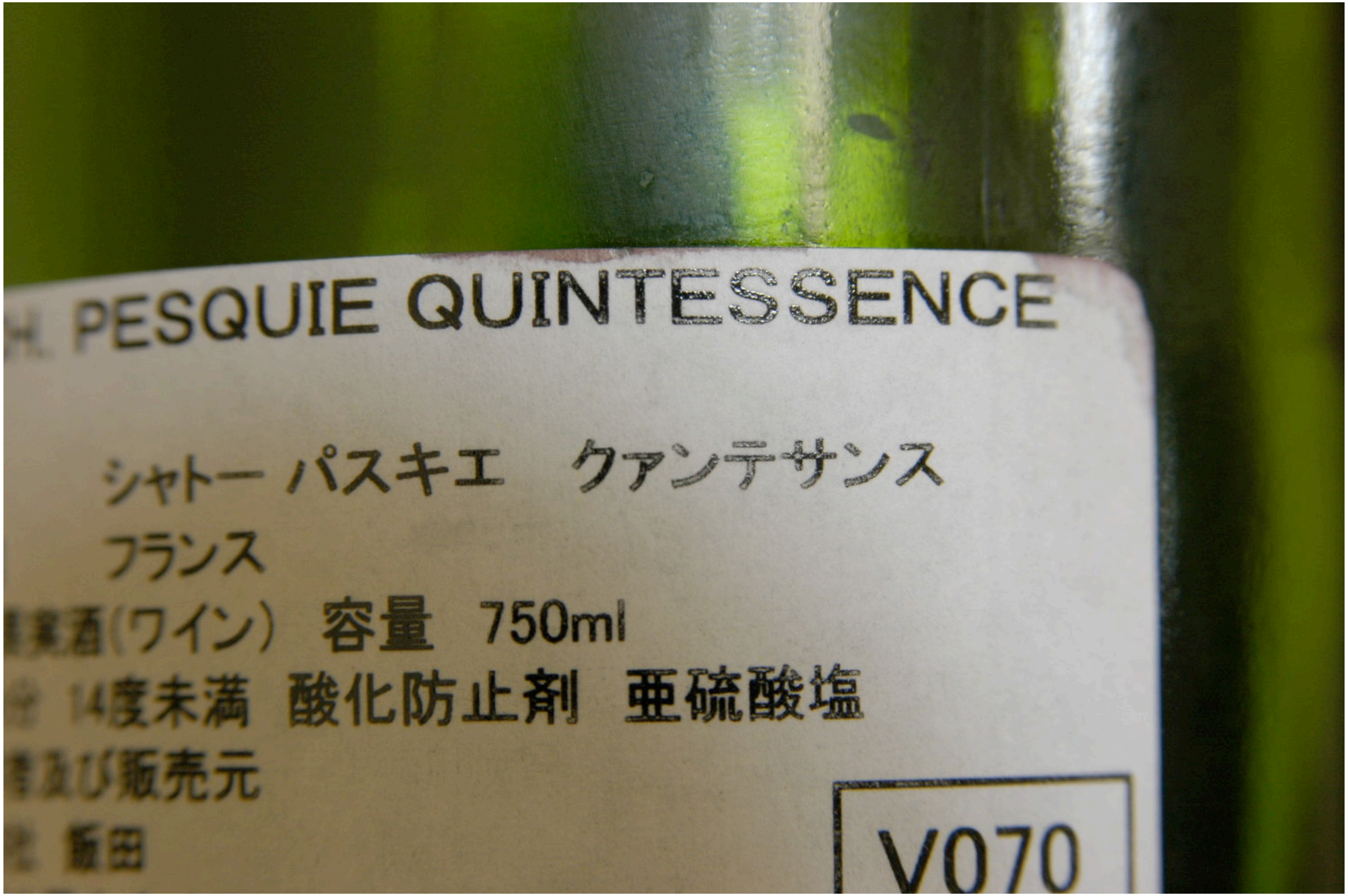


Models of dark energy

- Cosmo-illogical constant
- Quintessence (Potential energy of scalar fields)
- K-essence (Kinematic energy of scalar fields)
- Tachyon (Energy density : $\rho = V(\phi)/\sqrt{1-\dot{\phi}^2}$)
-  (Equation of state : w < −1)
- Ghost condensate etc.

➡ **Many scalar-field models were proposed to explain the origin of dark energy.**


La Quintessence
du
Château Pesquié
1998


PESQUIE QUINTESSENCE
シャトー パスキエ　クァンテサンス
フランス
(ワイン)　容量　750ml
14度未満　酸化防止剤　亜硫酸塩
及び販売元
V070

Cosmological scaling solutions

In constructing viable models of dark energy, it is convenient if we know cosmological scaling solutions.

Scaling solutions: $\rho_\varphi \propto \rho_m$ (Dark energy density is proportional to fluid energy density)

For a minimally coupled scalar field, the exponential potential

$$V = V_0 \exp(\lambda\varphi)$$

corresponds to scaling solutions. In this case

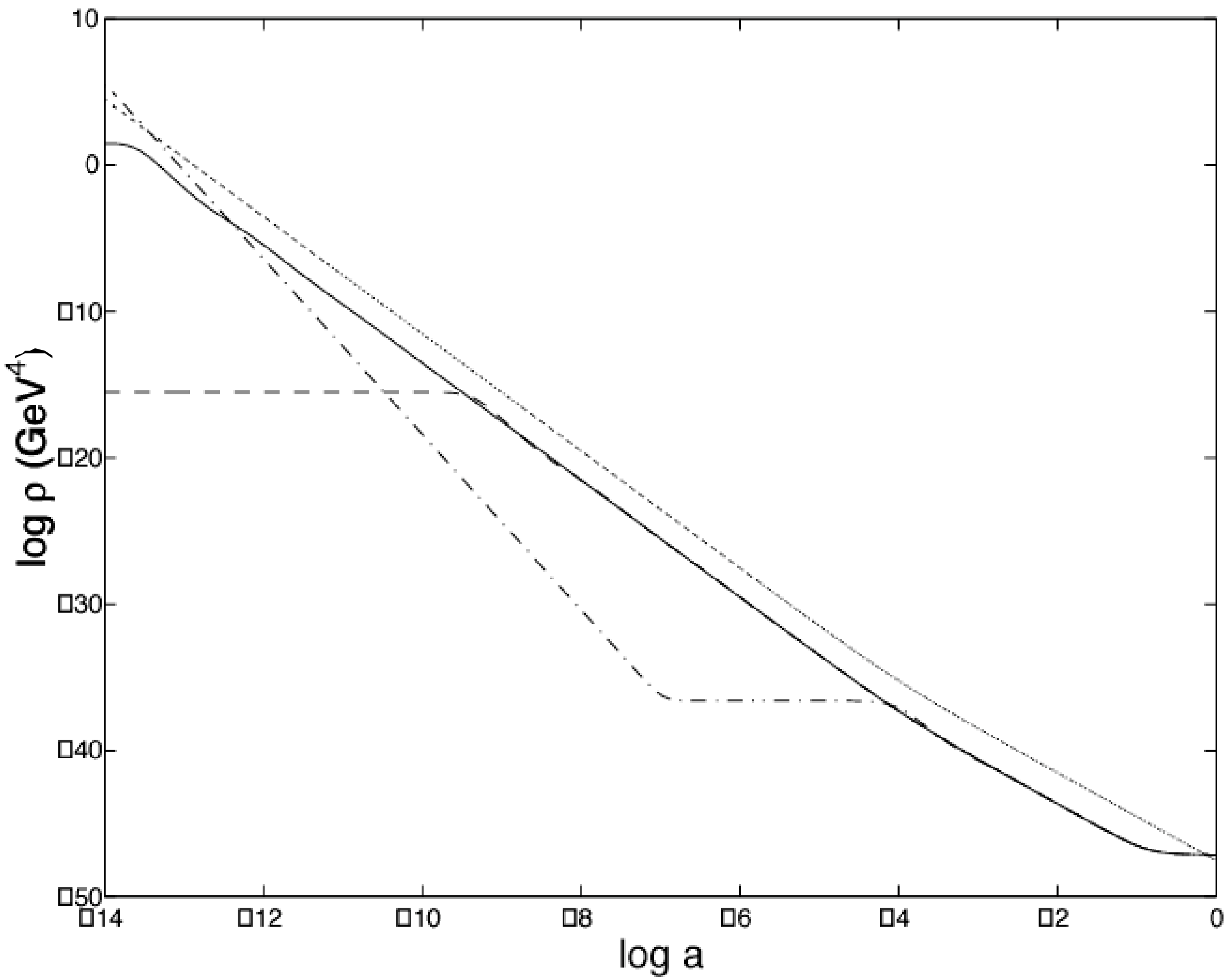
(i) $\lambda^2 > 3(1 + w_m)$ $\Rightarrow$ $w_\varphi = w_m$ (scaling solutions)

(ii) $\lambda^2 < 3(1 + w_m)$ $\Rightarrow$ $w_\varphi = -1 + \lambda^2/3$ and $\Omega_\varphi = 1$

It is possible to explain the late-time acceleration if the slope of the potential changes at late-times: e.g., $V = V_0[\exp(\alpha\varphi) + \exp(\beta\varphi)]$

Barreiro, Copeland and Nunes, PRD61, 127301 (2000)

$$V = V_0[\exp(\alpha\varphi) + \exp(\beta\varphi)]$$



α =20, β=0.5

A unified approach to scaling solutions

We wish to know the condition for a scalar-field Lagrangian for the existence of scaling solutions.

Let us start with a general Lagrangian:

$$S=\int d^4x\sqrt{-g}\left[R/2+p(X,\varphi)\right]+S_m \qquad X=-g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/2$$

This includes (coupled)-quintessence, phantom, ghost condensate, tachyon, k-essence, …

We consider a general cosmological background:

$$H^2=\beta^2\rho_T{}^n$$

(H: Hubble rate)

$n=1$: General Relativity (GR)

$n=2$: Randall-Sundrum (RS) braneworld

$n=2/3$: Gauss-Bonnet (GB) braneworld

Existence of scaling solutions

In FRW background we have

Scalar field: $\dot{\rho} + 3H(1 + w_{\varphi})\rho = -Q\rho_m\dot{\varphi}$

Fluid: $\dot{\rho}_m + 3H(1 + w_m)\rho_m = Q\rho_m\dot{\varphi}$

Q: coupling between dark energy and fluid (dark matter)

Scaling solutions: $\rho / \rho_m = const$

$$\Rightarrow \quad \frac{d\log\rho}{dN} = \frac{d\log\rho_m}{dN} = -3(1 + w_s)$$

where $w_s = w_m + \Omega_{\varphi}(w_{\varphi} - w_m)$ and $N = \log(a)$

and

$$\frac{d\varphi}{dN} = \frac{3\Omega_{\varphi}}{Q}\left(w_m - w_{\varphi}\right) = const$$

Lagrangian for the existence of scaling solutions

$$X = \frac{H^2}{2}\left(\frac{d\varphi}{dN}\right)^2 \propto H^2 \propto \rho^n \quad \Rightarrow \quad \frac{dX}{dN} = -3n(1+w_s)X$$

By using

$$\frac{\partial \log p}{\partial X}\frac{dX}{dN} + \frac{\partial \log p}{\partial \varphi}\frac{d\varphi}{dN} = -3(1+w_s)$$

we find

$$n\frac{\partial \log p}{\partial \log X} - \frac{1}{\lambda}\frac{\partial \log p}{\partial \varphi} = 1$$

$$\Rightarrow \quad p = X^{1/n} g(Xe^{n\lambda\varphi})$$

F. Piazza and S.T. (2004)
S.T. and M. Sami (2004)

where $\quad \lambda = Q\dfrac{1+w_m-\Omega_\varphi(w_m-w_\varphi)}{\Omega_\varphi(w_m-w_\varphi)}$

g : arbitrary function

One can also derive

$\boxed{w_\varphi, \Omega_\varphi}$

for scaling solutions.

Application to dark energy models

(A) Quintessence (sometimes known as 'French wine')

When p is written in the form: $p = f(X) - V(\varphi)$, we obtain

$$p = c_1 X^{1/n} - c_2 e^{-\lambda\varphi}$$

In the case of GR (n=1), this corresponds to an exponential potential with a canonical field.

Defining a new field: $\phi = \exp[(n-1)\lambda\varphi/2]$, we get

$$p = \frac{4c_1}{(n-1)^2\lambda^2}\tilde{X} - c_2\phi^{-2/(n-1)}$$

where $\tilde{X} = \dot{\phi}^2/2$

(corresponding to the choice $g = c_1 Y^{1-1/n} - c_2 Y^{-1/n}$)

Canonical field with potential

$$V(\phi) = c_2\phi^{-2/(n-1)}$$

$\propto \phi^{-2}$ for RS braneworld

$\propto \phi^{6}$ for GB braneworld

(B) Tachyon

The Lagrangian for tachyon is

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2}$$

Substituting $g(Y) = -cY^{-1/n}\sqrt{1-2Y}$ for $p = X^{1/n} g(Xe^{n\lambda\varphi})$, we get

$$p = -ce^{-\lambda\varphi}\sqrt{1-2Y} \qquad \text{where} \quad Y = Xe^{n\lambda\varphi}$$

Defining $\phi = (2/n\lambda)\exp(n\lambda\varphi/2)$, we find

$$p = -c\left(n\lambda\phi/2\right)^{-2/n}\sqrt{1-\dot{\phi}^2}$$

Therefore the tachyon has scaling solutions for the potential

$$V(\phi) = \phi^{-2/n} \qquad \propto \phi^{-2} \quad \text{for GR}$$

When Q=0, $w_m < 0$ for the existence of scaling solutions.
Scaling solutions exist for $w_m > 0$ in the presence of the coupling Q.

See: B. Gumjudpai, T. Naskar, M. Sami, S.T., JCAP 0506, 007 (2005)

(c) Dilatonic ghost condensate

Phantom is plagued by quantum vacuum instability, but this is overcome by accounting for higher-order kinematic terms:

$$p = -X + ce^{\lambda\varphi} X^2$$

F. Piazza and S.T. (2004)

This is obtained by substituting $g(Y) = -1 + cY$ for $p = X^{1/n} g(X e^{n\lambda\varphi})$.

For scaling solutions, we obtain

$$cY = \frac{3(1-w_m{}^2) - 2Q(Q+\lambda)}{3(1-3w_m)(1+w_m)} \qquad \Omega_\varphi = \frac{3(1+w_m)[1+w_m - Q(Q+\lambda)]}{(\lambda+Q)^2(1-3w_m)}$$

The vacuum is stable at quantum level for $cY > 1/2$.

$\Longrightarrow$ $Q(Q+\lambda) < 3/4$ when $w_m = 0$

Accelerated expansion occurs for $Q > \lambda/2$ when $w_m = 0$

The coupling Q can lead to a viable scaling solution.

Fixed points

B. Gumjudpai, T. Naskar, M. Sami, S.T., JCAP 0506, 007 (2005)

Defining

$$x = \frac{\dot{\varphi}}{\sqrt{6}H} \qquad y = \frac{e^{-\lambda\varphi/2}}{\sqrt{3}H}$$

we obtain the differential equations for $p = X^{1/n} g(X e^{n\lambda\varphi})$.

We find fixed points by setting $dx/dN = 0$, $dy/dN = 0$.

→ One can study the stability of fixed points as well.

We found the following results:

$(i)\, w_\varphi > -1$ → The final stable attractor is either a scaling solution or a scalar-field dominant universe with $\Omega_\varphi = 1$

$(ii)\, w_\varphi < -1$ → The final stable attractor is a scalar-field dominant universe with $\Omega_\varphi = 1$

In this case scaling solutions are unstable.

Conclusion

We derived the condition for scalar-field Lagangian
for the existence of scaling solutions:

$$p = X^{1/n} g(X e^{n\lambda\varphi})$$

This includes a wide variety of dark energy models:
(coupled)-quintessence, phantom, ghost condensate,
tachyon, k-essence, …

We applied the above Lagrangian to dark energy
and derived the effective potentials, the fixed points etc.

→ Useful for the construction of viable dark energy models