Dynamical Dark Energy

Constraints, Forecasts, and Milestones for Future Progress

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28 Aug. 2005

Outline

Concerning constraints on the dark energy equation of state, my goal is to answer the question,

How good is good enough?

- I will begin with a quick summary of the current state of affairs.
- Next, three milestones for measuring progress in constraining the dark energy will be discussed.
- Finally, forecasts will be used to assess our chances of reaching these milestones in the near future.

The state of dark energy constraints today

Available data:

CMB:	SN IA:	P(k):	H_0:
•ACBAR •BOOMERANG •CAPMAP •CBI •DASI •WMAP	“gold set” (Riess, et. al., 2004) compiles current data	•2dF •Lensing •Lyman α •SDSS	•HST Key Project

Equation of state constraints

Data sets used:

- CMB: 1 year of data from WMAP
- SNe IA: “gold set” from Riess, et. al., 2004.
- P(k) from SDSS galaxy survey

Cosmological parameters:

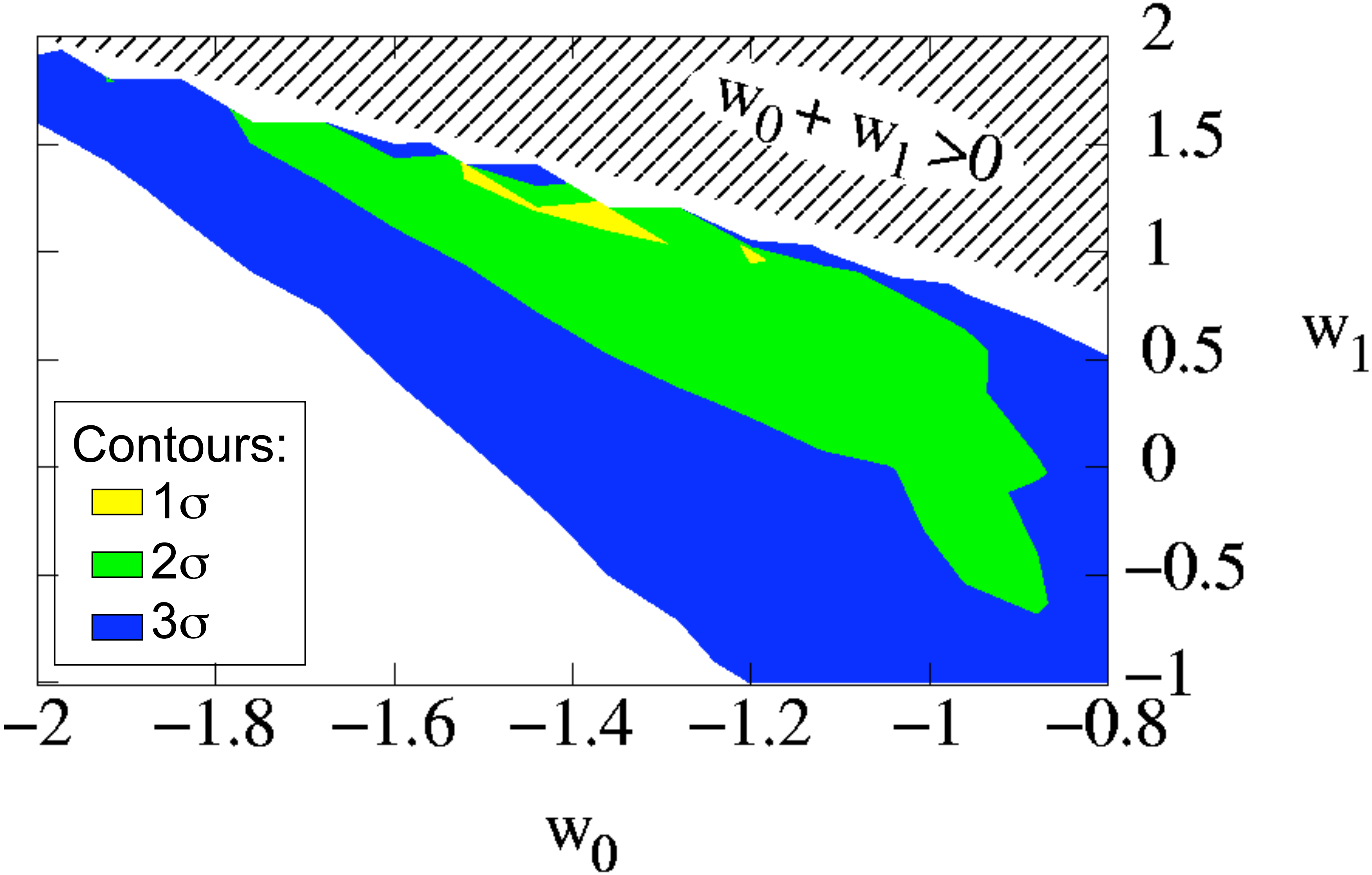
h, $\Omega_m h^2$, $\Omega_b h^2$, τ, n_s, A, w_0, w_1

Dark energy parameterization:

$$w(z) = \begin{cases} w_0 + w_1 z & \text{if } z < 1 \\ w_0 + w_1 & \text{if } z \geq 1. \end{cases}$$

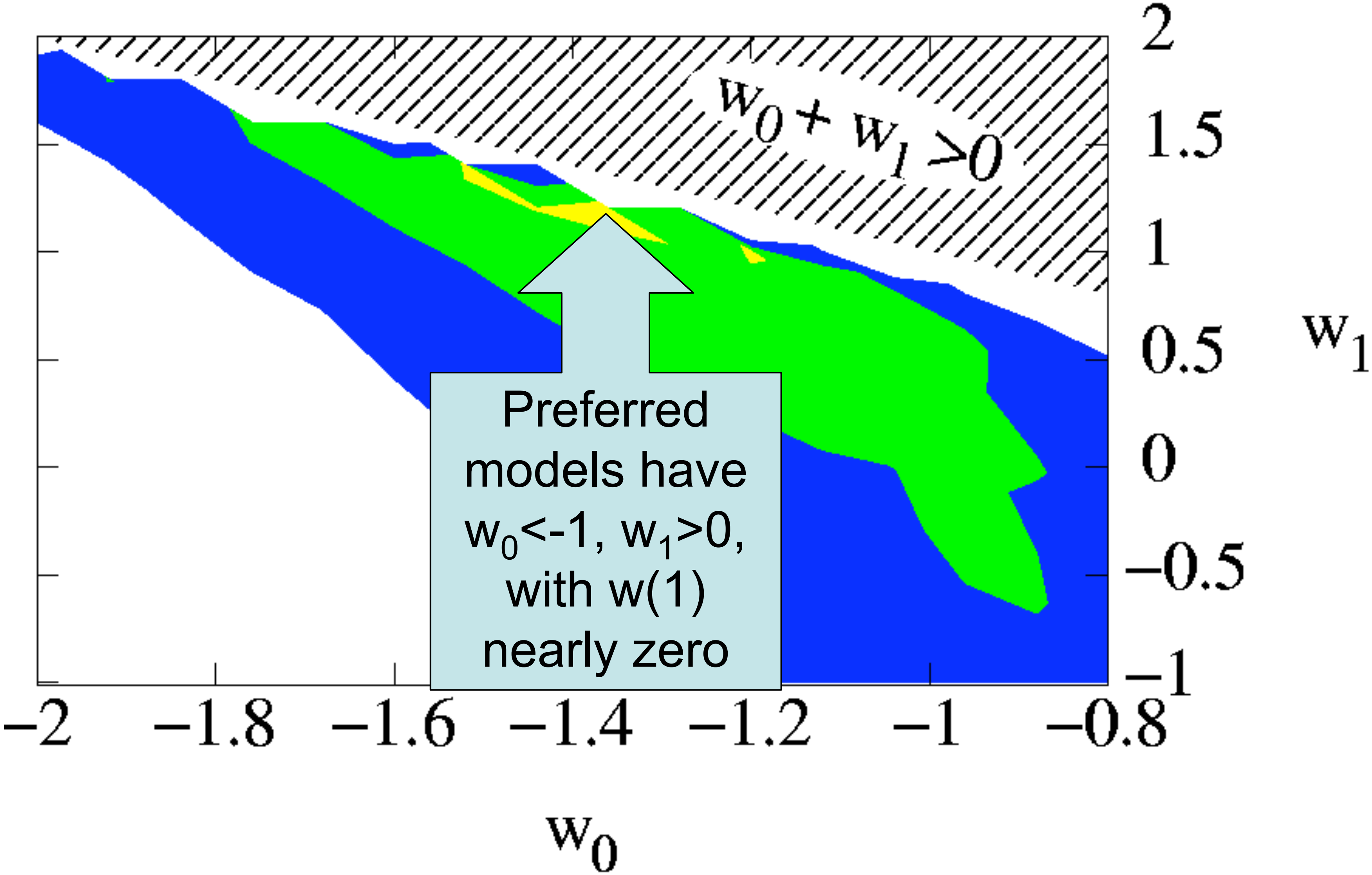
Equation of state constraints

(Upadhye, Ishak, and Steinhardt, 2004)



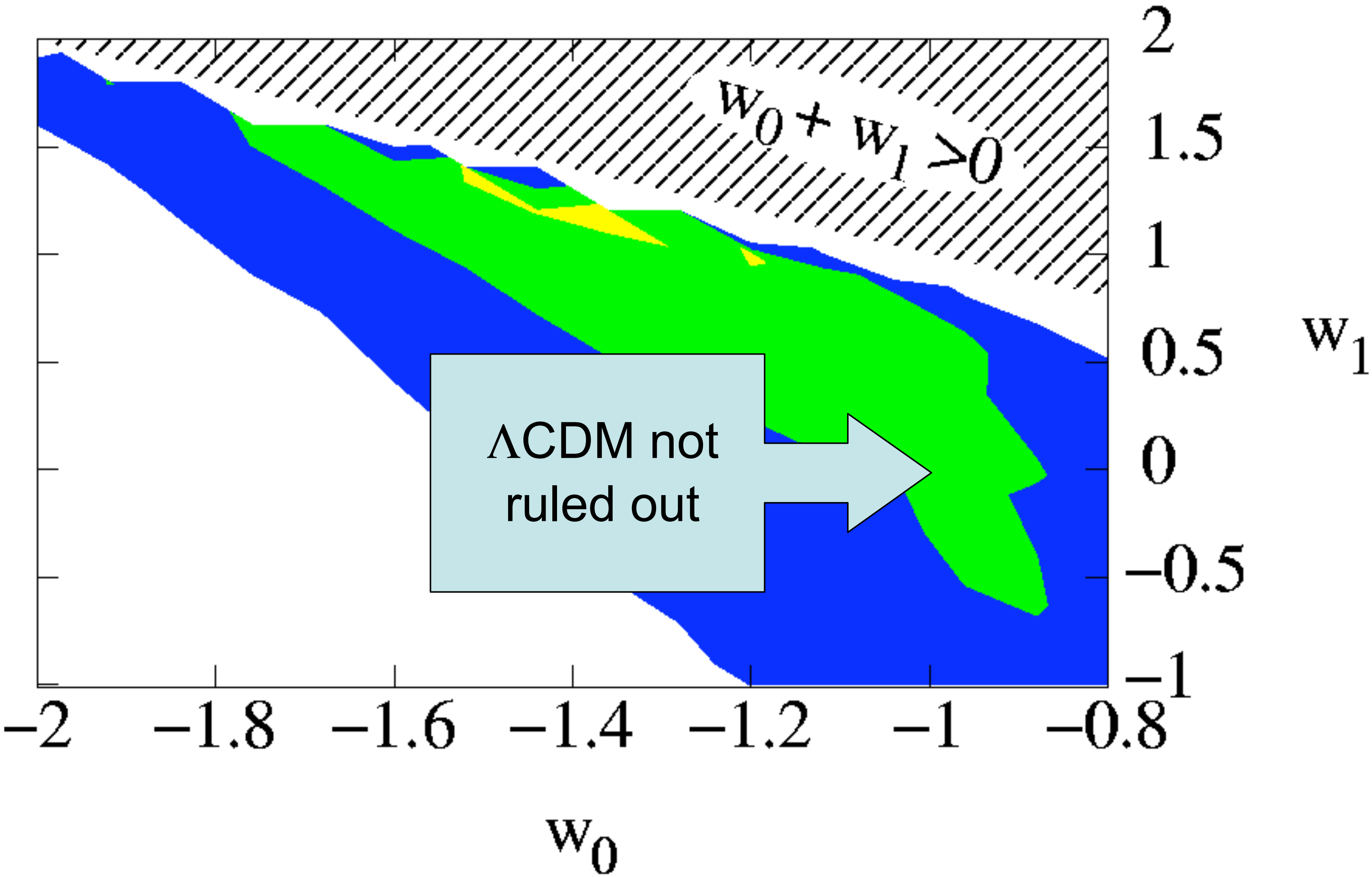
Equation of state constraints

(Upadhye, Ishak, and Steinhardt, 2004)



Equation of state constraints

(Upadhye, Ishak, and Steinhardt, 2004)



Distinguishing between different dark energy models

The first goal of the study of dark energy is to determine whether or not the dark energy is a cosmological constant. Since there is a continuum of $w(z)$ models around the ΛCDM model, the best that we can do is to distinguish between ΛCDM and other dark energy models with $w(z)$ reasonably far from -1.

We say that we can distinguish between two models if, regardless of the location of the best fit point in parameter space, the 2σ contours exclude at least one of the models.

Milestone #1

Current data prefer dark energy equations of state $w(z)$ with $w(0)$ near or less than -1, and $w(1)$ slightly less than 0. Such models are drastically different from the ΛCDM model.

The first milestone is to distinguish between models with $w(1)=-1$ and $w(1)=0$. In the worst case scenario, observers will measure $w(1)=-0.5$, meaning that the 2σ uncertainty in $w(z)$ at $z=1$ must be reduced below 0.5, $\Delta w(1)[2\sigma] < 0.5$ in order to reach this milestone.

Milestone #1

As I know of no theoretically predicted models with such behavior, this milestone is motivated purely by the current data.

If the equation of state is actually found to change this quickly, then a more complicated parameterization of $w(z)$ may be needed, in order to describe the red shift at which $w(z)$ begins its sharp increase and the red shift range over which it increases (see, *e.g.*, Bassett, Corasaniti, and Kunz, 2004; Hannestad and Mortsell, 2004).

Milestone #2

The second milestone is to distinguish between the ΛCDM model and SUGRA-motivated tracking quintessence models (see Zlatev, Wang, and Steinhardt, 1999; Brax and Martin, 1999).

In tracker models, w(z) evolves from a slightly negative value, in the late matter-dominated era, to -1 in the distant future. The value of w today is limited by the rate at which w can transition between these two limiting values. Surveys of a broad range of tracker models have shown that $w_0 \geq -0.8$. Since w is decreasing with time, $w_1 > 0$.

Milestone #2

A data set will be able to distinguish between ΛCDM and trackers if it can distinguish between ΛCDM and the marginal case (w_0=-0.8, w_1=0). In order to do this, the data set must be able to constrain w(z), at some red shift, to a 2σ uncertainty of 0.1. That is, $\Delta w(z_*)[2\sigma]=0.1$, for some red shift z_* at which w is best constrained.

Milestone #3

A third milestone is to distinguish between the ΛCDM model and brane world models, such as the DGP model (Dvali, Gabadadze, and Porrati, 2000; Deffayet, Dvali, Gabadadze, 2002).

The DGP model is a 5 dimensional brane world model with action

$$\begin{aligned} S_{DGP} &= \frac{M_{(5)}^3}{2}\int d^4x dy \sqrt{-g_{(5)}} R_{(5)} \\ &+ \frac{M_{(4)}^2}{2}\int d^4x \sqrt{-g_{(4)}} R_{(4)} + S_{(4)}^{(matter)} \end{aligned}$$

Milestone #3

The action on the brane is the standard 4 dimensional Einstein-Hilbert action, plus the action for matter, which is constrained to lie on the brane.

$M_{(4)}$ and $M_{(5)}$, the four and five dimensional Planck masses, define a characteristic scale

$$r_c = \frac{M_{(4)}^2}{2M_{(5)}^3}.$$

At distance scales below r_c, the universe appears four dimensional.

Milestone #3

On scales larger than r_c, gravity “leaks out” into the fifth dimension; gravity is weaker on these scales. If r_c is tuned to be of order H_0^{-1}, the effective Friedmann equations contain a term that mimicks the dark energy at late times,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\sqrt{\rho + \rho_{r_c}} + \sqrt{\rho_{r_c}}\right)^2 .$$

Here, $\rho_{rc}=3/(32\pi G r_c^2)$ is a constant, while r is the energy density of matter and radiation.

Milestone #3

Compared to dark energy models, the DGP model predicts a different relationship between the Hubble parameter $H(z)$ and the growth factor $G(a)$ of large scale structure. If either $H(z)$ or $G(a)$ were to be measured by itself, then the data could be fit using an effective dark energy equation of state $w_H(z)$ or $w_G(z)$.

The observational test for the DGP model is to measure both $H(z)$ and $G(a)$ well enough to distinguish between these two effective equations of state.

Milestone #3

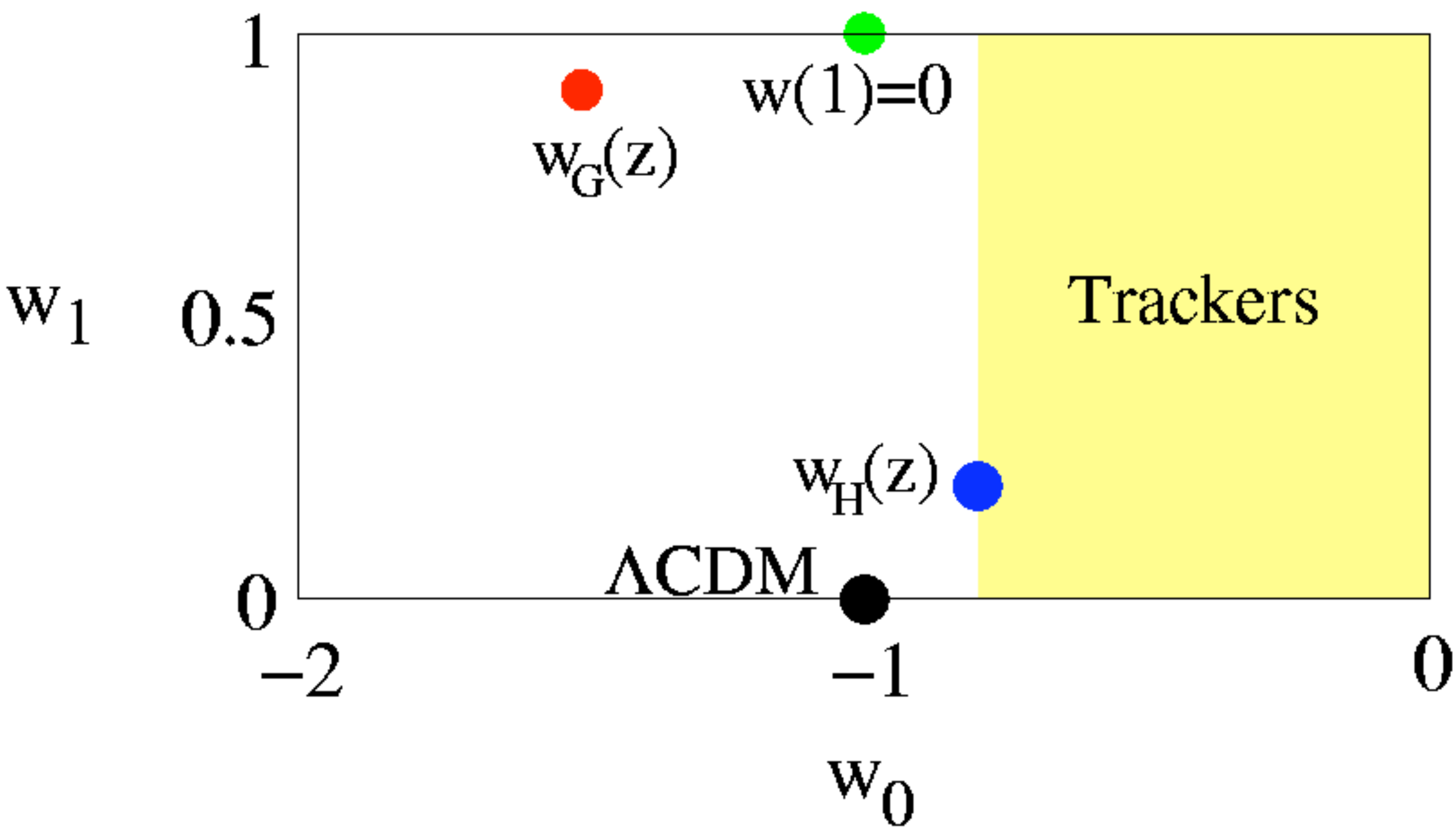
H(z) and G(a) cannot be measured directly. However, the supernova data depend only on H(z), while weak lensing depends on G(a) as well. Thus, we can test for the DGP model by attempting to distinguish between the effective dark energy equations of state found using SN IA and WL data (see Ishak, Upadhye, and Spergel, 2005). We find the effective equations of state

$$H(z)\text{: } w_{H,0}=-0.8,\ w_{H,1}=0.2$$
$$G(a)\text{: } w_{G,0}=-1.5,\ w_{G,1}=0.9.$$

For distinguishability, we need

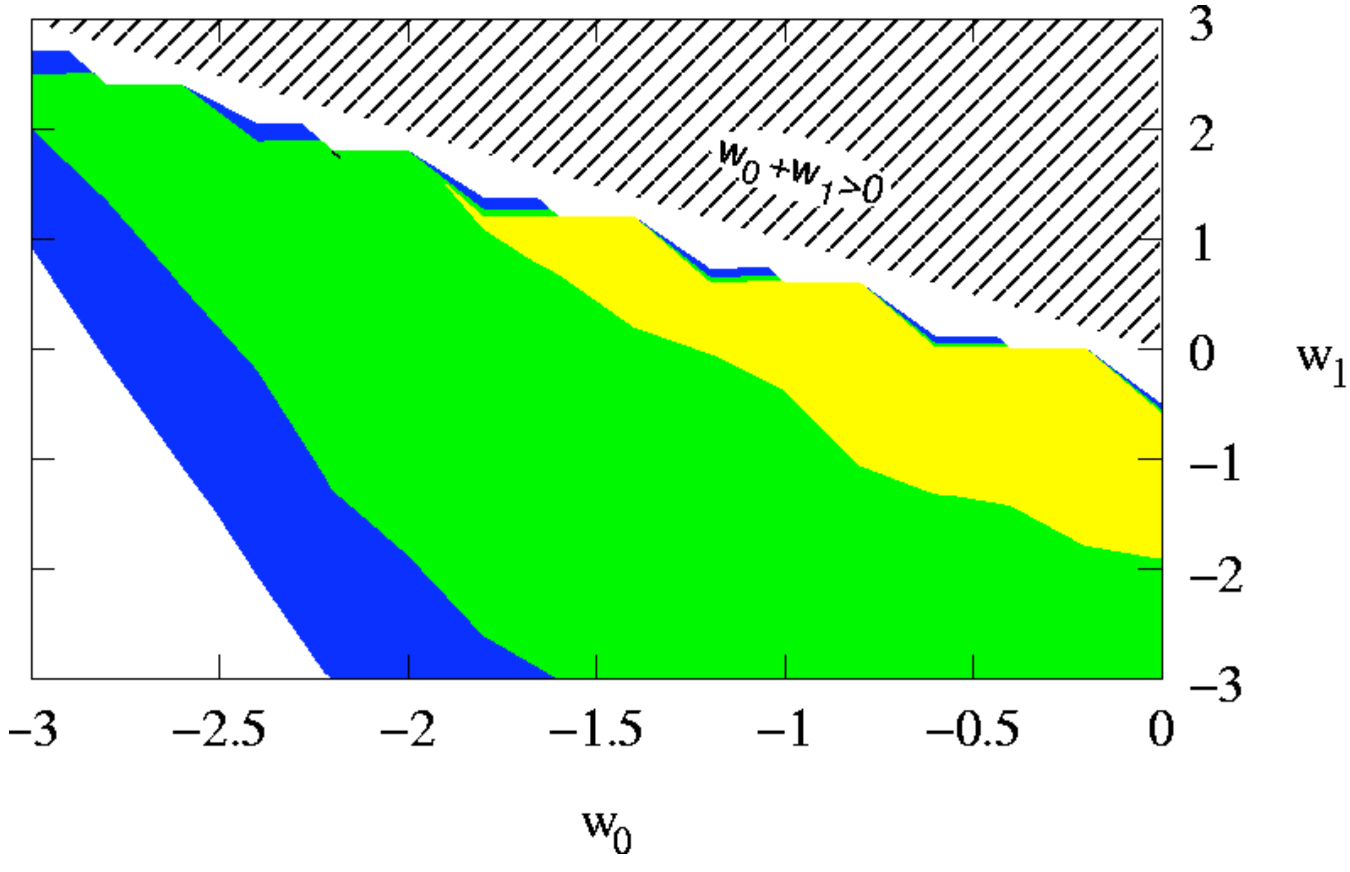
$$\Delta w_0[SN,2\sigma] + \Delta w_0[WL,2\sigma] = 0.7,$$
$$\Delta w_1[SN,2\sigma] + \Delta w_1[WL,2\sigma] = 0.7.$$

Summary of milestones

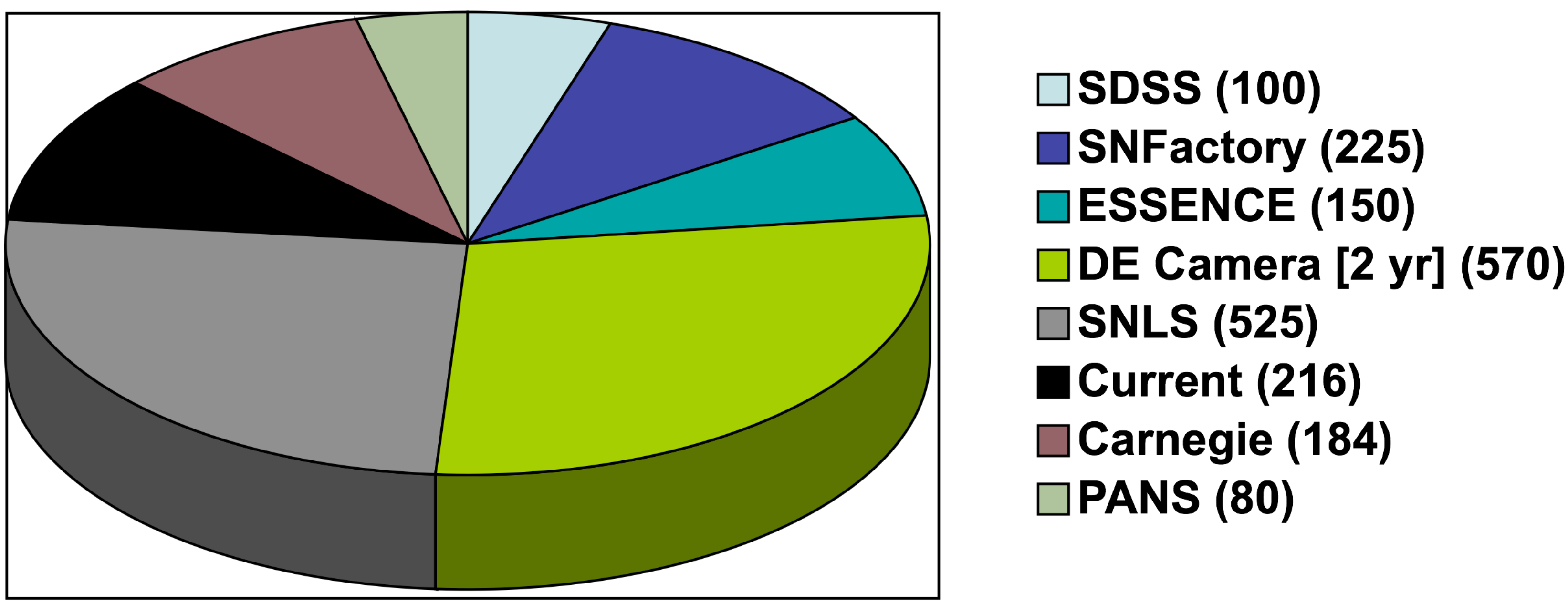


Can we reach these milestones over the next few years?

Start with 8 years of WMAP CMB data…

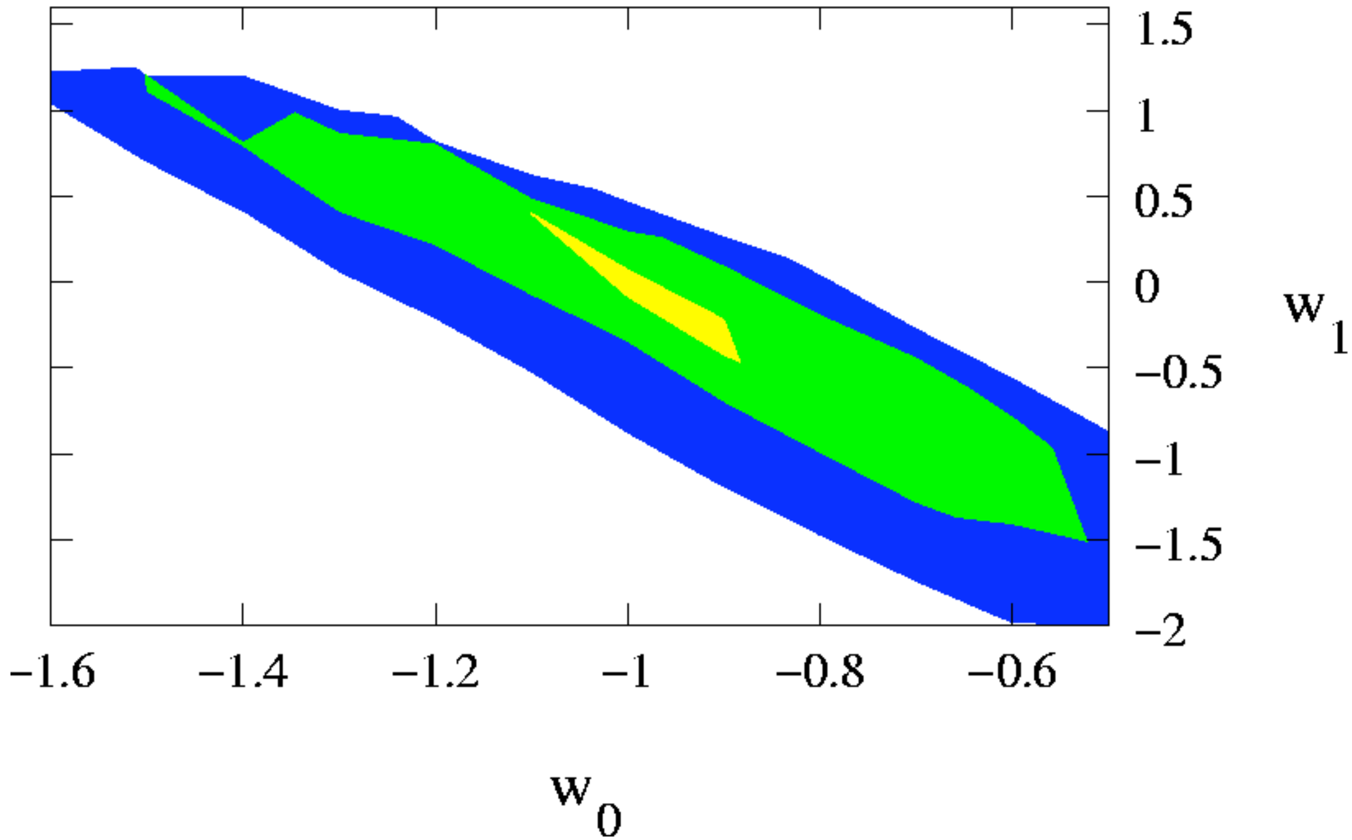


Add simulated SN IA data based on current and planned surveys…



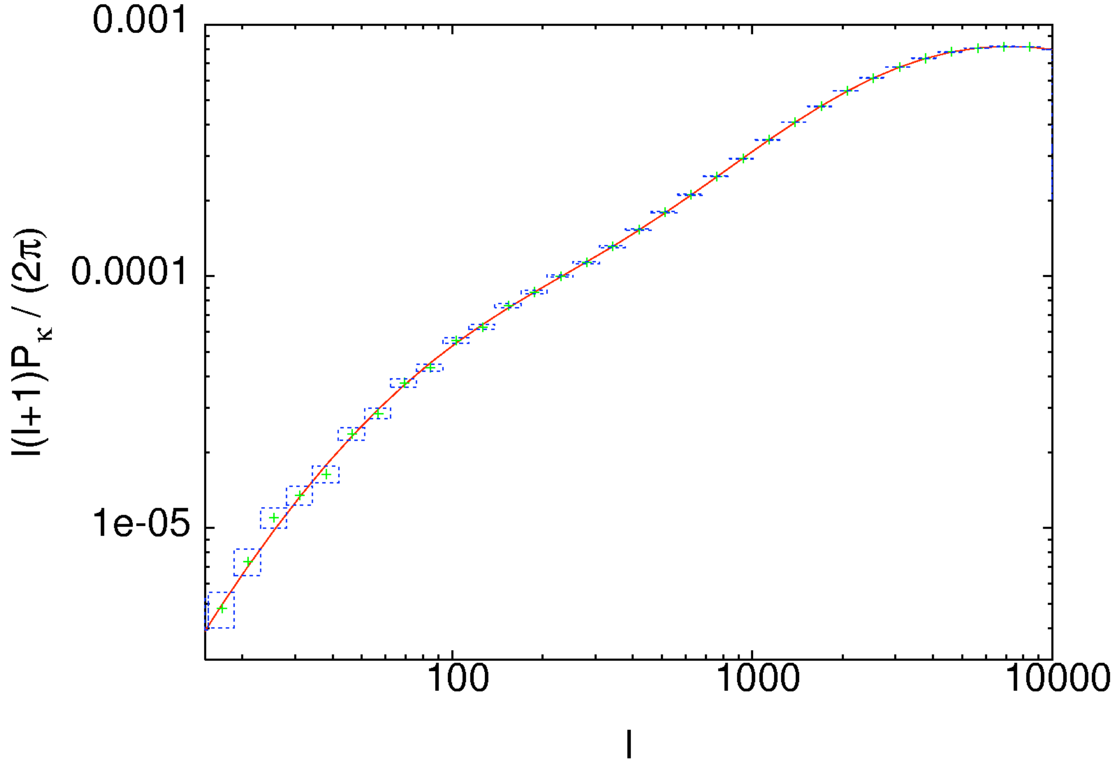
Total: 2050 Type Ia Supernovae

Add simulated SN IA data based on current and planned surveys…



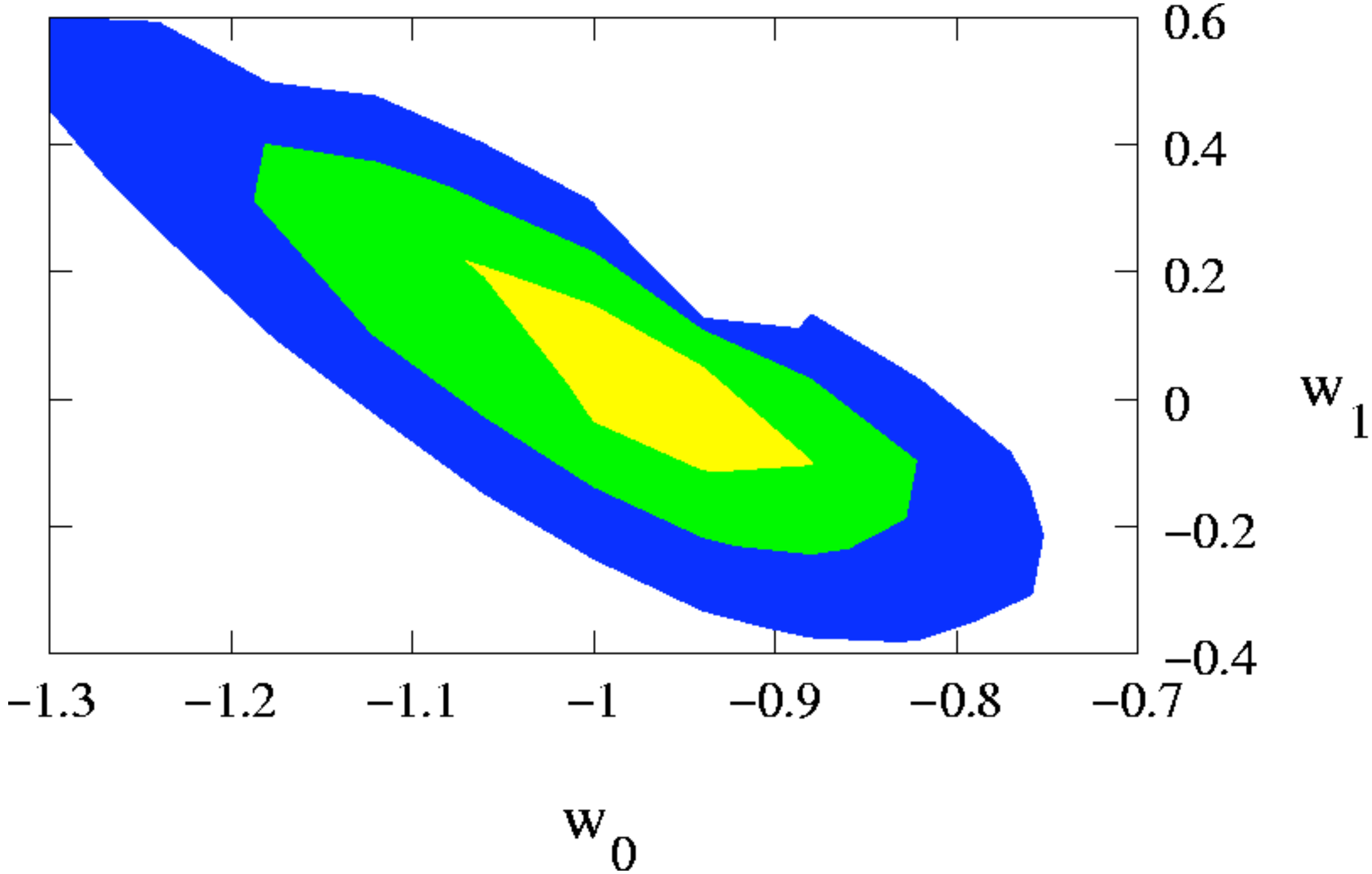
$\Delta w_0[2\sigma] = 0.50$, $\Delta w_1[2\sigma] = 1.39$

Add simulated weak lensing data from a Pan-STARRS-like reference survey…



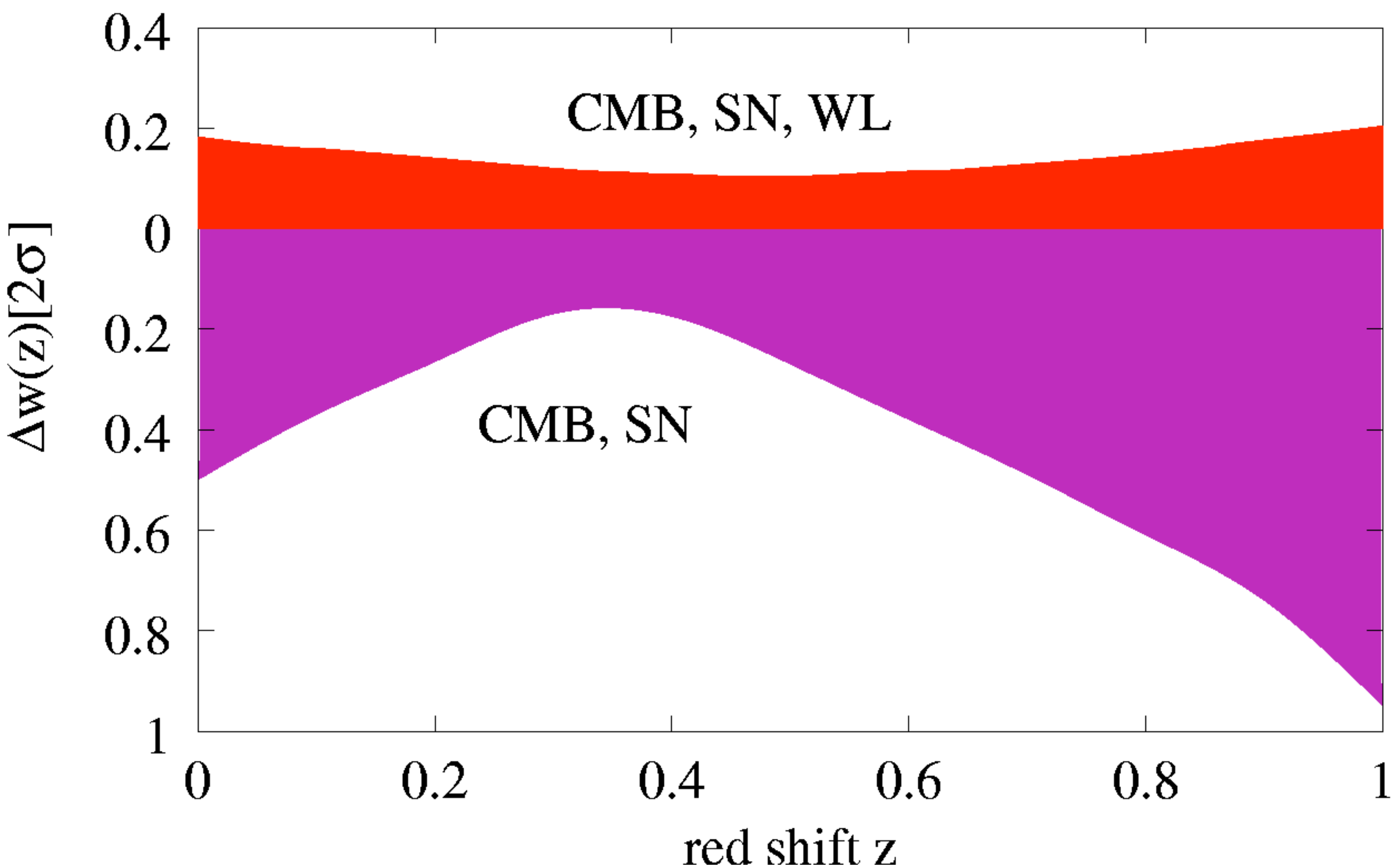
$f_{sky}=0.7$, $l_{max}=3000$, $<\gamma_{int}^2>^{1/2}=0.4$, and n=56 gal/arcmin^2

…and we end up with the constraints



$\Delta w_0[2\sigma] = 0.20$, $\Delta w_1[2\sigma] = 0.37$

Now we can check our milestones.



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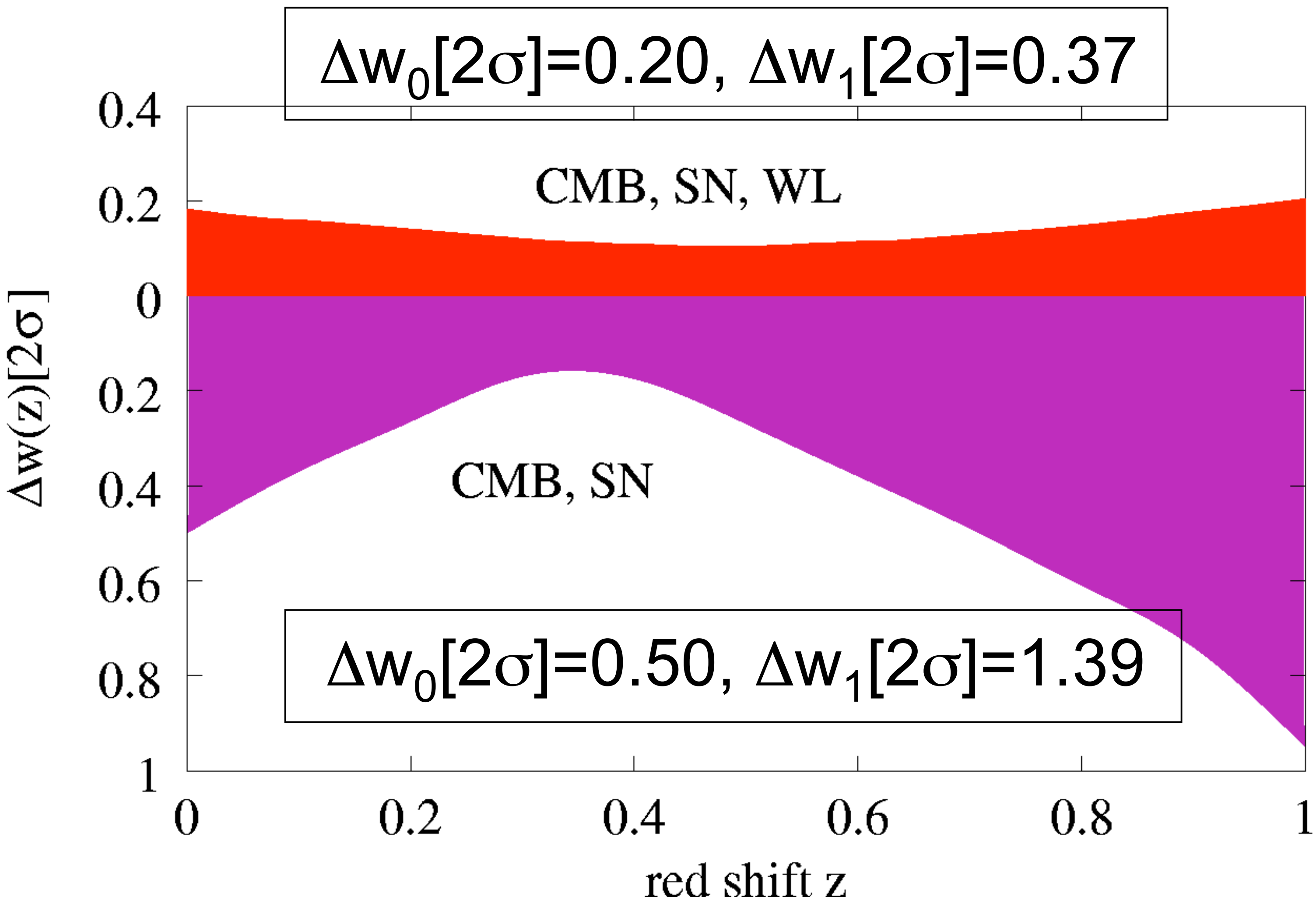

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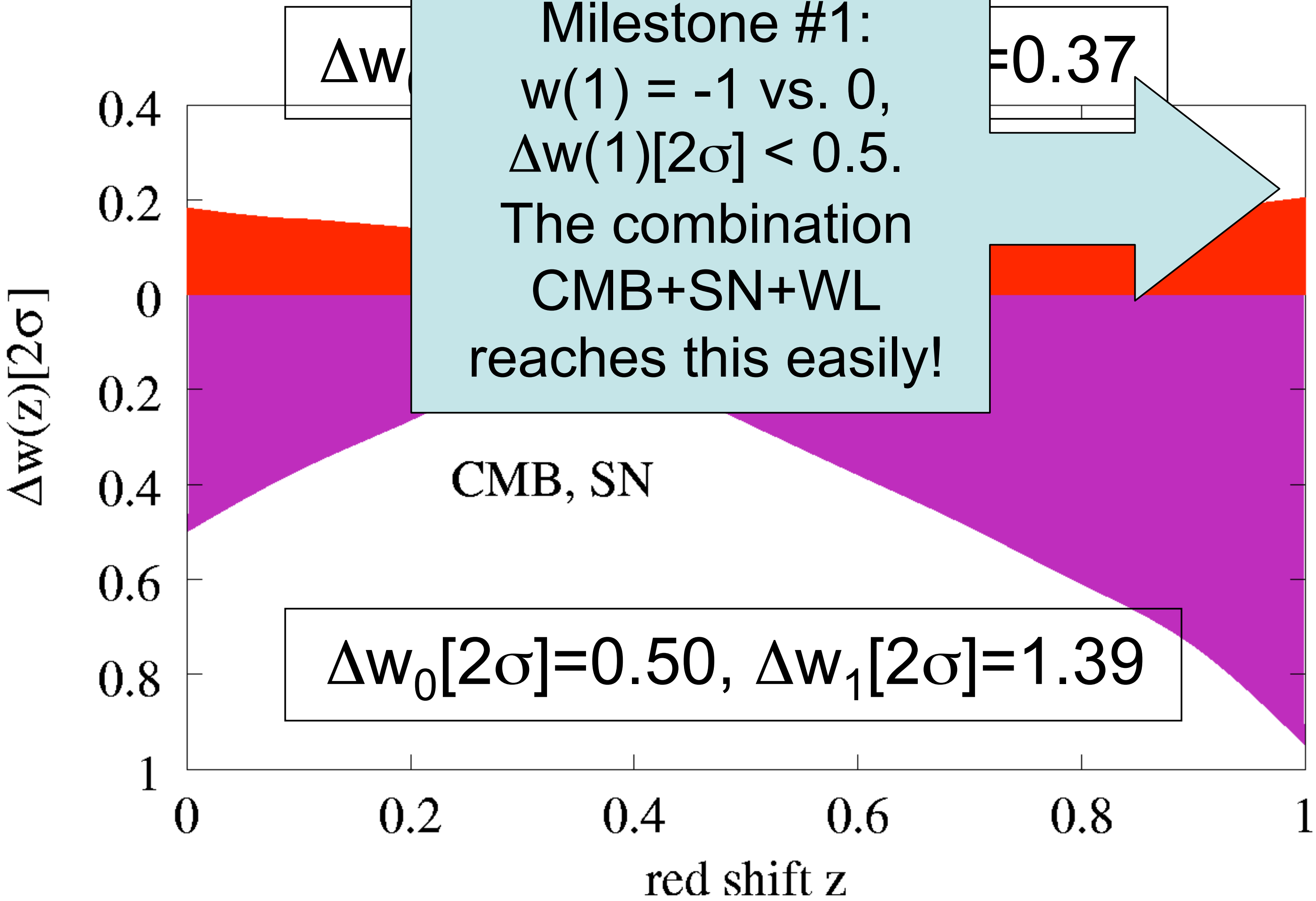
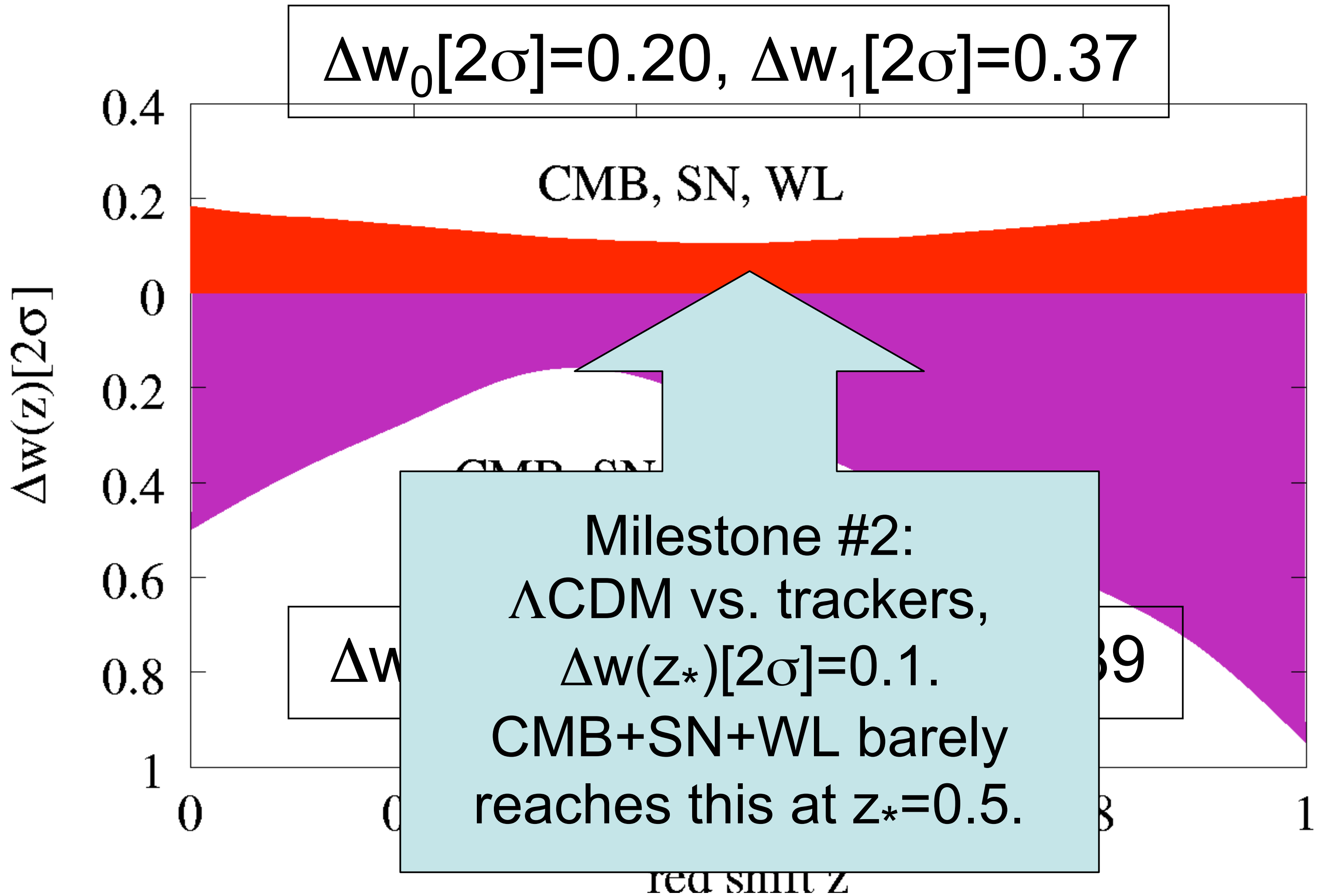


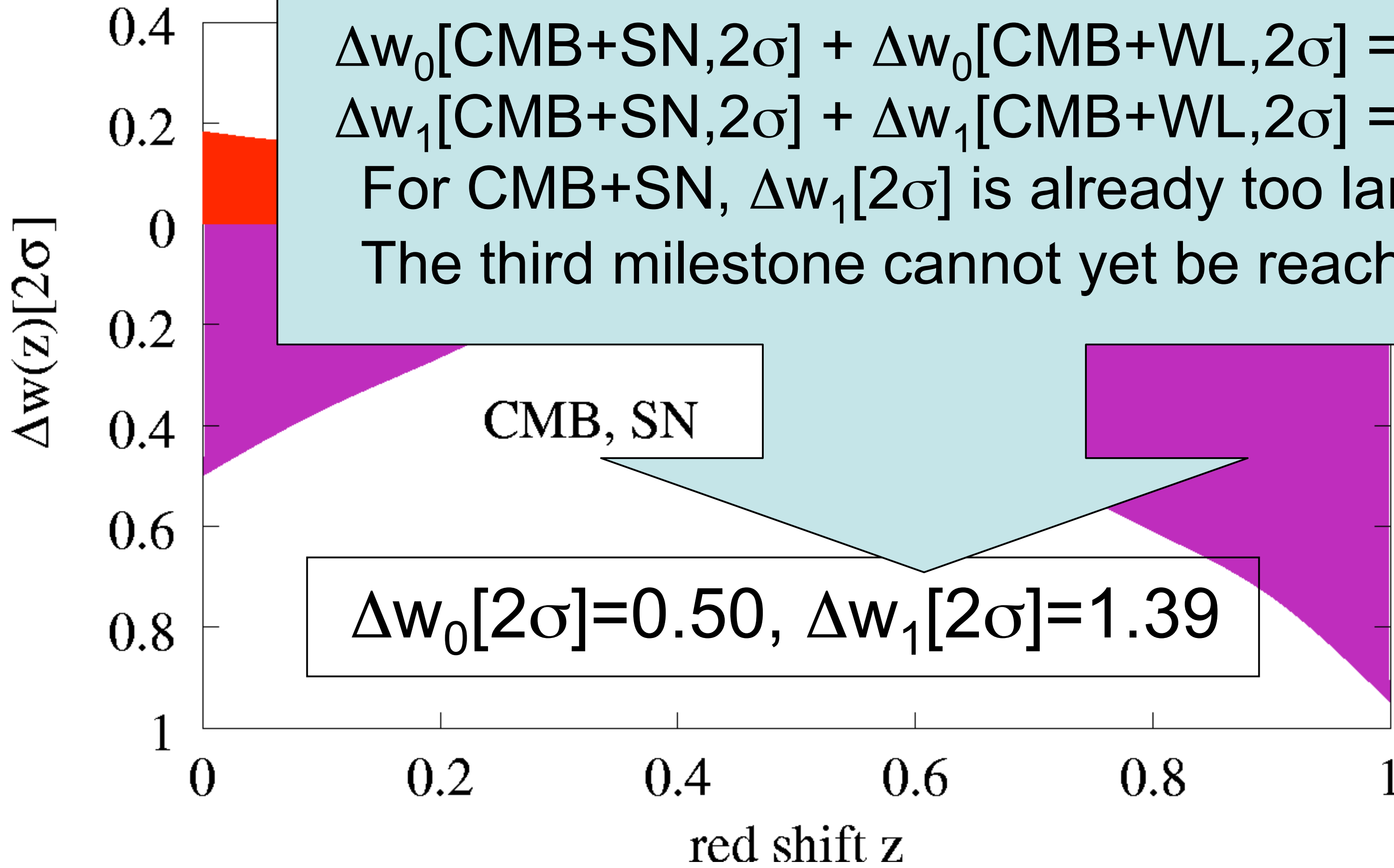
Milestone #1:
w(1) = -1 vs. 0,
Δw(1)[2σ] < 0.5.
The combination
CMB+SN+WL
reaches this easily!

Now we can check our milestones.



Now we can check our milestones.

Milestone #3: ΛCDM vs. DGP brane world,
Δw_0[CMB+SN,2σ] + Δw_0[CMB+WL,2σ] = 0.7,
Δw_1[CMB+SN,2σ] + Δw_1[CMB+WL,2σ] = 0.7.
For CMB+SN, $\Delta w_1[2\sigma]$ is already too large!
The third milestone cannot yet be reached.



$\Delta w_0[2\sigma]$=0.50, $\Delta w_1[2\sigma]$=1.39

What will it take to reach Milestone #3?

Ishak, Upadhye, and Spergel, 2005, showed that the third milestone can be reached by the following combination of data sets:

- CMB: Power spectrum from Planck survey
- SN IA: 2000 SNe IA with systematic $\delta m=0.02$ (*e.g.*, SNAP)
- WL: Space-based tomographic shear survey with 10 bins ($\Delta z=0.3$), $f_{sky}=0.1$, $l_{max}=3000$, $\langle\gamma_{int}^2\rangle^{1/2}=0.25$, and $n=100$ gal/arcmin^2.

Conclusions

- Current data favor dark energy models in which w(z) rises to just below zero at z=1.
- Data from the CMB, SN IA, and WL will be able to distinguish between such models and ΛCDM at the 4.5σ level.
- The combination CMB+SN+WL will also distinguish between ΛCDM and trackers at the 2σ level.
- Distinguishing between ΛCDM and the DGP brane world model will require a shear survey more ambitious than any yet planned.

END.

Variations on the analysis

Variation	w_0 (2σ)	w_1 (2σ)
	$-1.38^{+0.30}_{-0.55}$	$1.20^{+0.64}_{-1.06}$
$w = w_0 + w_1(1-a)$	$-1.30^{+0.39}_{-0.34}$	$1.25^{+0.40}_{-2.17}$
SN systematic $\delta m = 0.04$	$-1.36^{+0.52}_{-0.56}$	$1.20^{+0.60}_{-1.88}$
Ignore C_ℓ for ℓ<20	$-1.63^{+0.43}_{-0.65}$	$1.50^{+0.45}_{-1.67}$

The constraints today

- The data suggest $w_0 < -1$ and $w_1 > 0$, with $w(z)$ going nearly to 0 at $z=1$.
- The ΛCDM model ($w_0=-1$, $w_1=0$) is not conclusively ruled out.
- These conclusions are consistent with other analyses, using different data sets and $w(z)$ parameterizations (see, *e.g.*, Rapetti, Allen, and Weller, 2004; Hannestad and Mortsell, 2004; Riess, *et. al.*, 2004).