Cosmology with Neutrino Models of Dark Energy

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Main questions addressed in this talk:

- What are the cosmological implications of an interaction between dark energy and massive neutrinos?
- Can we test the interaction, using cosmological observations?
- What are the model-independent features of these theories?

Work based on

A. Brookfield, C. van de Bruck, D. Mota, D. Tocchini-Valentini: *Cosmology with massive neutrinos coupled to dark energy*; astro-ph/0503349 (PRL submitted) + work in progress

Outline

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Observational properties of dark energy (I)

- Equation of state $w = p_{DE}/\rho_{DE} \approx -1$.
- Energy density ($\rho_{cr} = 3H^2/8\pi G$):

$$\Omega_{\rm DE} = \frac{\rho_{DE}}{\rho_{cr}} \approx 0.7 \qquad \Omega_{\rm m} = \frac{\rho_m}{\rho_{cr}} \approx 0.3$$

Dark energy dominates dynamics of the universe!

- Writing $\rho_{DE} \approx V_0^4$ and using $H_0 \approx 10^{-33}$eV, one gets

$$V_0 \approx 10^{-3}\text{eV}$$

- What determines this small energy scale? Is this energy scale related to the neutrino mass scale (0.1 eV)?

Observational properties of dark energy (II)

- Strange coincidence: why does the universe starts to accelerate only recently? That is, why is $\rho_{DE} \approx \rho_{\rm matter}$?
- Dark energy seems not to couple to baryons (electrons, quarks): no fifth force has been observed! Why is the coupling so small?
- What about couplings to dark matter and neutrinos? (Theories with dark matter/dark energy coupling have been named ”coupled quintessence”).
- We would like to have better understanding of the properties of dark energy, such as $w(z)$ and its coupling to other matter!

Models of dark energy

- Cosmological constant (Einstein (1916)).
- Slowly evolving scalar field (Wetterich (1988), Ratra & Peebles (1988)).
- Modified gravity, such as extra dimensions (Dvali & Turner (2003), Carroll et al (2004),...)

Here: dark energy is due to a slowly evolving scalar field.

Outline

Neutrino Models of Dark Energy: The Idea of FNW

- Solve the coincidence $\rho_{DE} \approx \rho_{matter}$ by couple them!
- Fardon, Nelson, Weiner (FNW) (2003): couple quintessence field to neutrinos. In these models:

$$m_\nu = m_\nu(\phi)$$

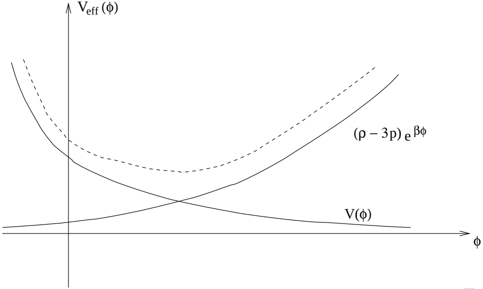
  Goal: we may hope to understand the dark energy scale + neutrino mass scale in one framework!.
- In FNW: specific "non-standard" quintessence potentials. Here: slight variation of the FNW model, based on standard quintessence potentials (more similar to Peccei (2005)).

Evolution of quintessence field in neutrino models of dark energy

Klein-Gordon equation + neutrino-density equation

$$\ddot{\phi} + 3H\dot{\phi} + \underbrace{\frac{\partial V}{\partial \phi} + \frac{\partial \ln m_\nu}{\partial \phi}(\rho_\nu - 3p_\nu)}_{\text{Effective Potential}} = 0$$

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu}{\partial \phi}(\rho_\nu - 3p_\nu)\,\dot{\phi}$$



An instructive example:

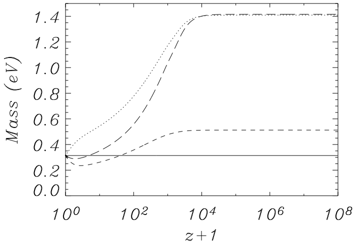
$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{3}{2}}\lambda\phi\right) \quad \text{and} \quad m_\nu = m_0 \exp(\beta\phi)$$

Minimum in the effective potential:

$$\phi_{\text{min}} = \sqrt{\frac{2}{3}}\frac{1}{\lambda}\ln\left(\sqrt{\frac{3}{2}}\frac{\lambda V_0}{\beta(\rho_\nu - 3p_\nu)}\right)$$

We consider here $\lambda > 0$. $\Rightarrow \beta > 0$ is required for ϕ_{min} to be finite.

Evolution of the neutrino mass



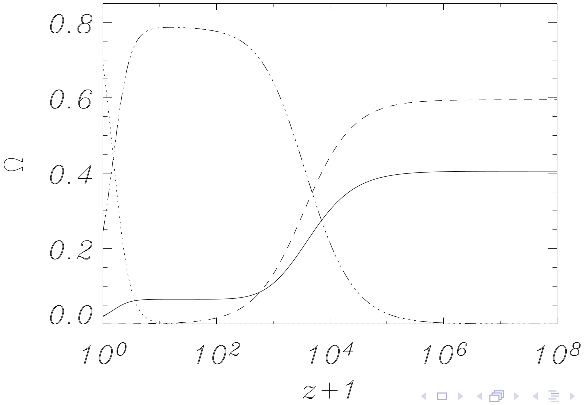
Here ($M_{\rm Pl} = 1$):

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{3}{2}}\lambda\phi\right) \qquad m_\nu = m_0 \exp\left(\beta\phi\right)$$

Cosmological Evolution: $\lambda = 1.0$, $\beta = 0.0$

Evolution of density parameter $\Omega_i = \frac{\rho_i}{\rho_c}$ with $\rho_c = 3H^2/8\pi G$.

Exponential potential $V = V_0 \exp(-\sqrt{\frac{3}{2}}\lambda\phi)$, $m_\nu = m_0 \exp(\beta\phi)$.



Cosmological Evolution: $\lambda = 1.0$, $\beta = 1.0$

Evolution of density parameter $\Omega_i = \frac{\rho_i}{\rho_c}$ with $\rho_c = 3H^2/8\pi G$.

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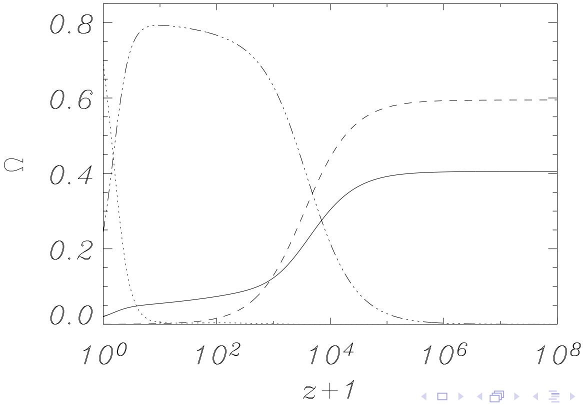
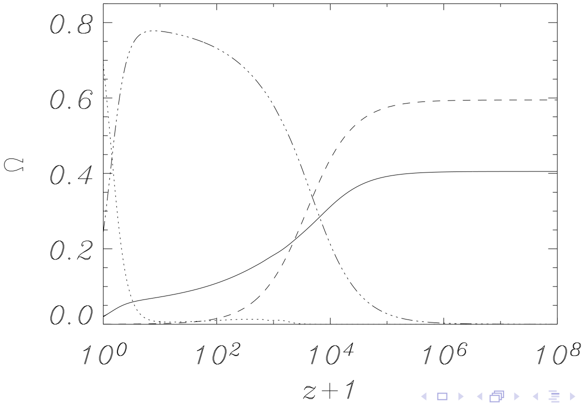


Cosmological Evolution: $\lambda = 0.5$, $\beta = 1.0$

Evolution of density parameter $\Omega_i = \frac{\rho_i}{\rho_c}$ with $\rho_c = 3H^2/8\pi G$.

Exponential potential $V = V_0 \exp(-\sqrt{\frac{3}{2}}\lambda\phi)$, $m_\nu = m_0 \exp(\beta\phi)$.



Outline

Aspects of Cosmological Perturbations

- Neutrinos feel an additional force, mediated by ϕ.
  $\Rightarrow$ They no longer follow geodesics. Geodesic equation

$$\frac{dP^\rho}{dt_p} + \frac{1}{m}\Gamma^\rho_{\mu\nu}P^\mu P^\nu = -\frac{dm}{d\phi}\frac{\partial\phi}{\partial x_\rho}$$

- Due to the coupling, neutrinos and dark energy perturbations influence each other via ϕ-mediated force *and* gravity.
- Effective gravitational constant felt by neutrinos is $(1+\beta^2)G_N$.

Boltzmann treatment: Background

Calculate density and pressure from distribution function $(\epsilon^2 = q^2 + m_\nu(\phi)^2 a^2)$:

$$
\begin{aligned}
\rho_\nu &= \frac{1}{a^4}\int q^2 dq d\Omega \epsilon f_0(q) \\
p_\nu &= \frac{1}{3a^4}\int q^2 dq d\Omega f_0(q)\frac{q^2}{\epsilon} \\
\Rightarrow \dot{\rho}_\nu &+ 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu}{\partial \phi}\dot{\phi}(\rho_\nu - 3p_\nu)
\end{aligned}
$$

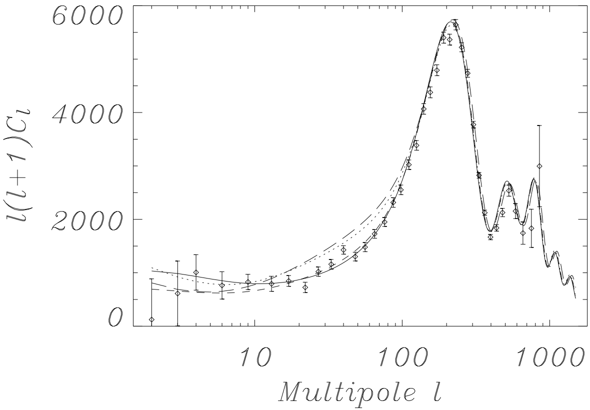
Boltzmann treatment: Perturbations

Calculate density and pressure fluctuations from distribution function $f = f_0(1 + \Psi)$, e.g.

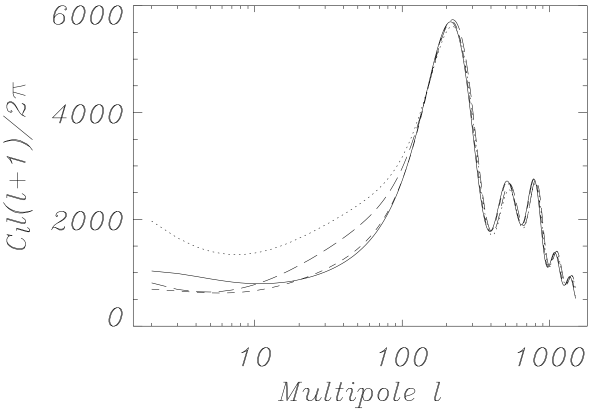
$$\delta\rho_\nu = \frac{1}{a^4}\int q^2 dq d\Omega \epsilon f_0(q)\Psi + (\rho_\nu - 3p_\nu)\frac{\partial \ln m_\nu}{\partial\phi}\delta\phi$$

Boltzmann hierarchy is solved in standard way (using CAMB).

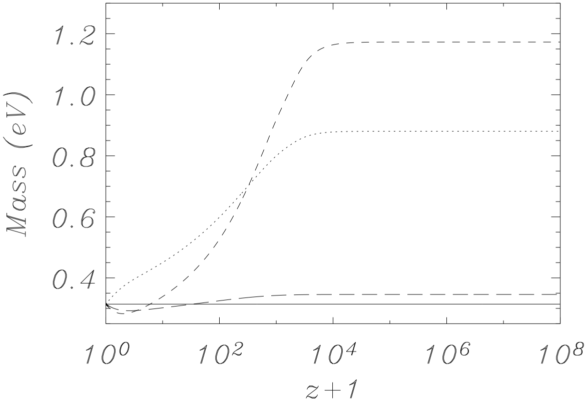
CMB Anisotropies: Exponential Potential



CMB Anisotropies: Exponential Potential



CMB Anisotropies: $V(\phi) = M^6/\phi^2$



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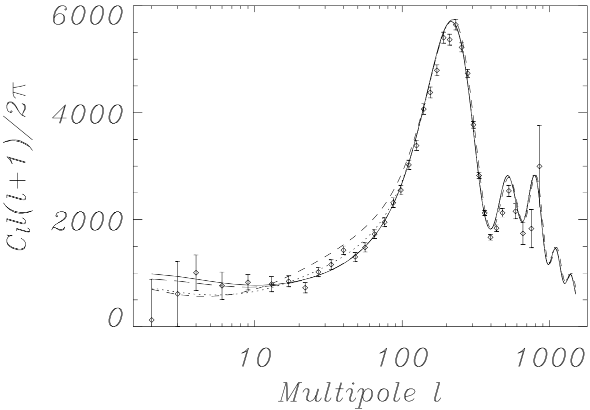
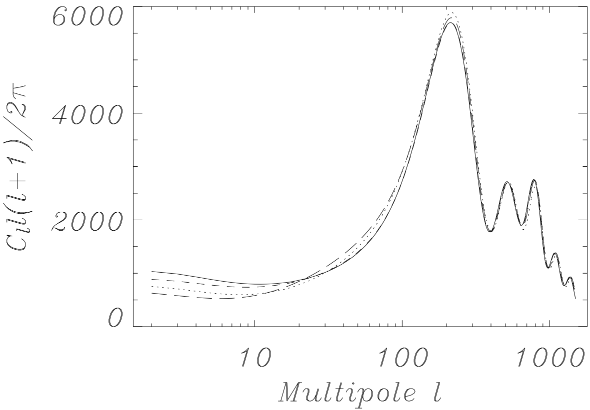


CMB Anisotropies: $V(\phi) = V_0 \exp(-\phi)$, $m_\nu(\phi) = m_0 \cosh(\beta\phi)$



Predictions:

- Changing β increases power in region $l \approx 10 - 100$.
- Decrease of power in the region $l = 2 - 10$!
- Some choices of parameters can lead to enhancement in the region $l = 2 - 100$!
- The CMB power spectrum in these models fits region $l = 2 - 10$ *better* than ΛCDM.

Reduction of power of quadrupole has been observed both by WMAP and COBE. But there are issues. E.g.:

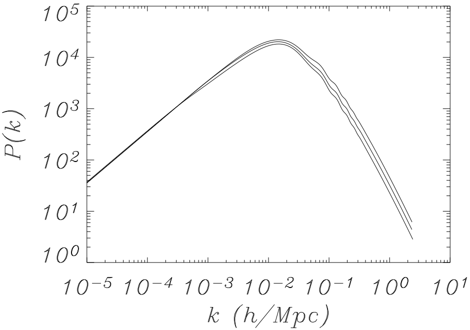
- Alignment of quadrupole and octupole; octupole is planar (de Oliveria-Costa et al. (2003)).
- Quadrupole-octupole alignment reported by other groups and other methods (seet talk by Dominik Schwarz on Tuesday).
- Northern and southern hemisphere asymmetrical (Eriksen et al. (2004)).
- If true, this would be very unlikely within standard cosmology!

Issues:

- Quadrupole and Octupole might be contaminated by non-cosmological signals or fluctuations might be non-gaussian!
- However: both *non-gaussianity* and *foreground effects* have not been observed (e.g. Land & Magueijo (2005))!
- Issue vital not only for neutrino models of dark energy but also for "normal" quintessence and coupled quintessence (perturbations influence C_l's considerably in region $l = 2 - 50$) (Bean & Dore (2004), Weller & Lewis (2004), Hannestad (2005)).

Need to understand large scale part better for data analysis!

What about large scale structures (LSS) ?



Damping of fluctuations due to neutrino freestreaming as usual. Large scale structures probe the neutrino mass at higher redshift (at time of structure formation).

Outline

Comparison with Fardon, Nelson and Weiner:

Basic Lagrangian at low energies (after integrating out a heavier sterile neutrino) of the form

$$\mathcal{L} = \frac{m_{lr}^2}{M(\phi)}\bar{\nu}\nu + \text{h.c.} + \Lambda^4 \ln\left(\frac{M(\phi)}{\mu}\right).$$

Couplings of the form

$$M(\phi) = M \exp\left(\frac{\phi^2}{f^2}\right), \quad or \quad M(\phi) = \lambda\phi.$$

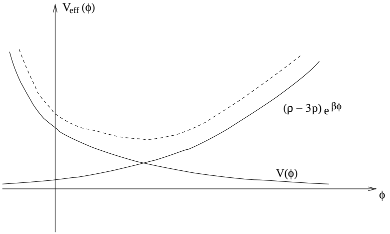
Comparison with Fardon, Nelson and Weiner:

Requiring that the neutrinos do not decay into $\phi-$quanta leads to $f > 100$keV.

$\Rightarrow$ Mass of field can be potentially of order $m_\phi \approx 10^{-4}$eV, compared to $m_\phi \approx H$ as in standard quintessence models! ϕ settles into the minimum of effective potential! And: β is very large (roughly $\beta = M_{\rm pl}/f$).

Potential is non-standard! (C.f. pseudo-Nambu-Goldstone boson: $V(\phi) \propto \cos(\phi/\alpha)$, with $\alpha \approx M_{\rm pl}$.)

Comparison with Fardon, Nelson, Weiner:



Field settles into minimum. Stationary condition (n_ν = neutrino number density):

$$0 = \frac{\partial V}{\partial m_\nu} + n_\nu$$

This equation leads to $m_\nu \propto n_\nu^{-1}$ for $w_\phi \approx -1$. $\Rightarrow$ Neutrino mass increases in time. Instability is possible (see talk by Afshordi on Wednesday)!

Remarks:

1. CMB + LSS useful probe for neutrino-dark energy coupling.
2. However, reduction of power on largest scales is not a smoking gun for these models. Other mechanism have been suggested:
   - Large fluctuations in dark energy field, giving rise to uncorrelated entropy perturbations. (Moroi & Takahashi (2003), Gordon & Hu (2004)).
   - Non-standard initial power spectrum from inflation. (Efstathiou (2003); Contaldi, Peloso, Kofman & Linde (2003)).
   - Non-trivial spatial topology (Silk, Levin, Barrow, Inoue, Sugiyama, Weeks, ...).
3. Supernovae (SN) probes: Coupling between DE and neutrinos change equation of state towards $w = -1$. $\Rightarrow$ SN an additional probe for effective potential.