Born-Infeld Cosmology

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- *The* Physical Guide

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- *Finale* ... well, not yet ...

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- *One* question and $e^{6.90775528...}$ or more responses
- *Mise em Scene*: Born-Infeld Theory
- *The* Physical Guide
- *Finale* ... well, not yet ...
- $e^{6.90775528...}$ reasons

Is there a good reason why . . .

high energy physicists should committ

to avant-garde cosmology?

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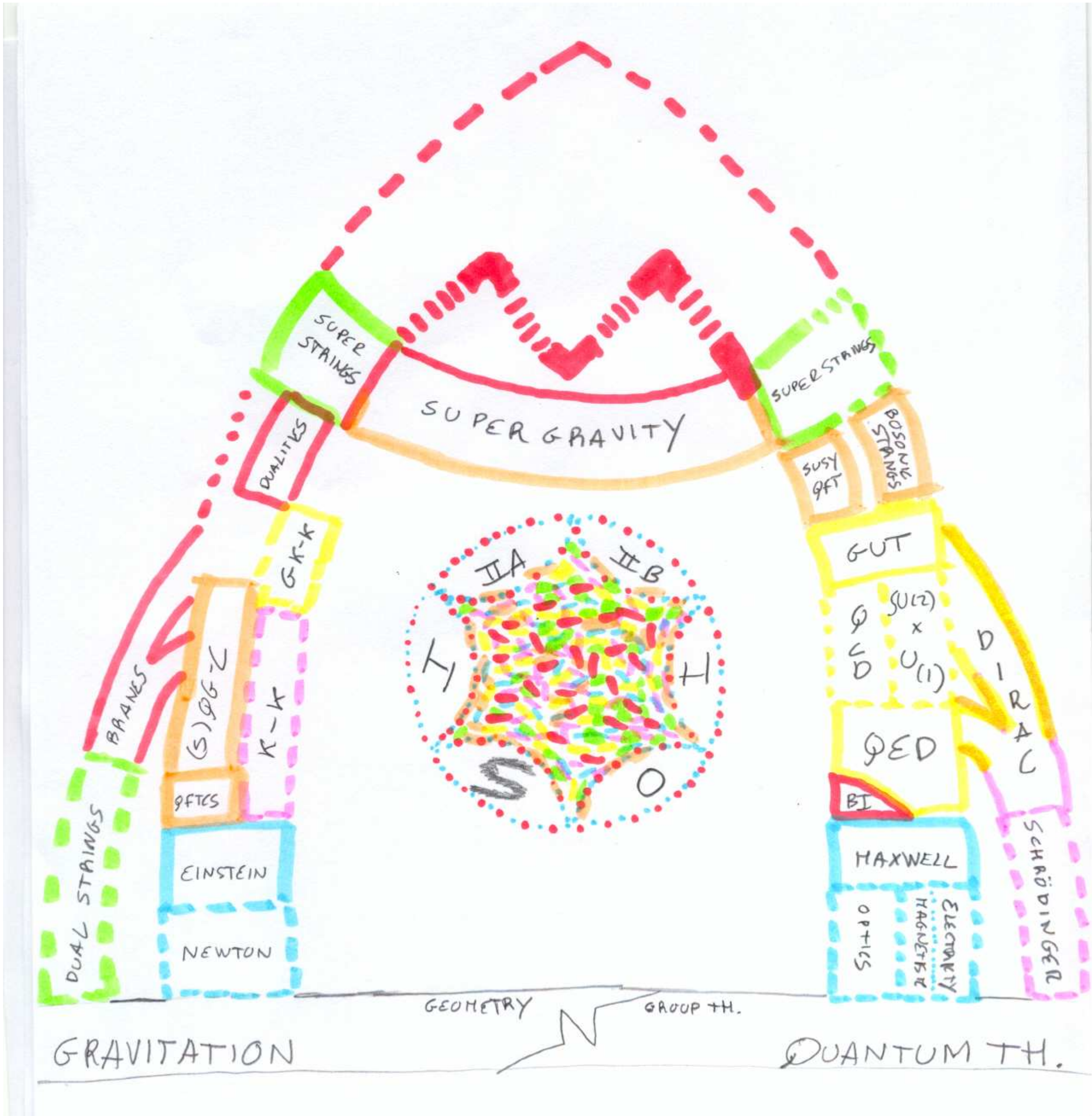
SUPER STRINGS
SUPER GRAVITY
SUPERSTRINGS
DUALITIES
SUSY QFT
BOSONIC STRINGS
G-K-K
GUT
IIA
IIB
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H
S
O
BRANES
(S)QG-C
K-K
Q C D
SU(2) × U(1)
DIRAC
QED
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BI
DUAL STRINGS
EINSTEIN
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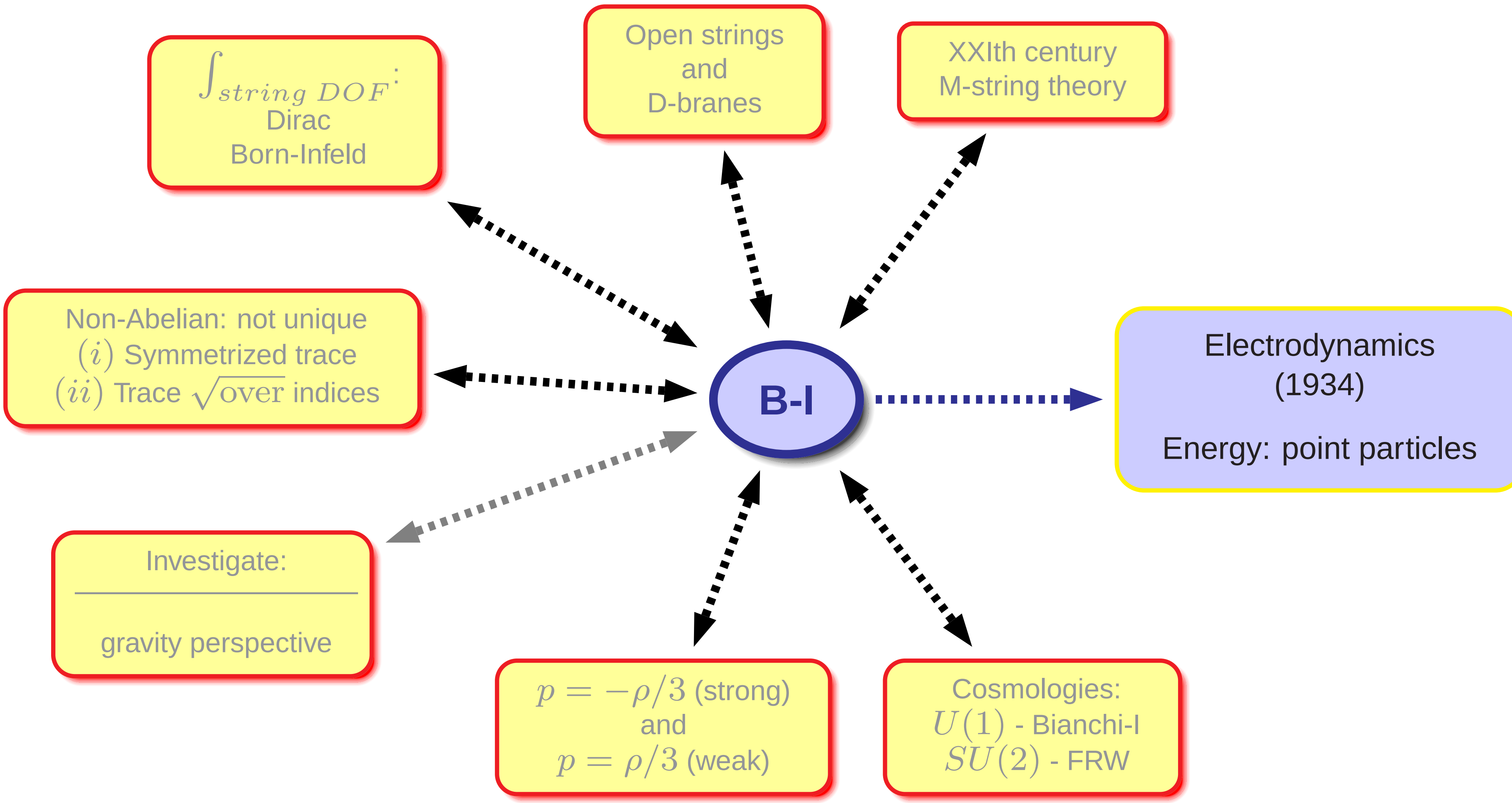
GROUP TH.
GRAVITATION
QUANTUM TH.

... and framework:



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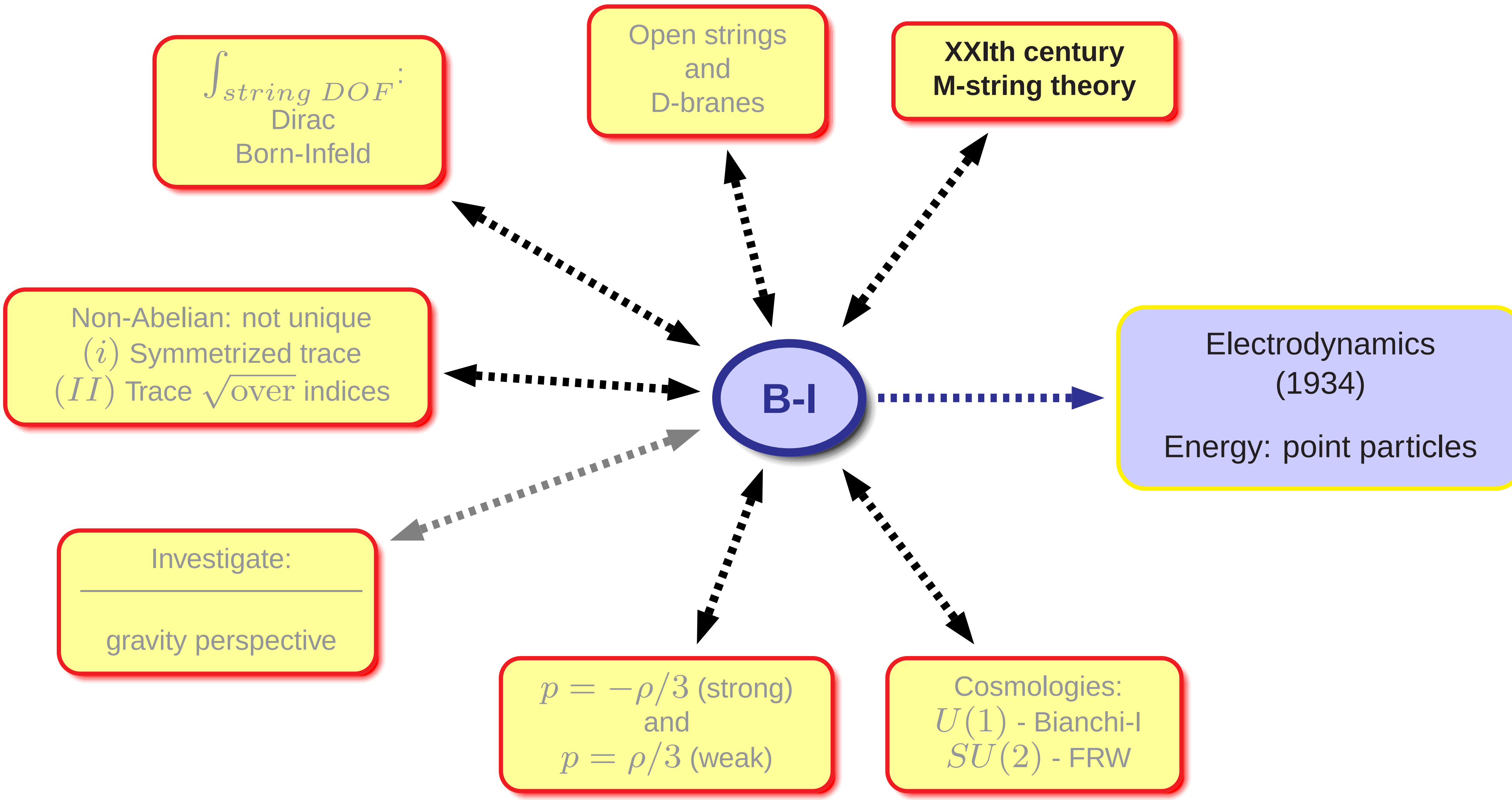

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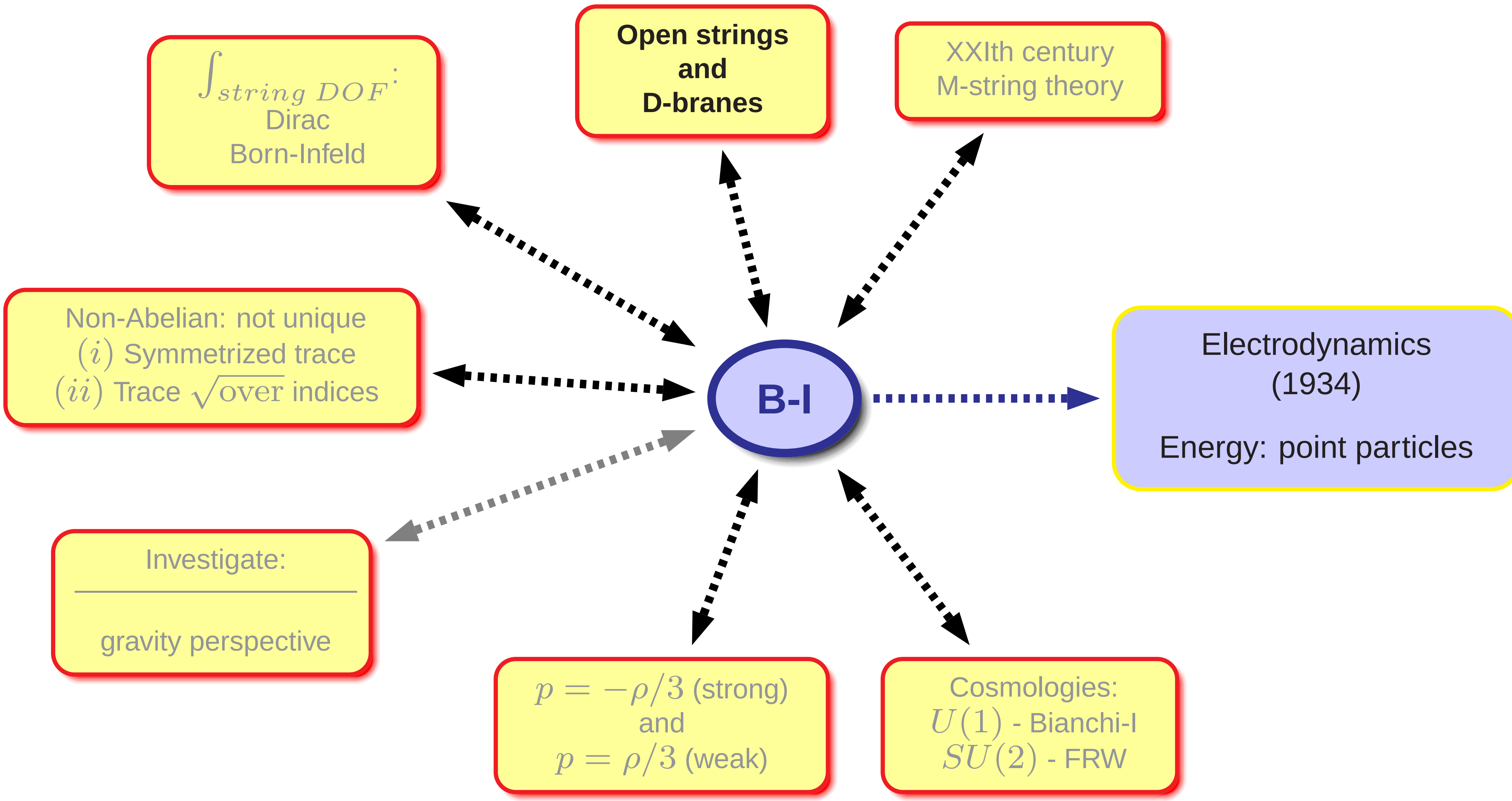

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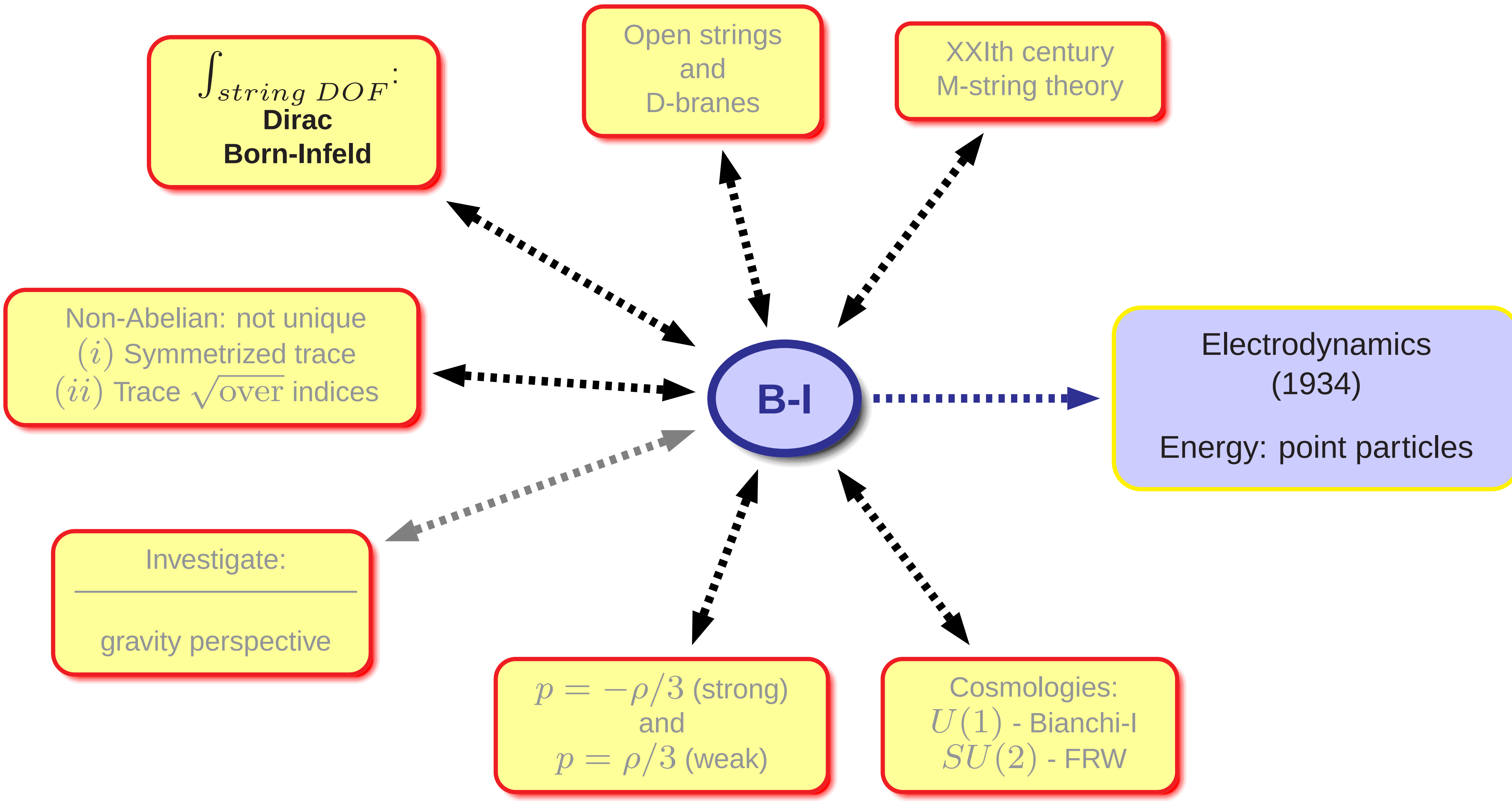

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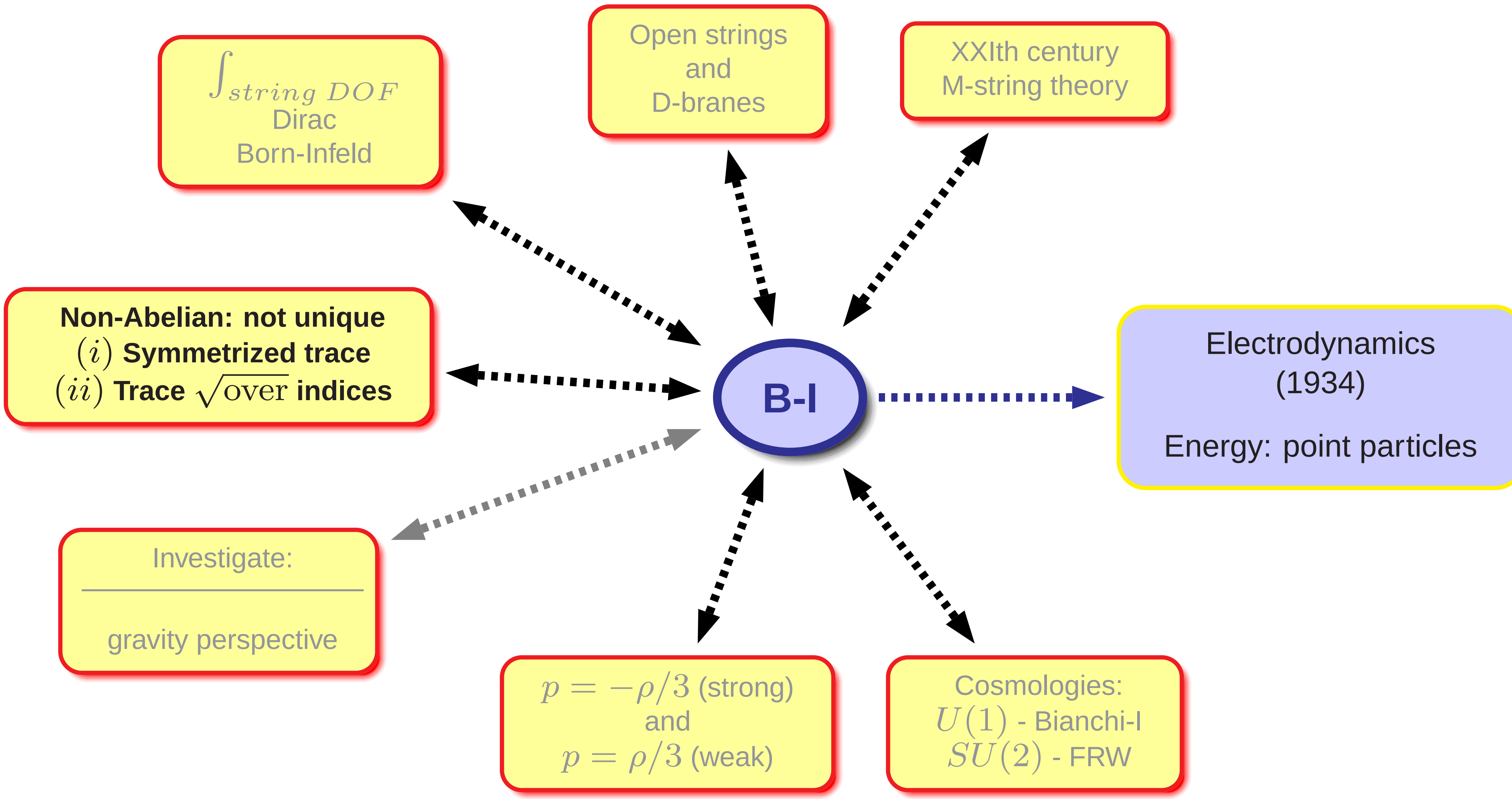

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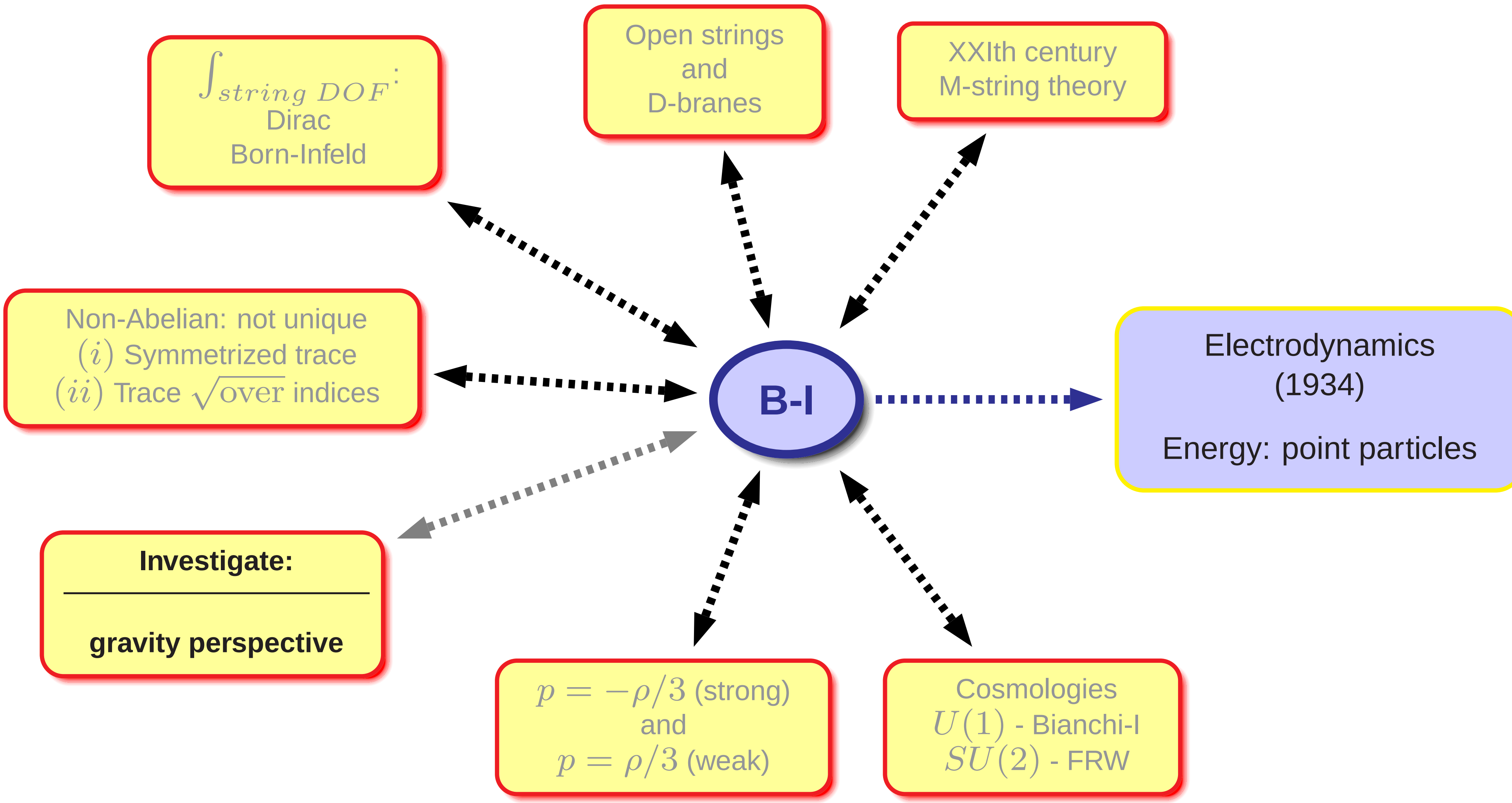

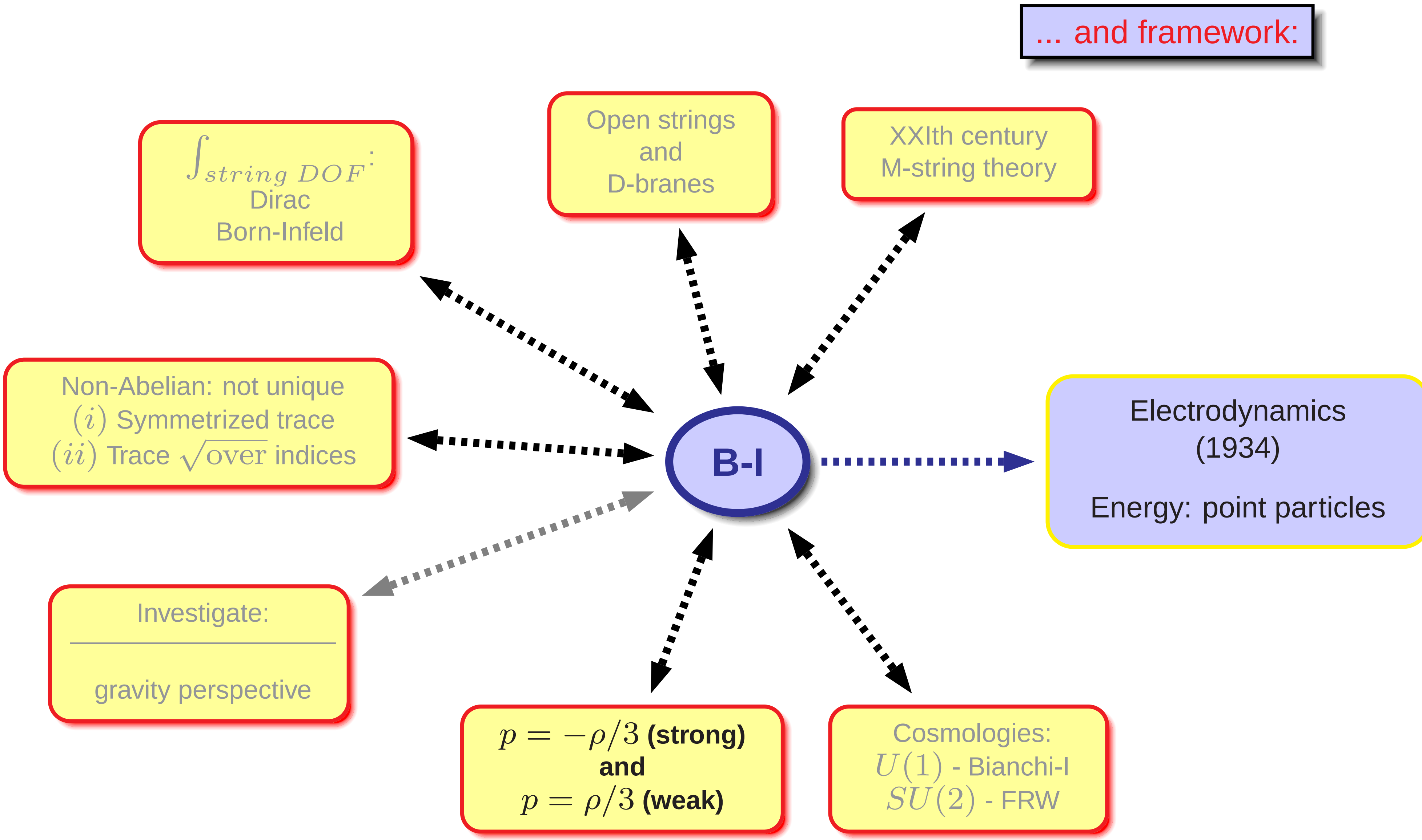
... and framework:
$\int_{string\ DOF}$:
Dirac
Born-Infeld
Open strings
and
D-branes
XXIth century
M-string theory
Non-Abelian: not unique
(i) Symmetrized trace
(ii) Trace $\sqrt{\text{over}}$ indices
B-I
Electrodynamics
(1934)
Energy: point particles
Investigate:
gravity perspective
$p = -\rho/3$ (strong)
and
$p = \rho/3$ (weak)
Cosmologies:
$U(1)$ - Bianchi-I
$SU(2)$ - FRW

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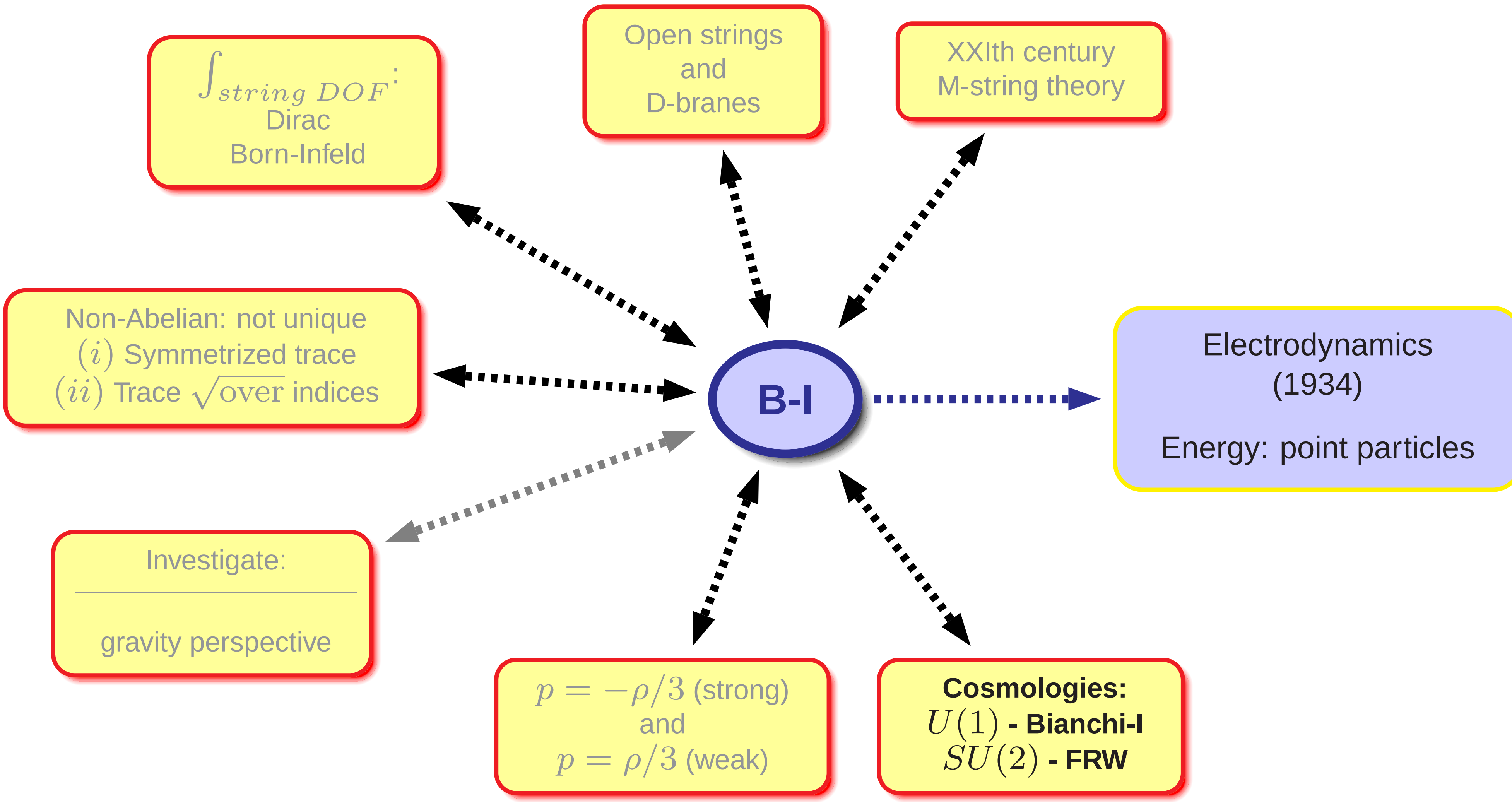
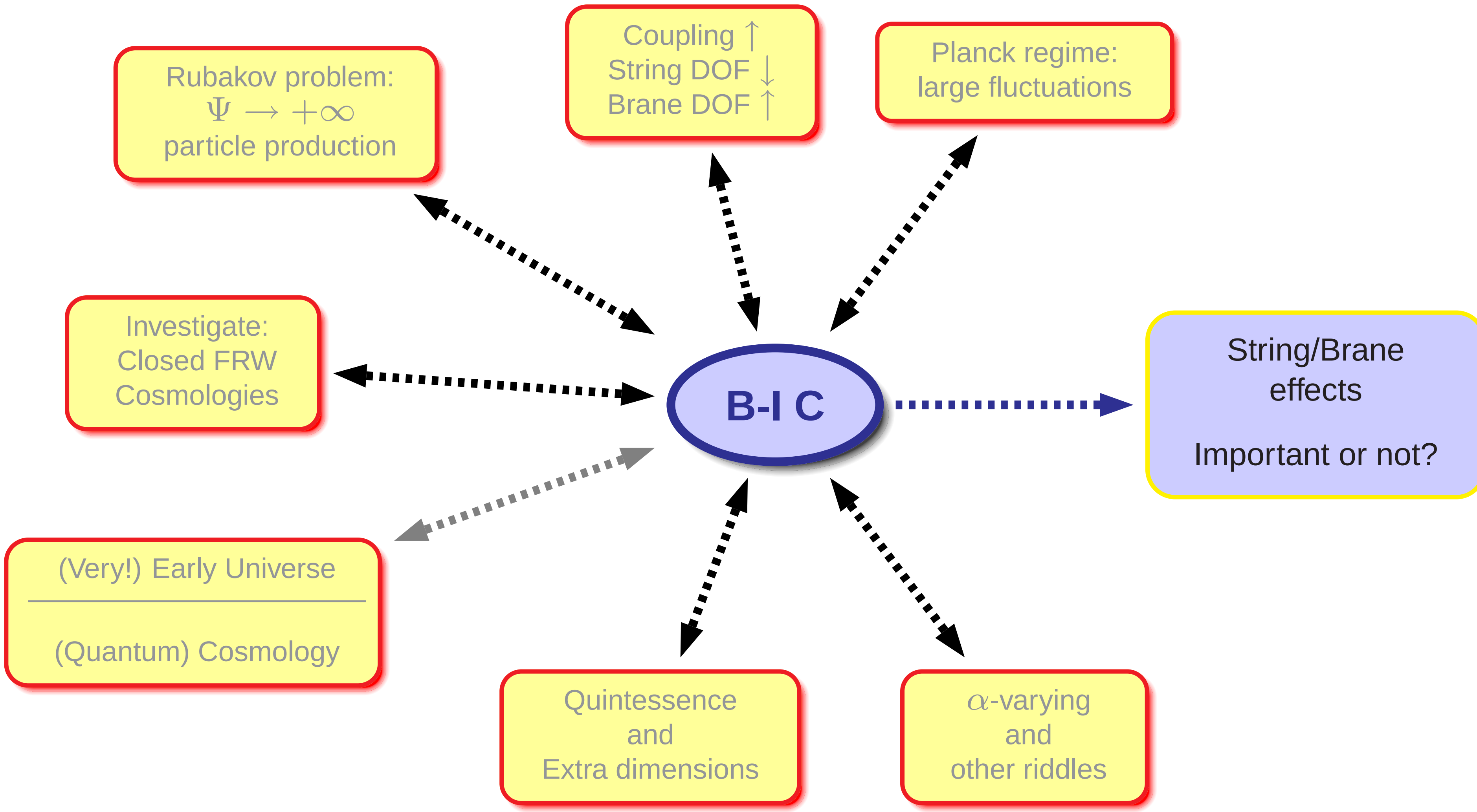
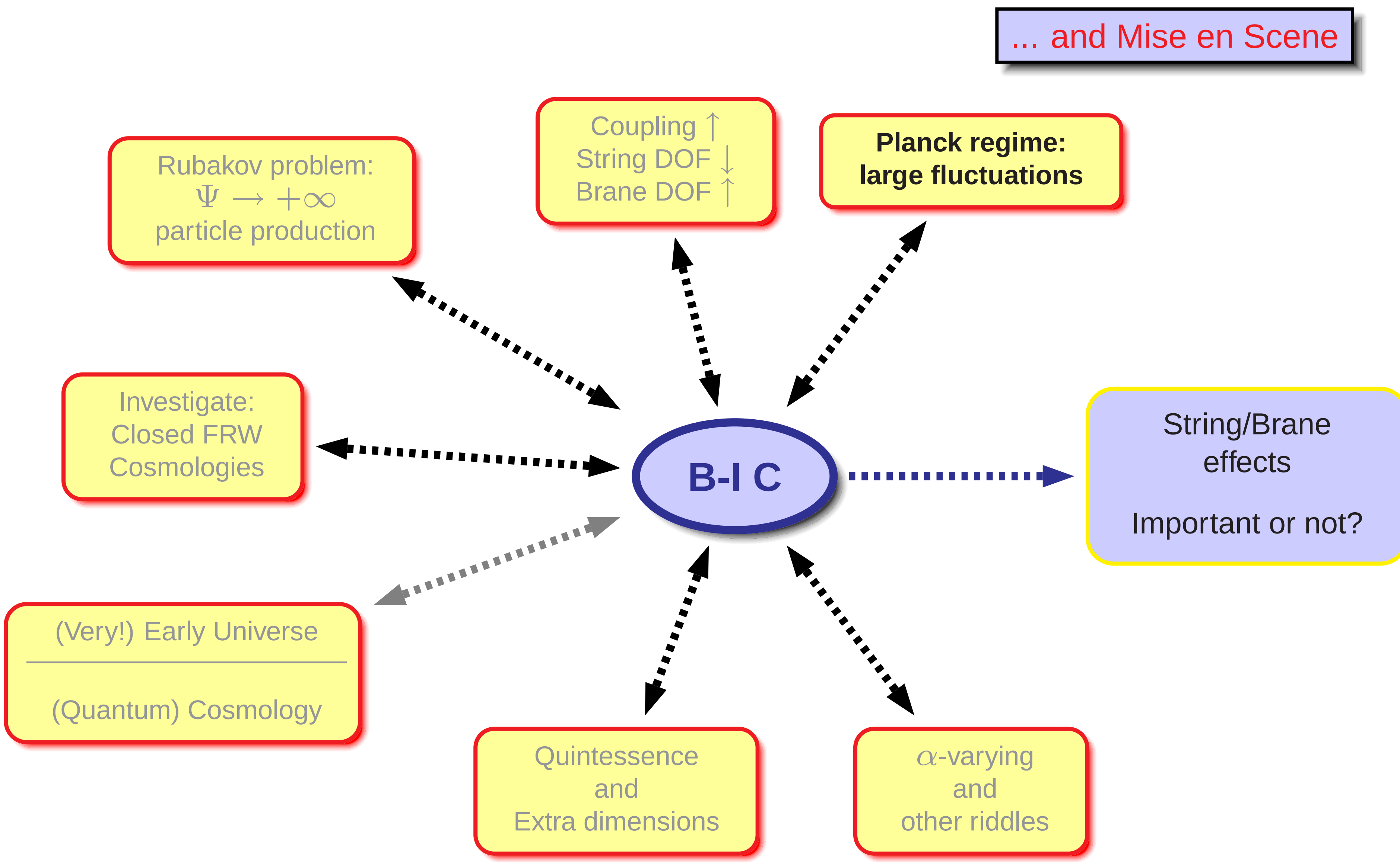


... and Mise en Scene




... and Mise en Scene
Rubakov problem:
$\Psi \longrightarrow +\infty$
particle production
Coupling ↑
String DOF ↓
Brane DOF ↑
Planck regime:
large fluctuations
Investigate:
Closed FRW
Cosmologies
B-I C
String/Brane
effects
Important or not?
(Very!) Early Universe
(Quantum) Cosmology
Quintessence
and
Extra dimensions
α-varying
and
other riddles

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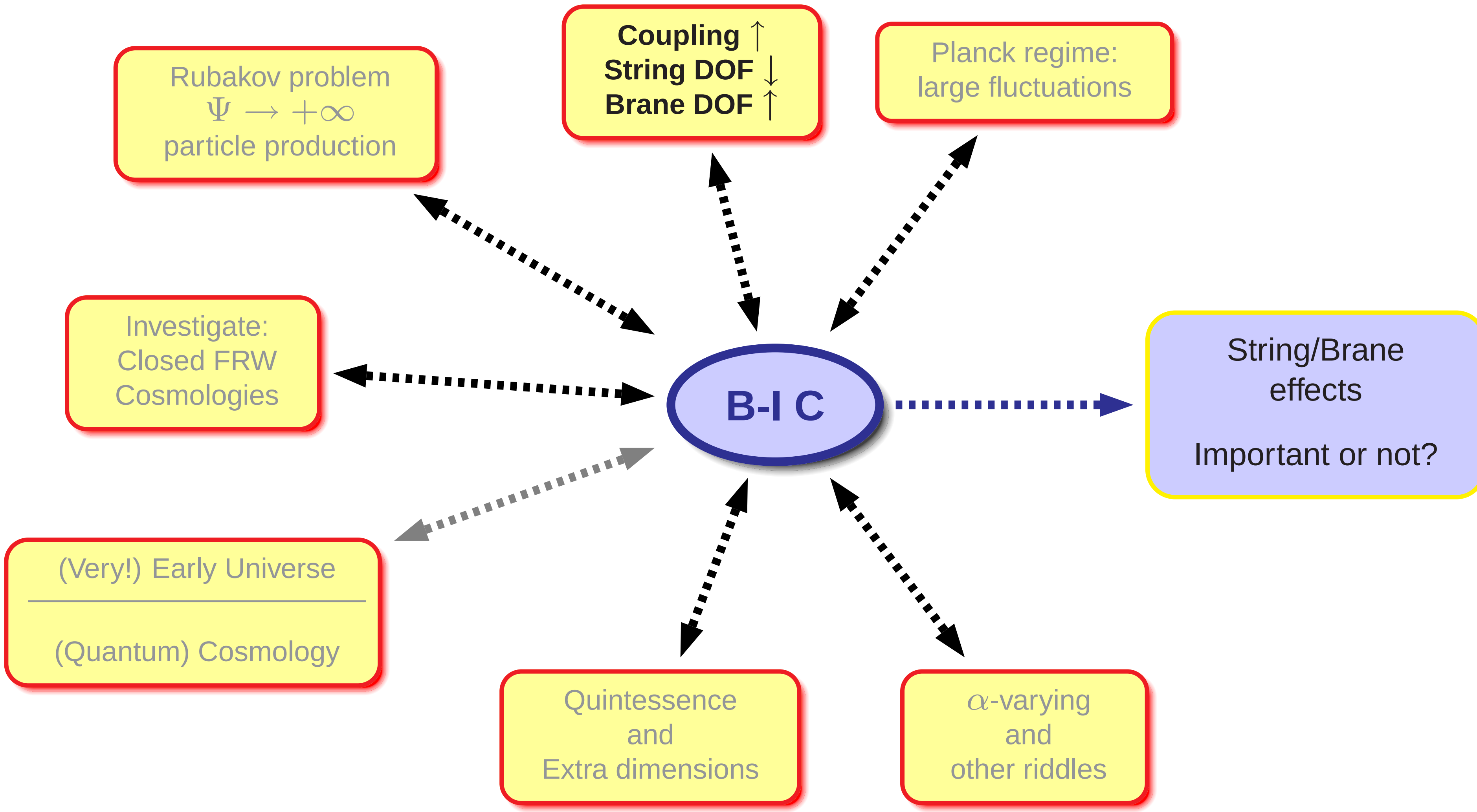

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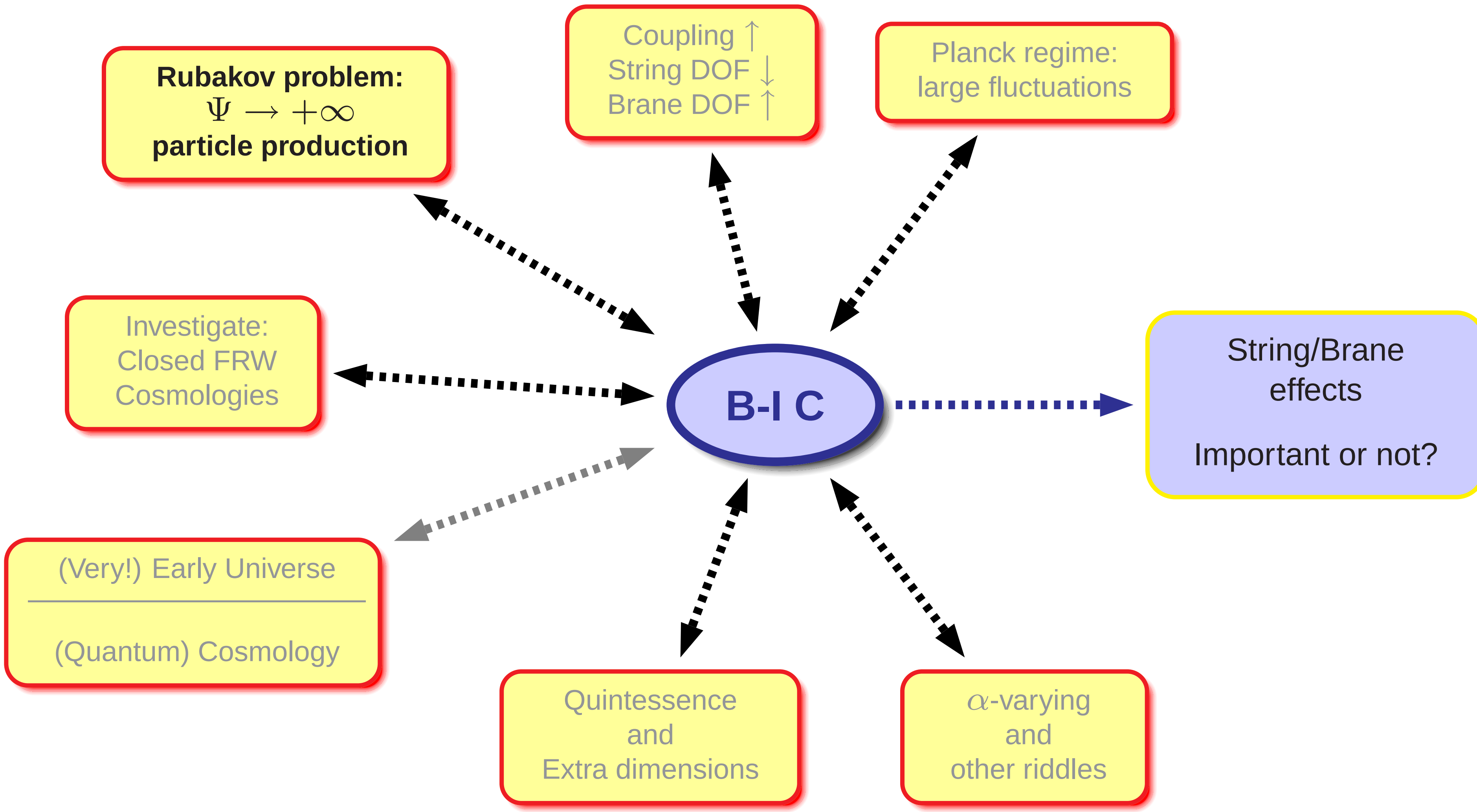

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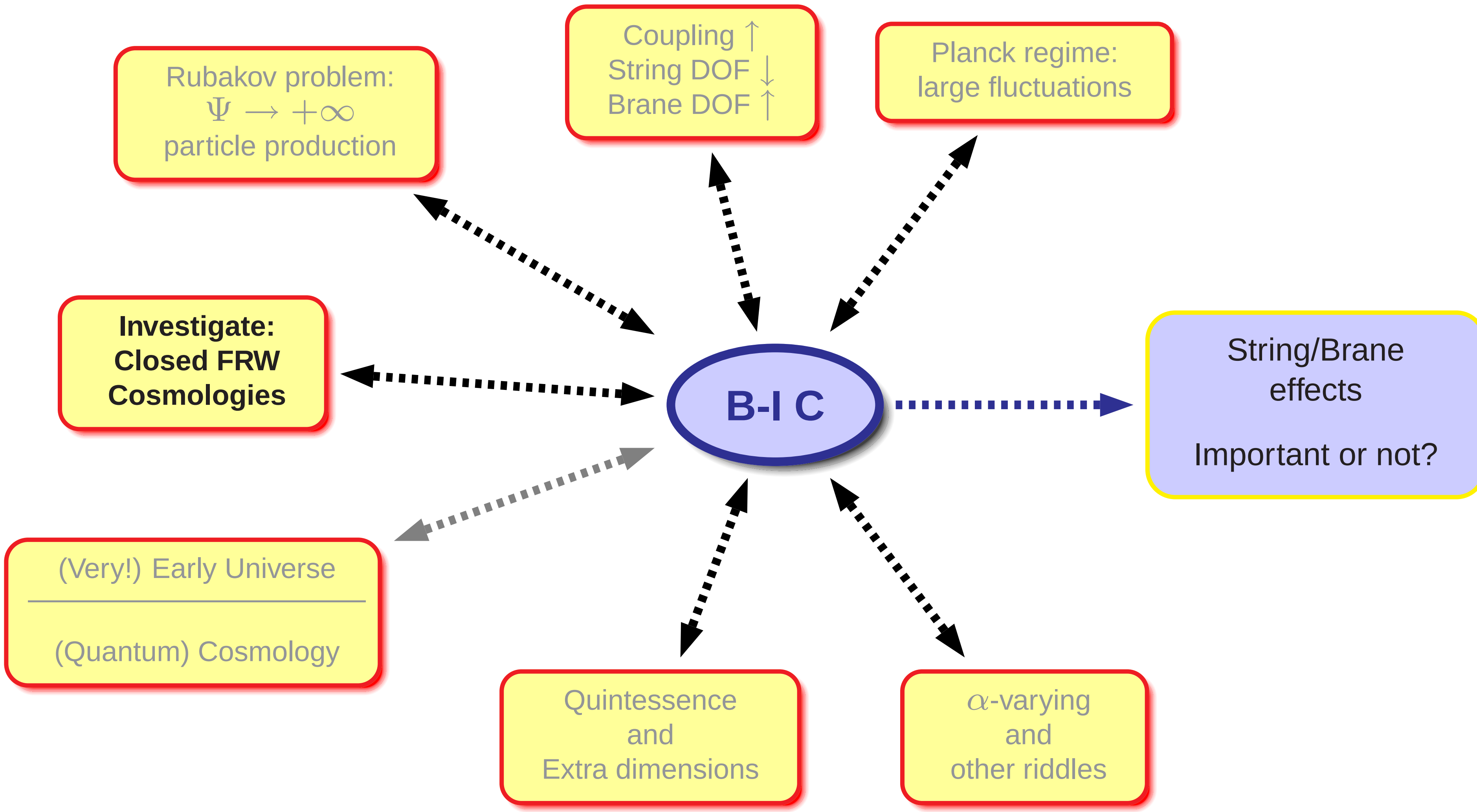

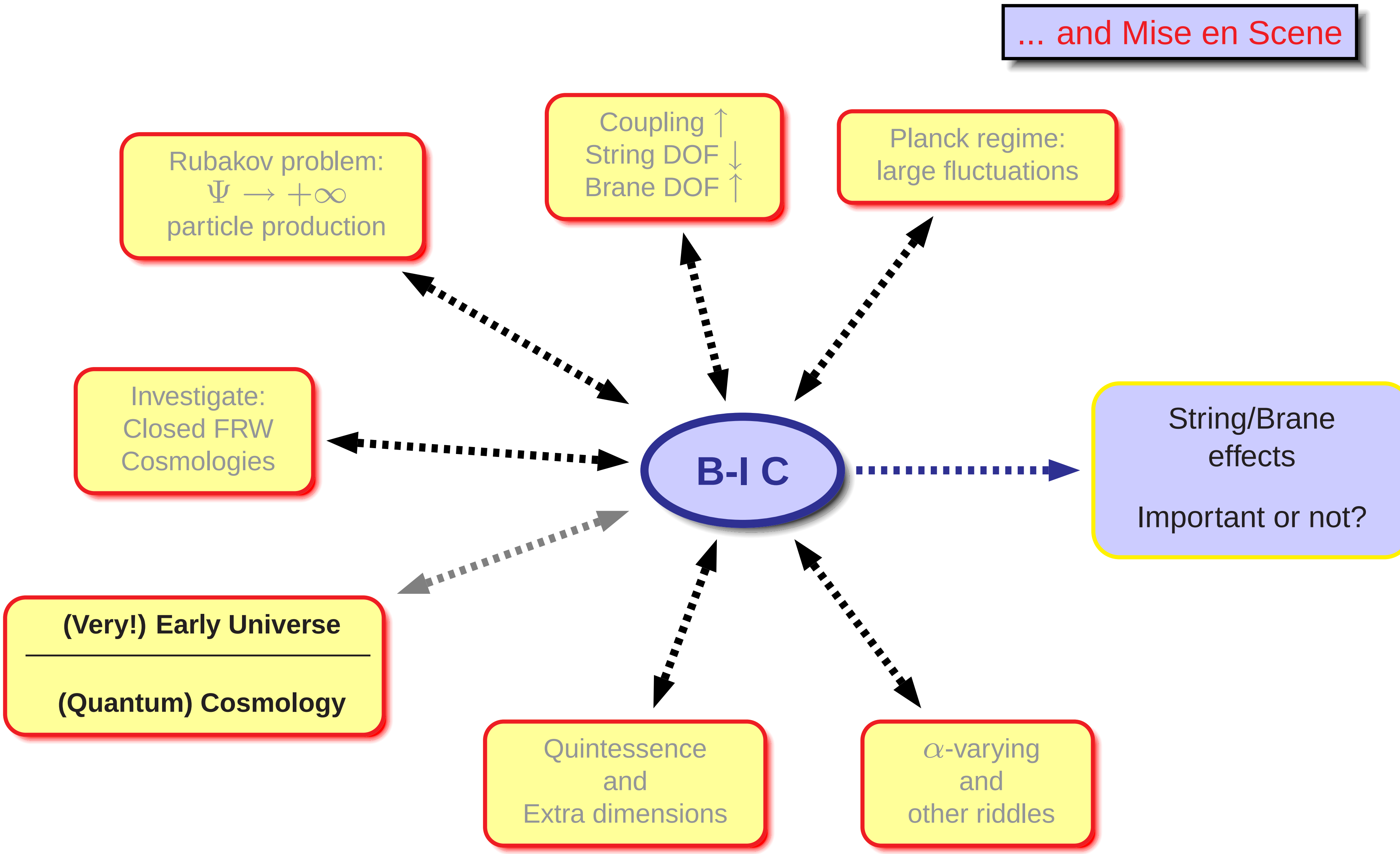
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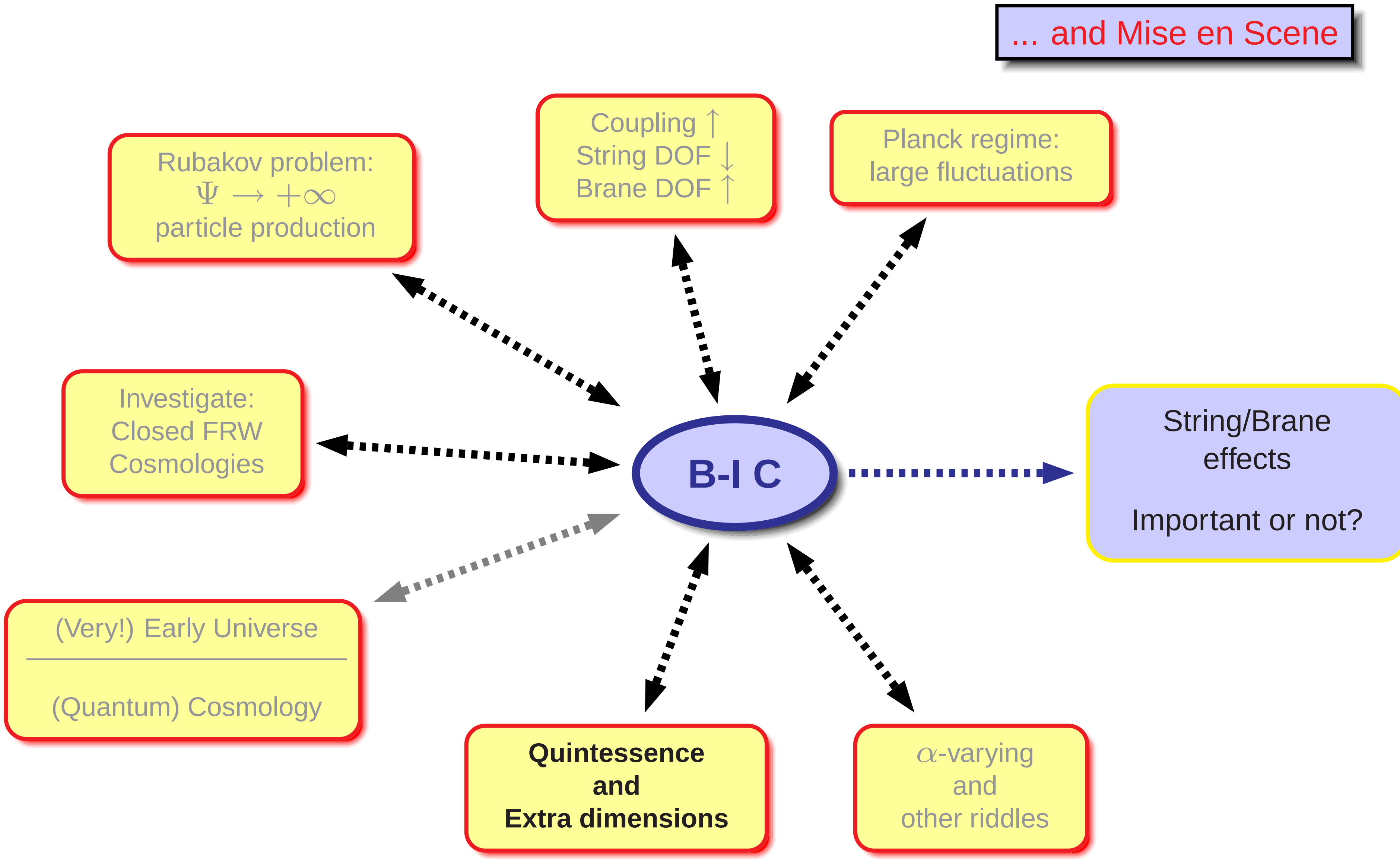
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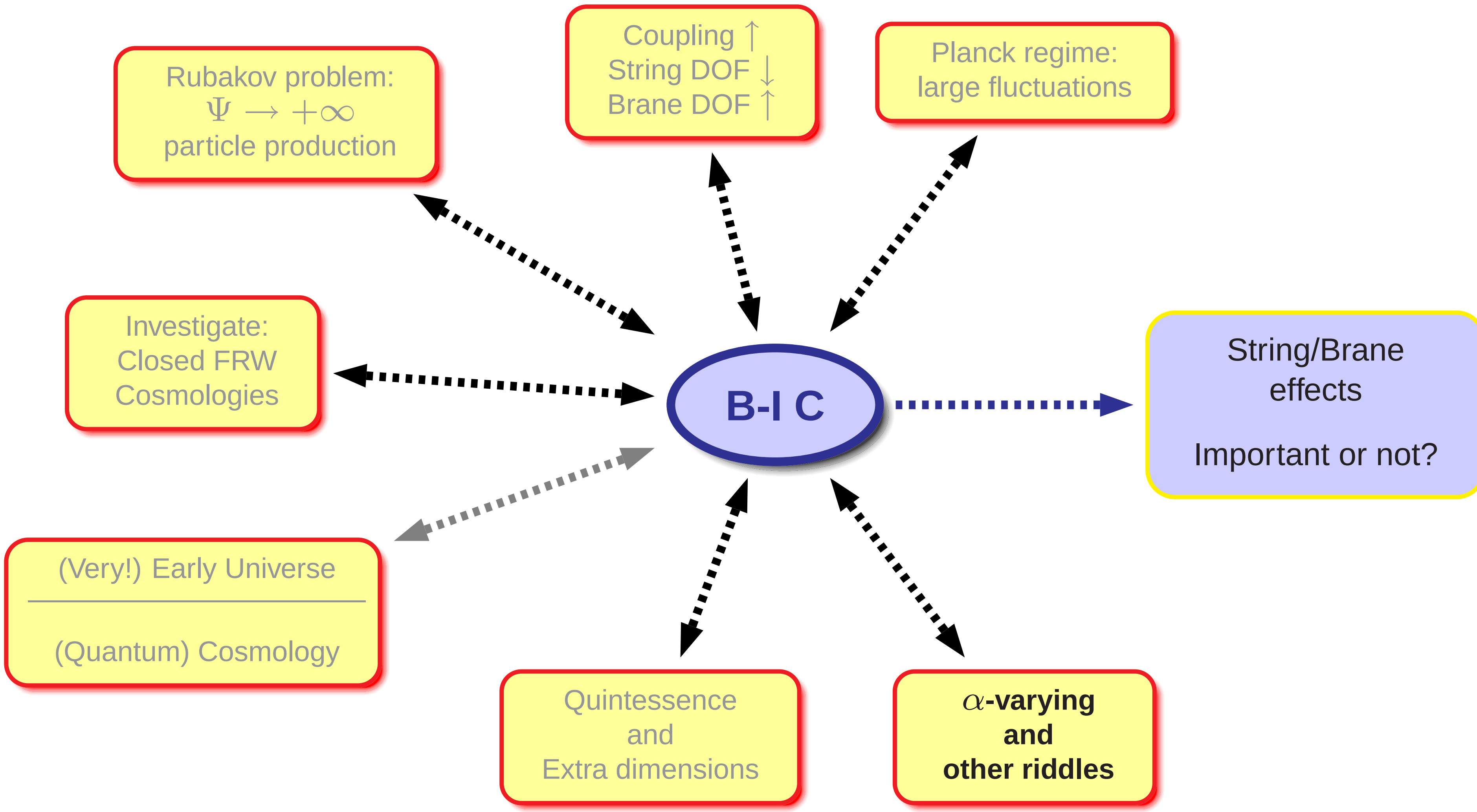

Action:

$$S = \int \frac{(R - 2\Lambda)}{16\pi G}\sqrt{(-g)}d^4x - \int \frac{\beta^2\,(\Re - 1)}{4\pi}\sqrt{(-g)}d^4x,$$

$$\Re = \left[1 - \frac{1}{2\beta^2}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{16\beta^4}\left(\widetilde{F}^a_{\mu\nu}F^{a\mu\nu}\right)^2\right]^{1/2}.$$

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Closed FRW metric

$\hookrightarrow SO(4)$: group of spatial homogeneity and isotropy.

$$g = -N^2dt^2 + a^2(t)\sum_{b=1,2,3}\omega^b \otimes \omega^b,$$

Class of gauge fields $\longleftarrow$ homogeneity/isotropy.

$\hookrightarrow$ $SO(4)$-invariance: too restrictive.

$\hookrightarrow$ ... transform under $SO(4)$ transformations *if* compensated by gauge!

- Useful class: $SO(4)-$*symmetric* fields

$$\mathbf{A}_\mu(t) = \sum_{1 \le k < i \le N-3} \Lambda^{ki}(t) T^{(3)}_{k+3,i+3} dt + \frac{1}{4}(1 + f_0(t))\varepsilon_{acb} T^{(3)}_{ab} \omega^c,$$

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Reduced Lagrangian:

$$L = \frac{1}{4\pi G\beta}\left[-\frac{3}{2}\frac{\dot{a}a^2}{N} + \frac{3}{8}aN - Na^3\frac{\Lambda}{2}\right] - \frac{gNa^3}{4\pi G\beta}\left[\sqrt{1-\frac{3\dot{f}_0^2}{2a^2N^2}} - 1\right],$$

$\hookrightarrow$ $V_{gauge\ field} \simeq 0$; $\dot{f}_0(t) \neq 0$; $g \equiv G\beta$.

Friedmann equation:

$$\frac{3}{2}\dot{a}^2 + \frac{3}{8} - \frac{1}{2}\Lambda a^2 - g a^2 \left(\frac{1}{\sqrt{1 - \frac{3\dot{f}_0^2}{2a^2N^2}}} - 1 \right) = 0$$

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Hamiltonian constraint

$$\pi_a^2 + \frac{9}{4}a^2 - 3\Lambda a^4 - 6 g a^4 \left(\sqrt{1 + \frac{2}{3g^2}\frac{1}{a^4}\pi_{f_0}^2} - 1 \right) = 0.$$

First, consider large a, $\pi_{f_0}^2 = C$, small $\dot{f}_0$ small

$$\frac{2}{3g^2}\frac{1}{a^4}\pi_{f_0}^2 \ll 1.$$

$\hookrightarrow$ Wheeler-DeWitt equation

$$\left[-\frac{\partial^2}{\partial a^2} + \frac{9}{4}a^2 - 3\Lambda a^4 + \frac{2}{g}\frac{\partial^2}{\partial f_0^2} + \frac{1}{3g^3 a^4}\frac{\partial^4}{\partial f_0^4} + \cdots\right]\Psi = 0.$$

$\hookrightarrow$ Ansatz: $\Psi(a, f_0) = Q(a)e^{\pm k f_0}$

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String/brane corrections: Solutions $\longrightarrow$ Bessel functions

$$Q = \sqrt{a}Z_{\frac{1}{\delta}}\left(\frac{2\sqrt{D}}{\delta}a^{\frac{\delta}{2}}\right), \delta = -2, -6, -10$$

$\hookrightarrow$ Interpretation: Σ perfect fluids $\longleftarrow \gamma = 5/3, \gamma = 3, \gamma = 13/3.$

Second, investigate case of κ^2 large, a^4 small.

$\hookrightarrow k^2 < 0$, i.e., $-k^2 \equiv \kappa^2 > 0$, $ik = \kappa$.

$\hookrightarrow$ Assume $\frac{2}{3g^2}\frac{1}{a^4}k^2$ dominates

- Hamiltonian constraint:

$$\pi_a^2 + a^2\left(\frac{9}{4} \mp 2\sqrt{6}\,|\kappa|\right) - a^4\,(3\Lambda - 6g) + \cdots \simeq 0,$$

$\hookrightarrow \mp 2\sqrt{6}a^2\,|\kappa|$ and $6ga^4 \longleftarrow$ BI string gas contributions

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Quantum solutions $\longleftarrow$ Wheeler-DeWitt equation

$\hookrightarrow$ Hartle-Hawking state $\longrightarrow Q^{HH} \sim Ai[z(a)]$

$\hookrightarrow$ Vilenkin state $\longrightarrow Q^V \sim C_1 Ai[z(a)] + C_2 iBi[z(a)]$

... **Calculations**! $\longrightarrow$ Physical Consequences !? ...

- Effective Friedmann equation

$$\dot{a}^2 + \left(\frac{1}{4} - \hat{b}\right) - \left(\frac{\Lambda}{3} - \hat{c}\right) a^2 \equiv \dot{a}^2 + f - ga^2 = 0.$$

$\hookrightarrow$ Solution: DeSitter-like universe $a = \sqrt{\frac{f}{g}} \cosh \sqrt{g} t$

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Wheeler-DeWitt equation ($R_0 \equiv \sqrt{\frac{\Lambda}{3}}$)

$$\frac{\partial^2 Q}{\partial a^2} - \frac{9}{4} R_0^2 \left[a^2 R_0^{-2} \tilde{f} - a^4 R_0^{-4} \tilde{g} \right] Q = 0,$$

$\hookrightarrow$ Potential $V(a) = \frac{9}{4} R_0^2 \left[a^2 R_0^{-2} \tilde{f} - a^4 R_0^{-4} \tilde{g} \right]$

- Turning points: $a_1 = 0; \quad a_2 = \sqrt{\frac{\tilde{f}}{\tilde{g}}} \sqrt{\frac{3}{\Lambda}}.$

$\hookrightarrow$ Shorter distance between the turning points $\leftrightarrow \pi_{f_0}$ increases

Potential extremum at $a_0 = \frac{1}{\sqrt{2}}\sqrt{\frac{\tilde{f}}{\tilde{g}}}R_0 \Rightarrow V(a_0) = \frac{\tilde{f}^2}{4\tilde{g}}$,

★ **Similar** effect: Friedmann eq. – original action (wo/ approximations)

$$\dot{a}^2 + 1 - a^2\left(\frac{\Lambda}{3} + D\right) - D\sqrt{a^4 + 16d\pi_{f_0}^2} = 0,$$

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$$\dot{a}^2 + 1 - a^2\left(\frac{\Lambda}{3} + D\right) - D\sqrt{a^4 + 16d\pi_{f_0}^2} = 0,$$

Probability of tunneling: ”increased”...!...?

$\hookrightarrow \Gamma \simeq e^{-S_E}$, $S_E = -\frac{2\Lambda}{16\pi G}V_4$: Euclidean action

$\hookrightarrow V_4$: Volume of the 4-sphere $\leftarrow a \sim \sqrt{\frac{\tilde{f}}{\tilde{g}}}\sqrt{\frac{3}{\Lambda}}$.

- Smaller Euclidean action $\leftrightarrow \neq$ no Born-Infeld matter sector.

Wormholes: *non*-singular solutions $\leftrightarrow$ Euclidean equations

$\hookrightarrow$ Only some types of geometry/matter

$\hookrightarrow$ Ricci ($R_{\mu\nu}$) eigenvalues: negative

$\hookrightarrow$ Connect two asymptotically flat regions

$\hookrightarrow$ Transition: $r_{min} \leftrightarrow S^3 \longleftrightarrow r_{max} \leftrightarrow S^3$

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Action ($\longleftarrow$ Euclidean)

$$S = -\frac{1}{4\pi}\left[\int \frac{(R-2\Lambda)}{4G}\sqrt{(-g)}d^4x - \beta^2\left(\Re - 1\right)\right]$$

$$\Re = \left[1 + \frac{1}{2\beta^2}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{16\beta^4}\left(\widetilde{F}^a_{\mu\nu}F^{a\mu\nu}\right)^2\right]^{1/2}.$$

$\hookrightarrow$ Metric and fields: $t \rightarrow \tau$

Lagrangian:

$$L = \frac{1}{4\pi G e}\left[-\frac{3}{2}\frac{\dot{a}a^2}{N} - \frac{3}{8}aN + Na^3\frac{\Lambda}{2} + gNa^3\left(\Re - 1\right)\right],$$

$\hookrightarrow g \equiv G\beta$, $t \rightarrow \beta^{-1/2}t$, $a \rightarrow \beta^{-1/2}a$.

$$\begin{aligned}\Re &= \left[1 + \frac{3\dot{f}_0^2}{2a^2N^2} + \frac{3V_1}{a^4} + \frac{9\dot{f}_0^2V_1}{2a^6N^2}\right]^{1/2} \\ &= \left[\left(1 + K^2\right)\left(1 + V^2\right)\right]^{1/2},\end{aligned}$$

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$$K^2 \equiv \frac{3\dot{f}_0^2}{2a^2N^2}, V^2 = \frac{3V_1}{a^4}, V_1 = V_{gauge\ field} = \frac{1}{8}(1 - f_0^2)^2.$$

Equations of motion:

$\hookrightarrow$ Friedmann equation

$$\frac{\dot{a}^2}{a^2} - \frac{1}{4a^2} + \frac{\Lambda}{3} = -\beta G \frac{2}{3} \left(\mathsf{P} - 1 \right) \equiv -\frac{8\pi G}{3}\varepsilon, \quad \varepsilon = \varepsilon_c \left(\mathsf{P} - 1 \right), \varepsilon_c = \frac{\beta}{4\pi}.$$

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Einstein equation

$$\frac{\ddot{a}}{a} = -\frac{\Lambda}{3} + \frac{4\pi G}{3}\varepsilon - \frac{g}{3}\left(\frac{\varepsilon}{\varepsilon_c} + 1\right) - \frac{2g}{3}\left(\frac{\varepsilon}{\varepsilon_c + \varepsilon}\right) + g = -\frac{\Lambda}{3} + \frac{4\pi G}{3}(\varepsilon + 3P),$$

Analysis: Interpolating regimes

$$\dot{a}^2 - \frac{1}{4} + \frac{\Lambda}{3}a^2 = -\frac{8\pi G}{3}\begin{cases} C \leftarrow string\, effects \\ \frac{C}{a^2} \leftarrow YM - Radiation \end{cases} .$$

Analysis: Interpolating regimes

$$\dot{a}^2 - \frac{1}{4} + \frac{\Lambda}{3}a^2 = -\frac{8\pi G}{3}\begin{cases} C \leftarrow string\,effects \\ \frac{C}{a^2} \leftarrow YM - Radiation \end{cases}.$$

Wormhole ... possible in YM-radiation limit

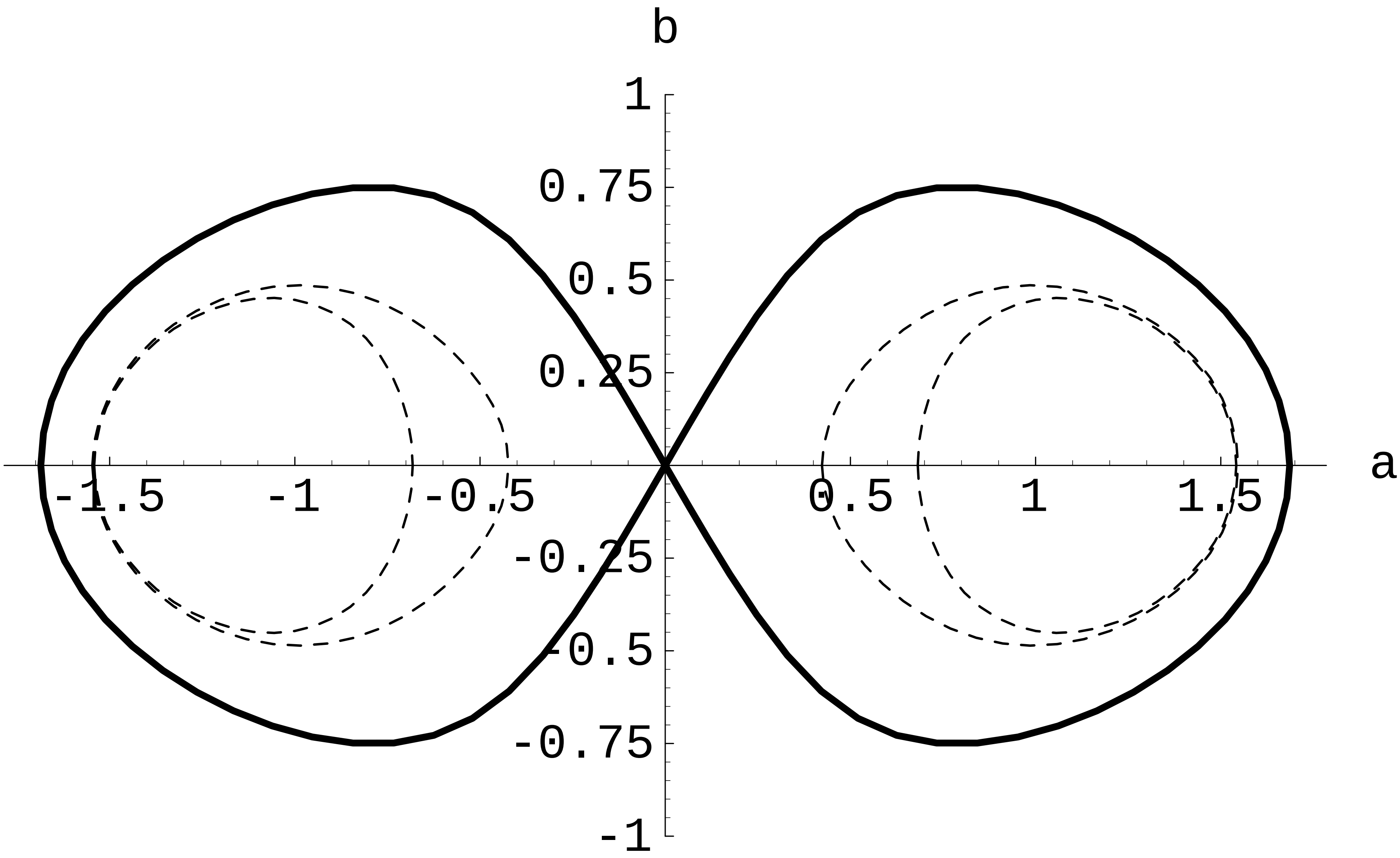
$\hookrightarrow$ Wormhole ... NOT possible in the string/brane limit

$\hookrightarrow$ **Investigate:** BI perturbations in wormhole scenario

- Wormhole solutions and BI modifications

$\hookrightarrow$ Analytical (approximated ...)

$$\tau - \tau_0 = \pm\frac{1}{2}\frac{1}{\sqrt{H}}\left(\arcsin\frac{2\frac{H}{\frac{1}{4}-F}a^2 - 1}{\sqrt{1 - 4\frac{H}{\frac{1}{4}-F}\frac{E}{\frac{1}{4}-F}}}\right).$$


b
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
-1
a
-1.5
-1
-0.5
0.5
1
1.5

Integral trajectories (curves)

$$3\left(b^2+\frac{\Lambda}{3}-1\right)^2-4ga^2\left(b^2+\frac{\Lambda}{3}-1\right)=\mathtt{C}.$$

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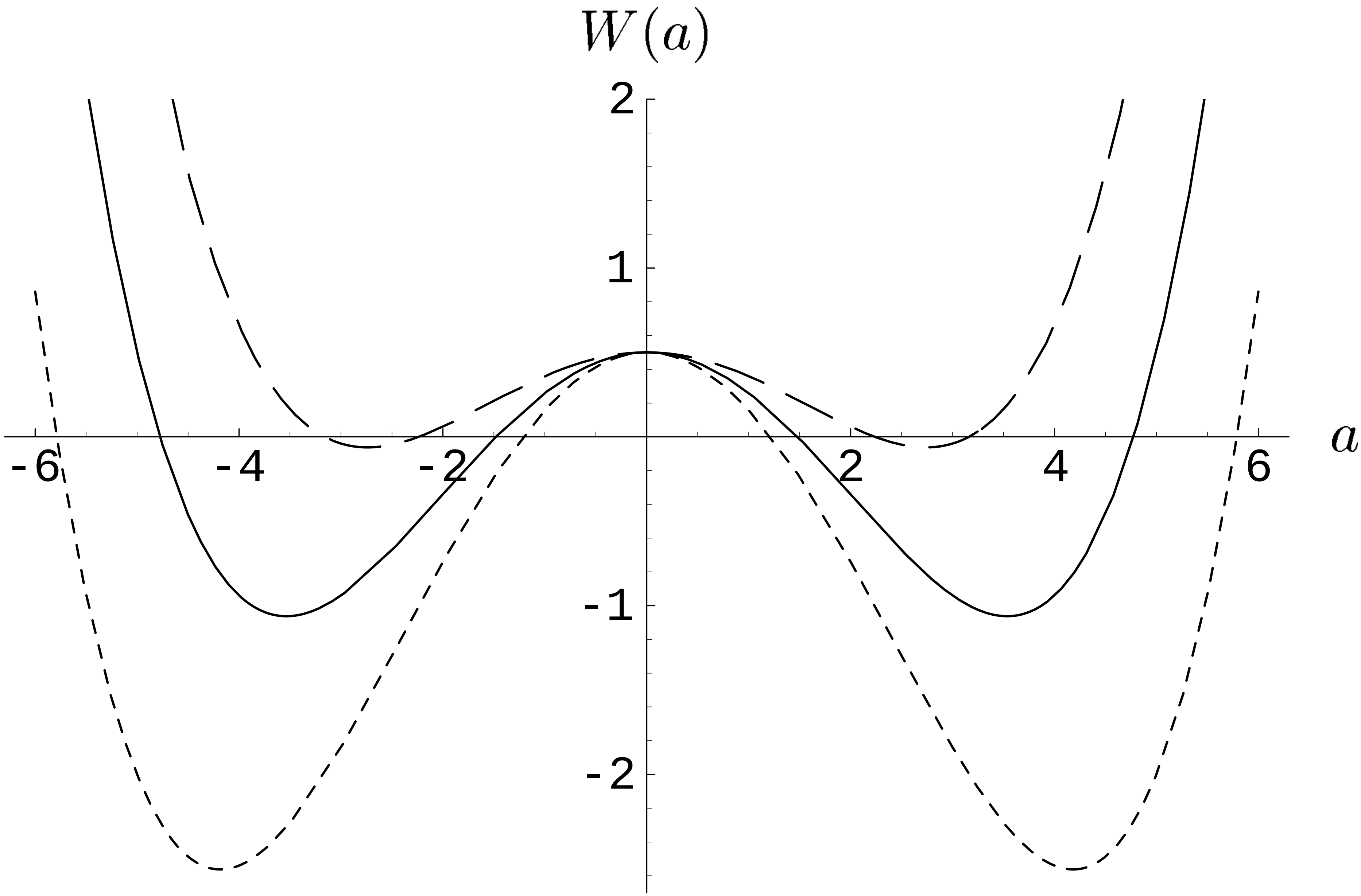
$$3\left(b^2+\frac{\Lambda}{3}-1\right)^2-4ga^2\left(b^2+\frac{\Lambda}{3}-1\right)=\mathtt{C}.$$

Physical consequences (BI modifications):

$\hookrightarrow$ Widening/Shortening between turning points

$\hookrightarrow$ Energy level quantization modified

$\hookrightarrow$ Gauge field vacuum $\leftrightarrow$ source: fermionic current


$W(a)$
2
1
-1
-2
-6
-4
-2
2
4
6
a

Quintessence & Extra Dimensions:

$\ast$ Recent measurements

$\hookrightarrow$ Expanding universe $\longrightarrow$ currently *accelerating* $\leftrightarrow q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2} < 0$

$\hookrightarrow$ SN-type Ia, CMBR peaks, mass spectrum

Quintessence & Extra Dimensions:

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... ”and the nominees are”

$\hookrightarrow \langle 0 \rangle \leftrightarrow \Lambda_{eff}$

$\hookrightarrow$ Dynamical energy (quintessence) $\leftrightarrow \phi$

$\hookrightarrow V(\phi) \longleftarrow$ shallow potential (e.g., exponential)

$\hookrightarrow$ ... damped until recently by expansion

$\hookrightarrow$ Non-minimal coupling, ... , ..., ...

Explore:

- gauge fields (BI) & extra dimension $\longleftarrow \mathcal{M}^D = M^4 \times I^d$

$$S[\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{A}_{\hat{\mu}}] = -\int_{\mathcal{M}^D} d\hat{x}\sqrt{-\hat{g}}\left[\frac{\hat{R}-2\hat{\Lambda}}{16\pi\hat{k}} + \frac{\hat{\beta}^2}{4\pi}\left(\hat{\Re}-1\right)\right] ,$$

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Metric:

$$\hat{g} = -\tilde{N}^2(t)dt^2 + \tilde{a}^2(t)\Sigma_{i=1}^{3}\omega^i\omega^i + b^2(t)\Sigma_{m=4}^{d+3}\omega^m\omega^m ,$$

Gauge field:

$$\hat{A} = \frac{1}{2}\Sigma_{p,q=1}^{N-3-d} B^{pq}(t)\mathcal{T}^{(N)}_{3+d+p\,3+d+q}dt + \frac{1}{2}\Sigma_{1\leq i<j\leq 3}\mathcal{T}^{(N)}_{ij}\omega^{ij} + \frac{1}{2}\Sigma_{4\leq m<n\leq 3}\mathcal{T}^{(N)}_{mn}\tilde{\omega}^{m-}$$

$$+ \Sigma_{i=1}^{3}\left[\frac{1}{4}f_0(t)\Sigma_{j,k=1}^{3}\epsilon_{jik}\mathcal{T}^{(N)}_{jk} + \frac{1}{2}\Sigma_{p=1}^{N-3-d}f_p(t)\mathcal{T}^{(N)}_{i\,d+3+p}\right]\omega^i + \Sigma_{m=4}^{d+3}\left[\frac{1}{2}\Sigma_{q=1}^{N-3-d}\right.$$

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Effective action:

$$S_{\text{eff}} = -16\pi^2\int dtNa^3\left\{-\frac{3}{8\pi k}\frac{1}{a^2}\left[\frac{\dot{a}}{N}\right]^2 + \frac{3}{32\pi k}\frac{1}{a^2} + \frac{1}{2}\left[\frac{\dot{\psi}}{N}\right]^2 - \Omega\right\},$$

$$\Omega(a,\psi) = \frac{1}{4\pi}v_d b_0^d e^{d\psi\gamma}\hat{\beta}^2 e^{-2d\gamma\psi}\left(\left[1 + e^{2d\gamma\psi}\frac{3}{\hat{\beta}^2\hat{\epsilon}^2a^4}v_1 + e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2\right]^{1/2} -\right.$$

$$+ e^{-d\gamma\psi}\left[-e^{-2\gamma\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2} + \frac{\Lambda}{8\pi k}\right],$$

$\hookrightarrow k = \hat{k}/v_d b_0^d\ \epsilon^2 = \hat{\epsilon}^2/v_d b_0^d,\ \gamma = \sqrt{16\pi k/d(d+2)},\ \psi = \gamma^{-1}\ln(b/b_0),\ \Lambda = v_d b_0^d\hat{\Lambda}.$

Equations of motion & **Analysis**:

$\hookrightarrow$ Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{4a^2} + \frac{8\pi k}{3}\left(\frac{\dot{\psi}^2}{2} + \Omega(a,\psi) + \rho\right),$$

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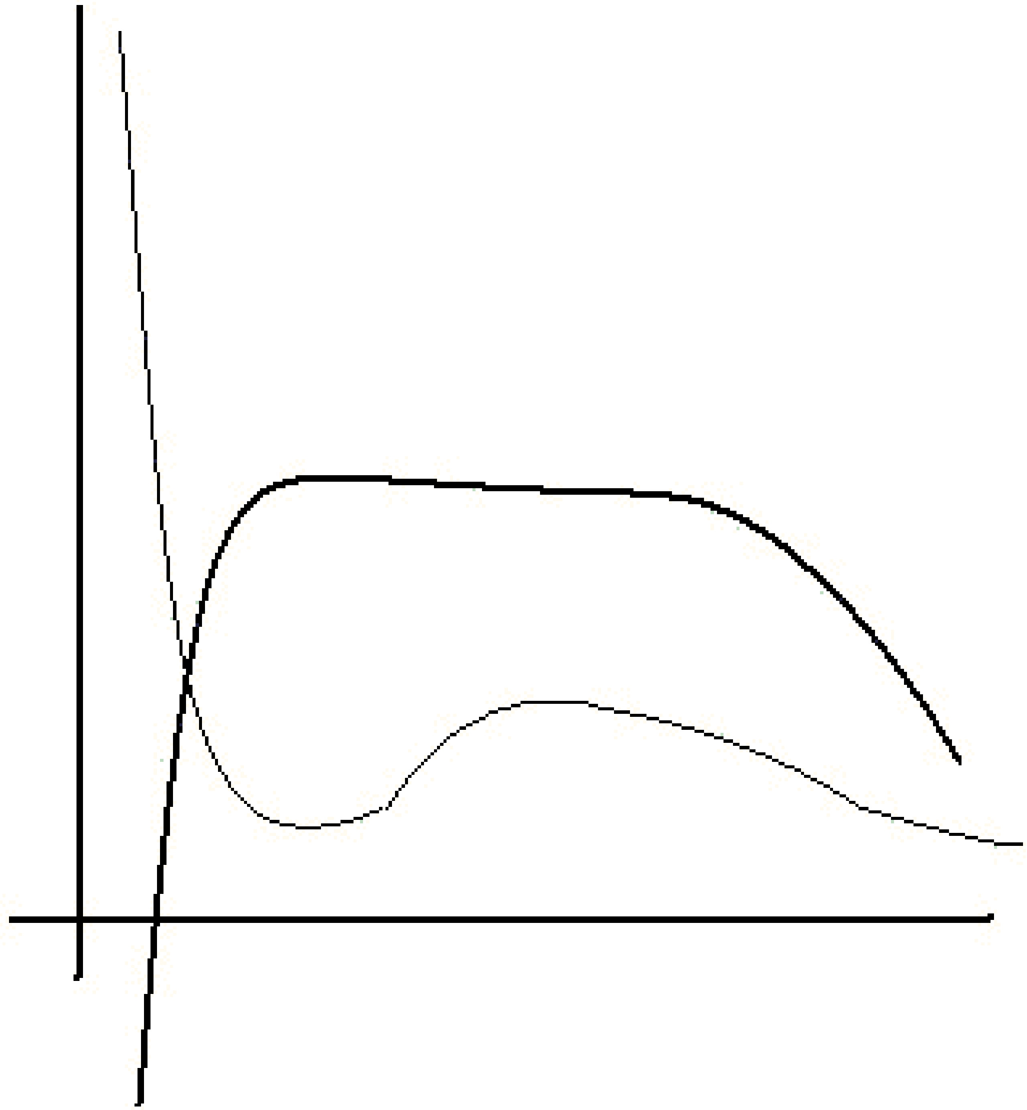
Raychaudhuri equation

$$\ddot{a} = -\frac{8\pi k}{3}a\dot{\psi}^2 - \frac{4\pi k}{3}a\left(\frac{\rho_0 a_0^3}{a^3}\right) + \frac{8\pi k}{3}a\Omega + \frac{4\pi k}{3}a^2\frac{\partial\Omega}{\partial a}.$$

$\hookrightarrow$ Deceleration parameter

$$q = \frac{\frac{8\pi k}{3}a^2\dot{\psi}^2 + \frac{4\pi k}{3}\frac{\rho_0 a_0^3}{a^3} - \frac{8\pi k}{3}a^2\Omega}{-\frac{1}{4} + \frac{8\pi k}{3}a^2\dot{\psi}^2 + \frac{8\pi k}{3}\frac{\rho_0 a_0^3}{a^3} + \frac{8\pi k}{3}a^2\Omega}$$



Case 1: $e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2 \gg 1,$

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Cosmological potential

$$\begin{aligned}\Omega(a,\psi) &\simeq \frac{1}{4\pi}\frac{\hat{\epsilon}}{\hat{\beta}}e^{-d\psi\gamma}e^{-2\gamma\psi}\frac{1}{b_0^2}\frac{\sqrt{d(d-1)}}{\sqrt{2}\epsilon^2}\sqrt{v_2} \\ &- e^{-d\gamma\psi}\left[e^{-2\gamma\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2} - \frac{\Lambda}{8\pi k}\right].\end{aligned}$$

Compactification exists $\leftrightarrow \psi_{ext} \simeq 0$ with

$$\Lambda = \frac{8\pi k(d+2)}{b_0^2}\left(\frac{d(d-1)}{8} - \sqrt{2}\frac{\hat{\epsilon}}{\hat{\beta}}\frac{\sqrt{d(d-1)}}{\epsilon^2}\sqrt{v_2}\right)$$

$\hookrightarrow$ Local *maximum* $\leftrightarrow$ unstable $\longleftarrow$ Transient effect ... !? ...

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Admiting negative values ... fine tuning

$$q_0 \simeq \frac{\frac{\Omega_0^m H_0^2 a_0^2}{2} - a_0^2\frac{\Lambda}{3}\left(\frac{d-1}{d+2}\right)}{-\frac{1}{4} + \Omega_0^m H_0^2 a_0^2 + a_0^2\frac{\Lambda}{3}\left(\frac{d-1}{d+2}\right)},$$

Case 2: $e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2 \ll 1,$

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Cosmological potential

$$\Omega \simeq e^{-d\gamma\psi}\left[Ae^{-4\gamma\psi} + Be^{-8\gamma\psi} - Ce^{-2\gamma\psi} + D\right],$$

$\hookrightarrow A = \frac{1}{16\pi}\frac{1}{b_0^4}\frac{d(d-1)}{\hat{\epsilon}^2\hat{\beta}^2}v_2$, $B = \frac{1}{4\pi}\frac{1}{32}\frac{1}{b_0^8}\frac{d^2(d-1)^2}{\hat{\epsilon}^2\hat{\beta}^2}\frac{v_2^2}{\varepsilon^2}$, $C = \frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2}$,
$D = \frac{\Lambda}{8\pi k}$.

Compactification exists $\leftrightarrow \psi_{ext} \simeq 0$ with

$$\Lambda < \frac{d-1}{16k}\frac{(d+2)^2}{d+4}\frac{\epsilon^2 v_2}{64}\frac{1}{\xi^2}, \quad \Lambda \simeq \frac{d(d-1)}{16b_0^2}\frac{(1+\delta)}{\xi},$$

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$$q_0 \simeq \frac{\frac{\Omega_0^m H_0^2 a_0^2}{2} - \frac{a_0^2}{b_0^2}\frac{d(d-1)}{48}\frac{\delta}{\xi}}{-\frac{1}{4} + \Omega_0^m H_0^2 a_0^2 + \frac{a_0^2}{b_0^2}\frac{d(d-1)}{48}\frac{\delta}{\xi}}.$$

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α-varying BI cosmology

- Recent observations – QSO$\longrightarrow$ α change in cosmological time

$\ast$ ... **BI framework** $\longleftarrow$ "early" string/brane inprints (?...)

$$L = \sqrt{-g}\left[\frac{1}{16\pi G}(R-2\Lambda) + L_M - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi + L_{Max}e^{-2\psi} + \frac{1}{\beta^2}L^2_{Max}e^{-4\psi}\right]$$

$$L_{BI} = -\frac{1}{2}\beta^2\left(1-\sqrt{1-\frac{1}{\beta^2}F^2}\right) \rightarrow L_{Max}(\beta\rightarrow\infty)$$

($e_0 \rightarrow e = e_0\varepsilon(x) \rightleftharpoons A_\mu \rightarrow \varepsilon A_\mu \leftrightarrow \varepsilon A_\mu + \chi_{,\mu}, F_{\mu\nu} = \frac{1}{\varepsilon}\left[(\varepsilon A_\nu)_{,\mu} - (\varepsilon A_\mu)_{,\nu}\right]$)

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FRW:

$$\begin{aligned} L \quad = \quad & -\frac{3\pi}{4G}\left(\frac{\dot{a}^2 a}{N} - Nka\right) + 2\pi^2 a^3\varepsilon\frac{\dot{\psi}^2}{2N} - 2\pi^2 a^3 N\frac{\Lambda}{8\pi G} - 2\pi^2 a^3 N\rho_0 a_0^{3(1+\gamma)}a^{-3(1+\gamma)} \\ & -2\pi^2 a^3 N\rho_{0r}a_0^4 a^{-4}e^{-2\psi} - 2\pi^2 a^3 N\rho_{0m}\xi a_0^3 a^{-3}e^{-2\psi} \\ & -\frac{1}{\beta^2}2\pi^2 a^3 N\rho_r^2 e^{-4\psi} - \frac{1}{\beta^2}2\pi^2 a^3 N\xi^2\rho_m^2 e^{-4\psi} - \frac{1}{\beta^2}4\pi^2 a^3 N\xi\rho_r\rho_m e^{-4\psi} \end{aligned}$$

Equations of motion: Dilaton

$$\ddot{\psi} + 3H\dot{\psi} \simeq \frac{2}{\omega}\xi\rho_m e^{-2\psi} + \frac{1}{\beta^2}\frac{4}{\omega}\xi^2\rho_m^2 e^{-4\psi}$$

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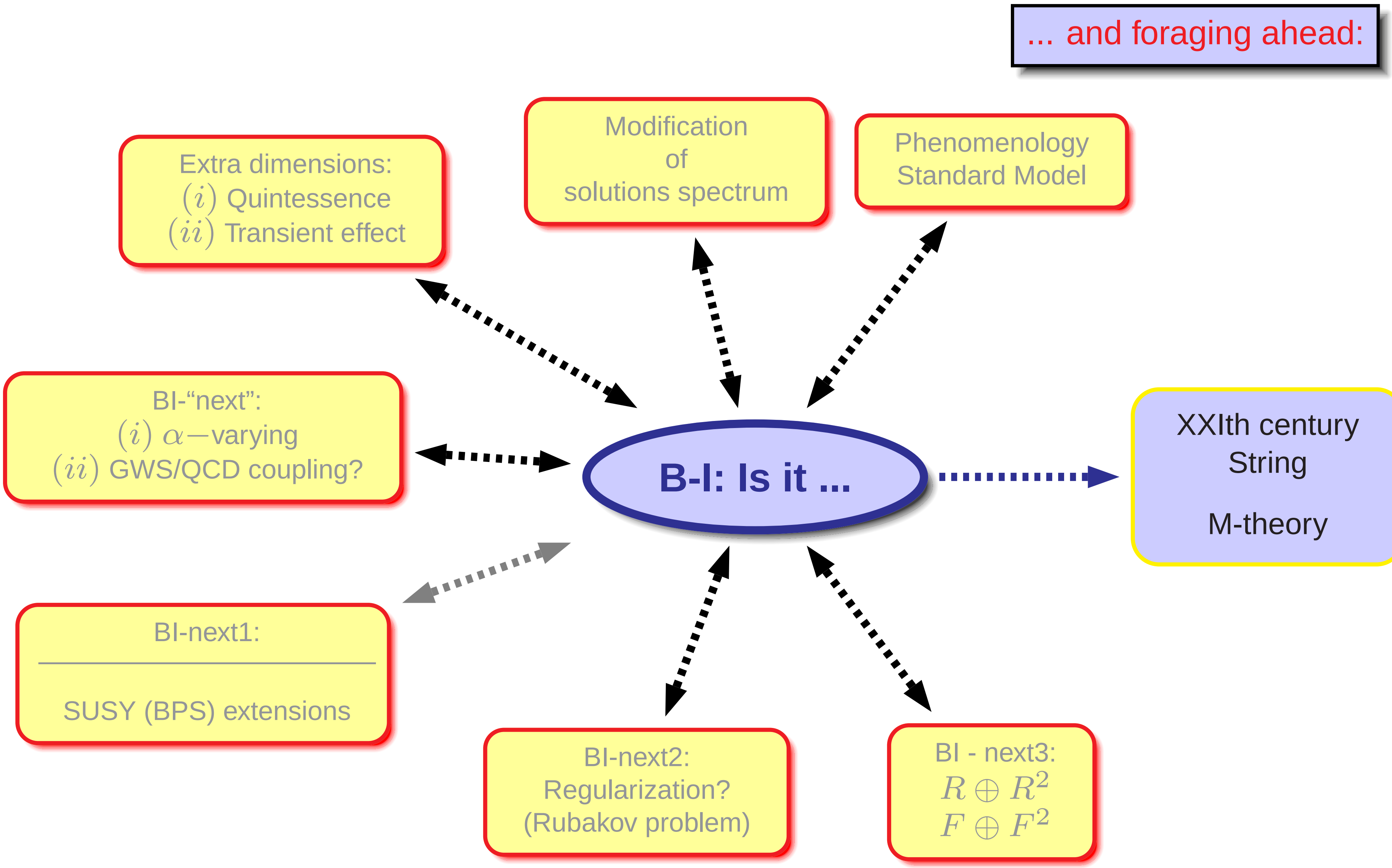
Equations of motion: Friedmann and Einstein

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{8\pi G}{3}\left[\omega\frac{\dot{\psi}^2}{2} + \rho_m + \rho_r e^{-2\psi} + \xi\rho_m e^{-2\psi} + \frac{1}{\beta^2}\xi^2\rho_m^2 e^{-4\psi}\right]$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \Lambda - 8\pi G\left[\omega\frac{\dot{\psi}^2}{2} + \frac{\rho_r}{3}e^{-2\psi} + \frac{1}{\beta^2}\xi^2\rho_m^2 e^{-4\psi}\right]$$

$$\begin{aligned} \rho_T &= \omega\frac{\dot{\psi}^2}{2} + \rho_m + \rho_r e^{-2\psi} + \xi\rho_m e^{-2\psi} + \frac{1}{\beta^2}\xi^2\rho_m^2 e^{-4\psi} \\ P_T &= \omega\frac{\dot{\psi}^2}{2} + \frac{\rho_r}{3}e^{-2\psi} + \frac{1}{\beta^2}\xi^2\rho_m^2 e^{-4\psi} \end{aligned}$$


... and foraging ahead:
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(ii) Transient effect
Modification
of
solutions spectrum
Phenomenology
Standard Model
BI-“next”:
(i) $\alpha-$varying
(ii) GWS/QCD coupling?
B-I: Is it ...
XXIth century
String
M-theory
BI-next1:
SUSY (BPS) extensions
BI-next2:
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(Rubakov problem)
BI - next3:
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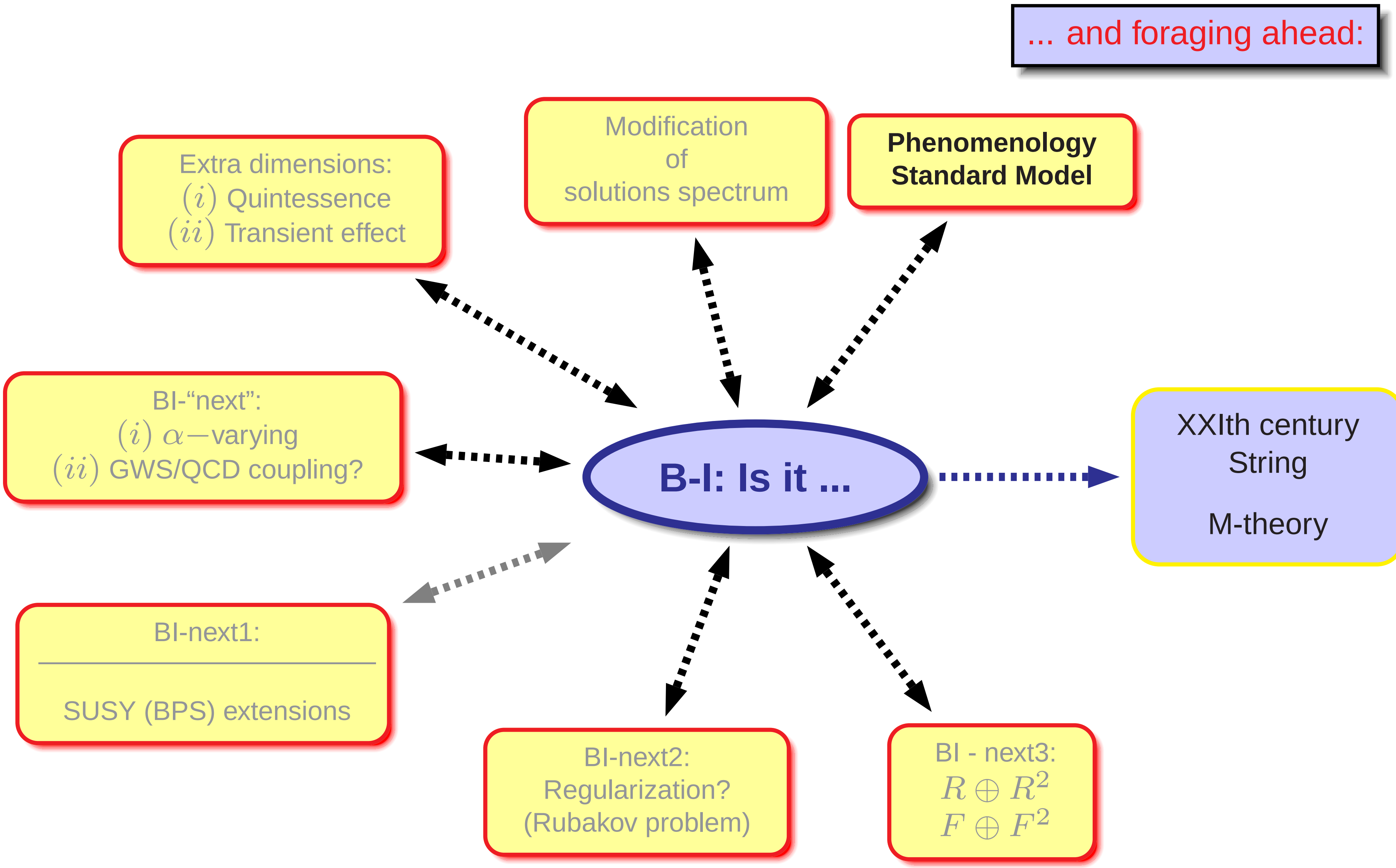
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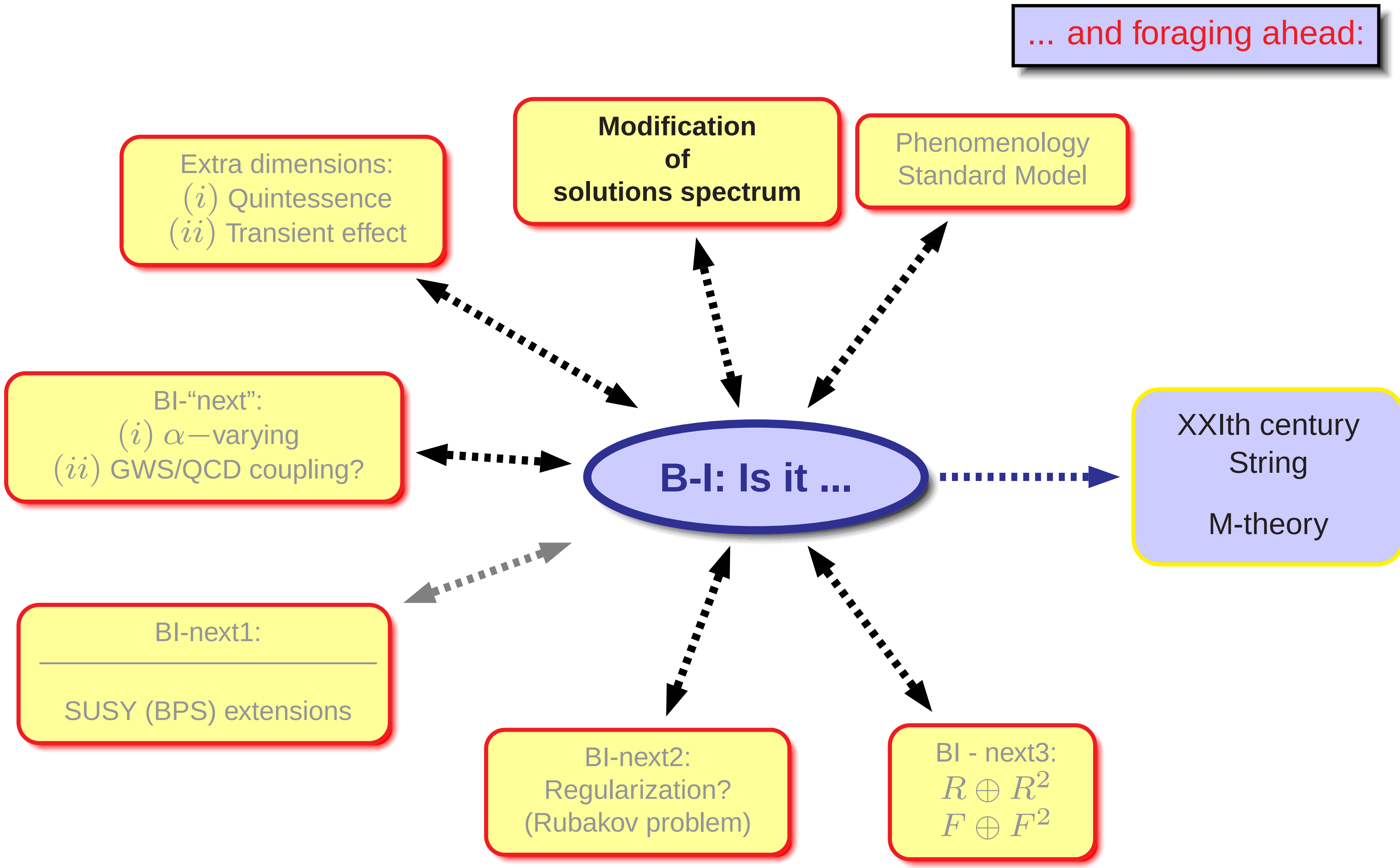
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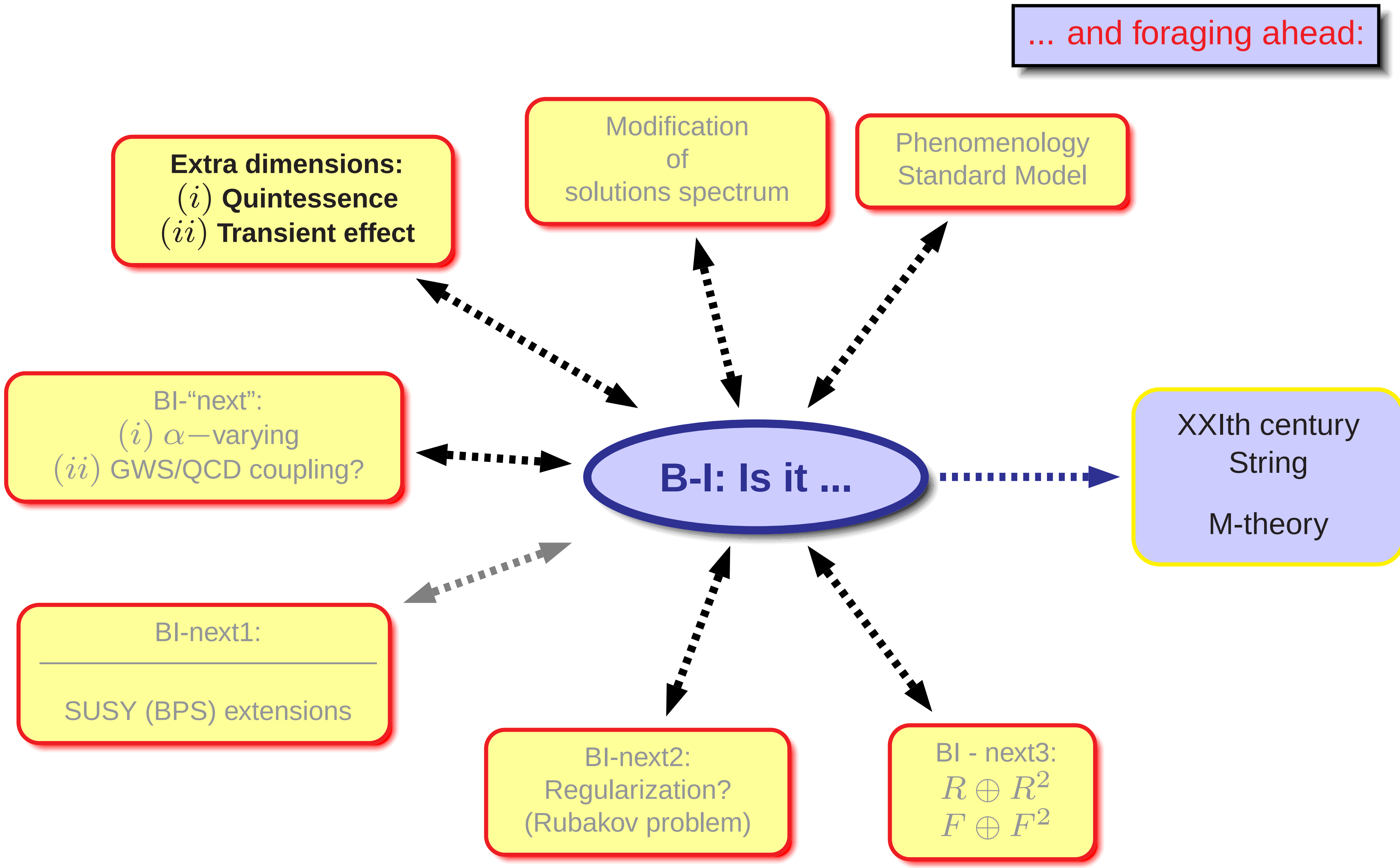

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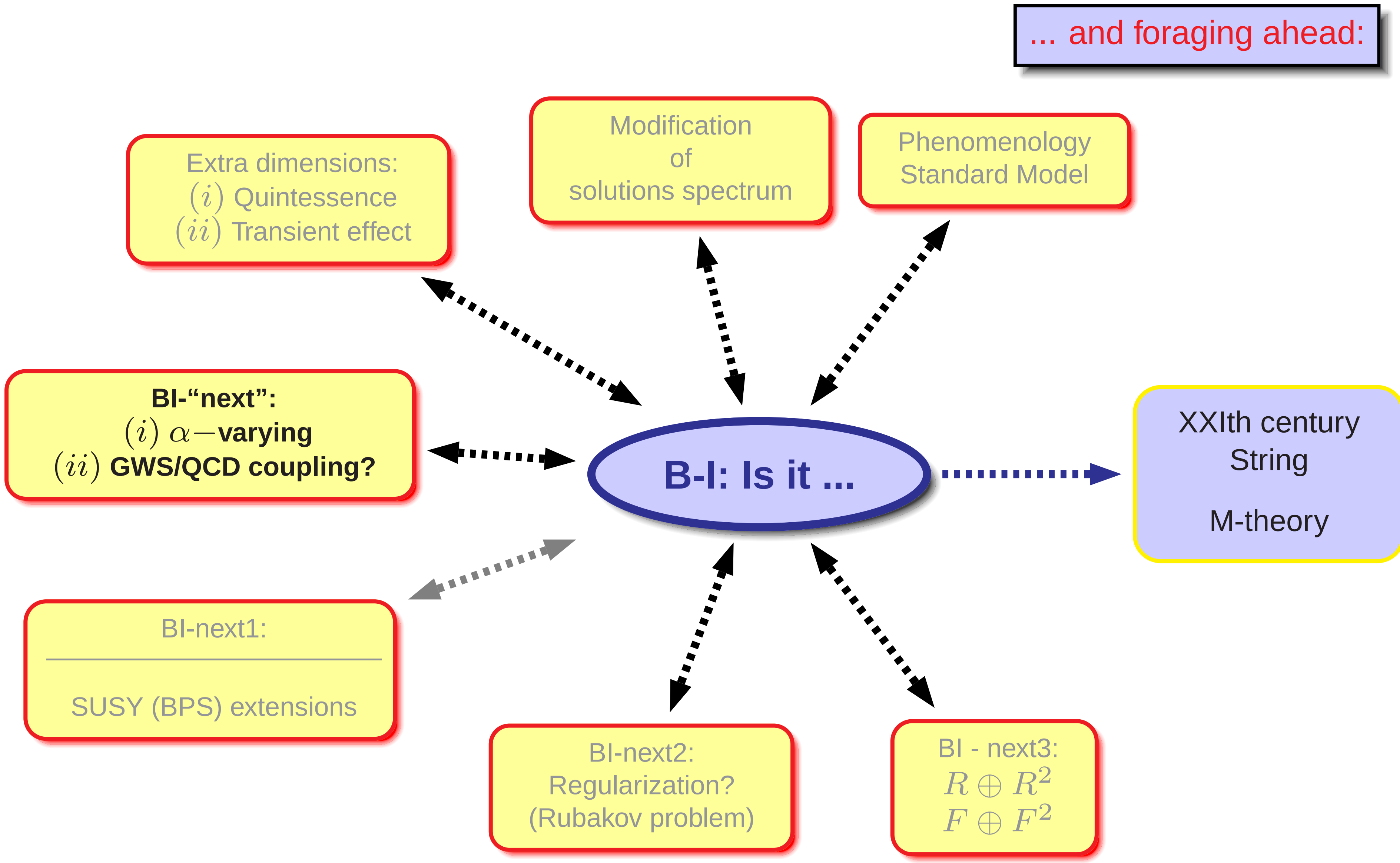
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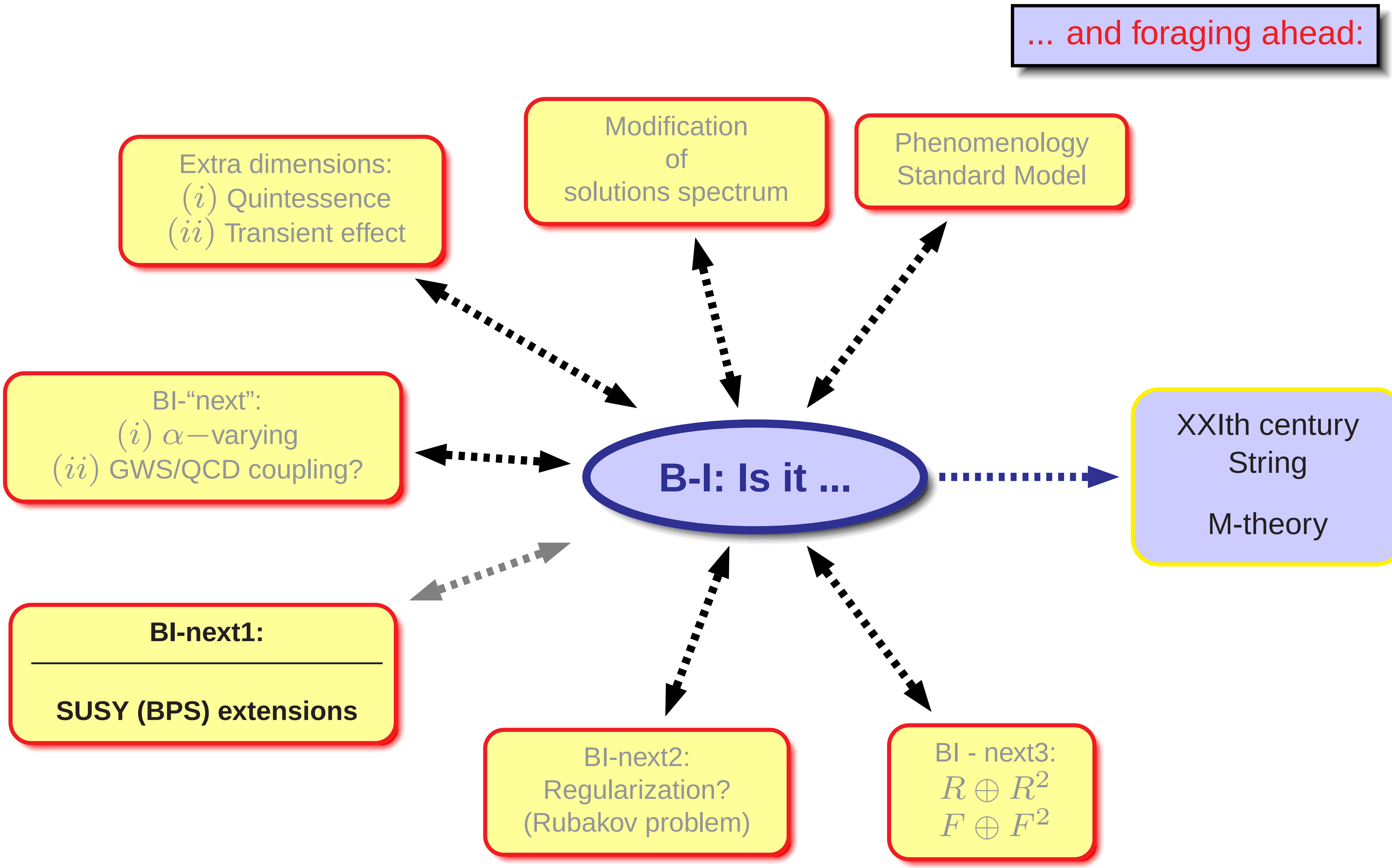
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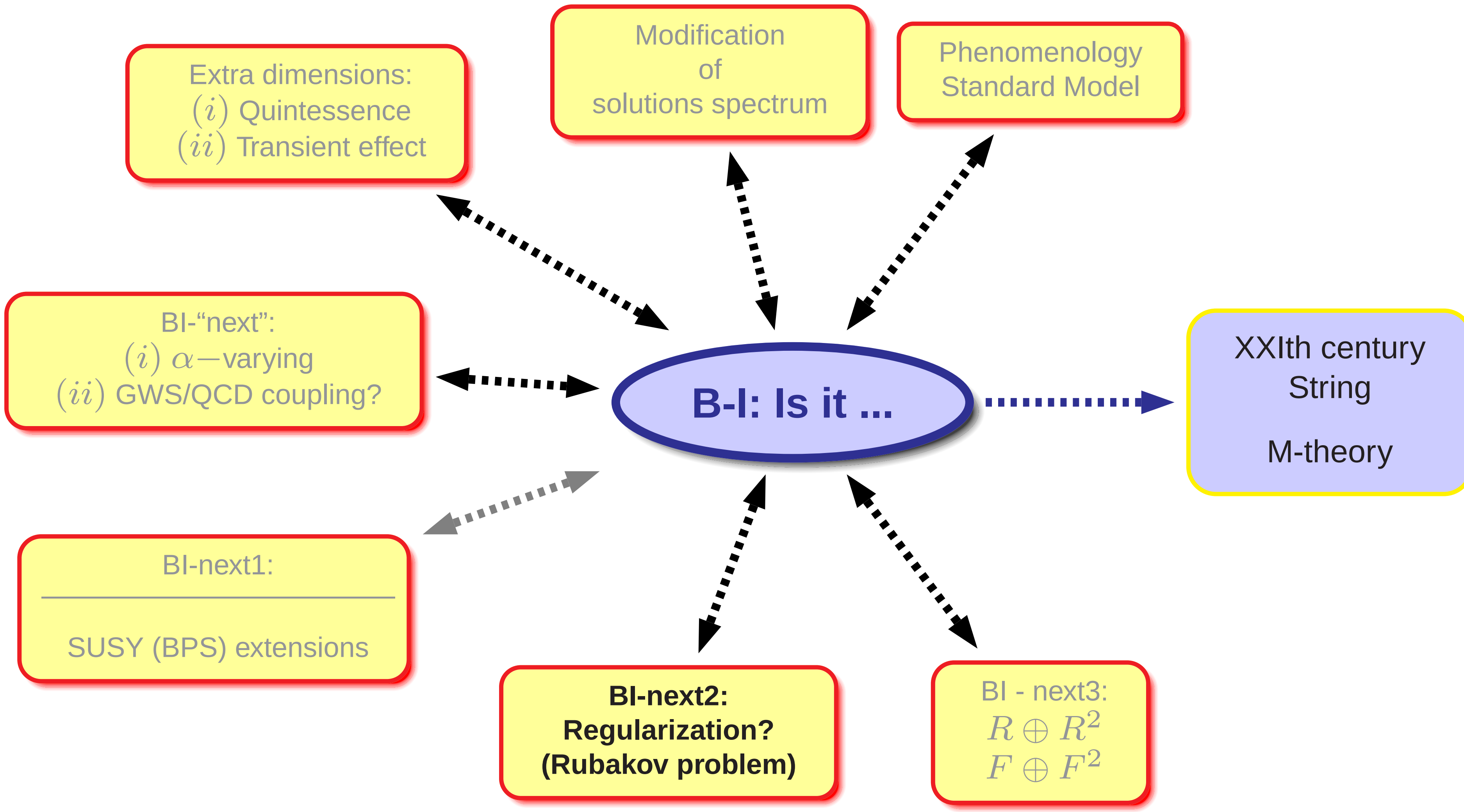

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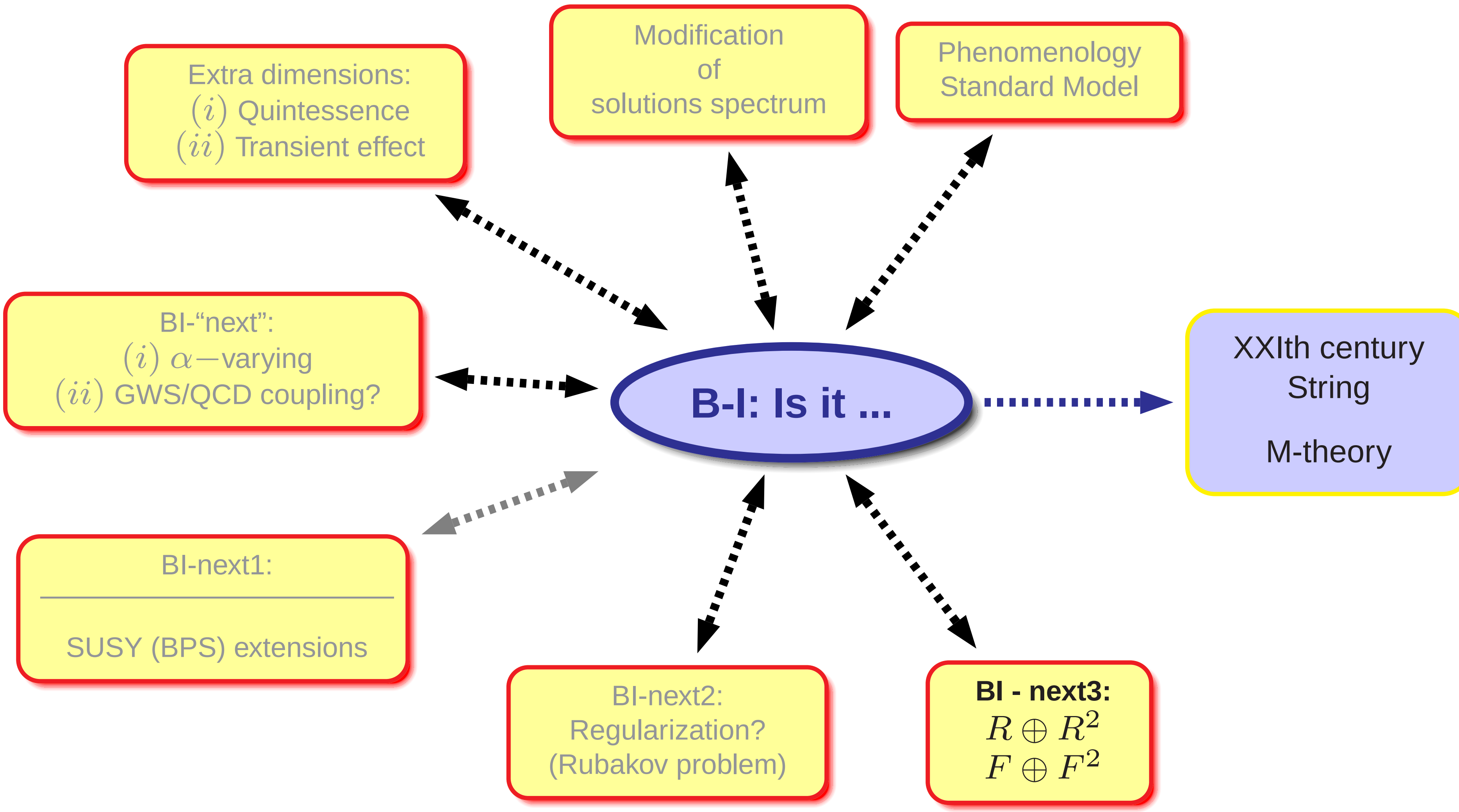

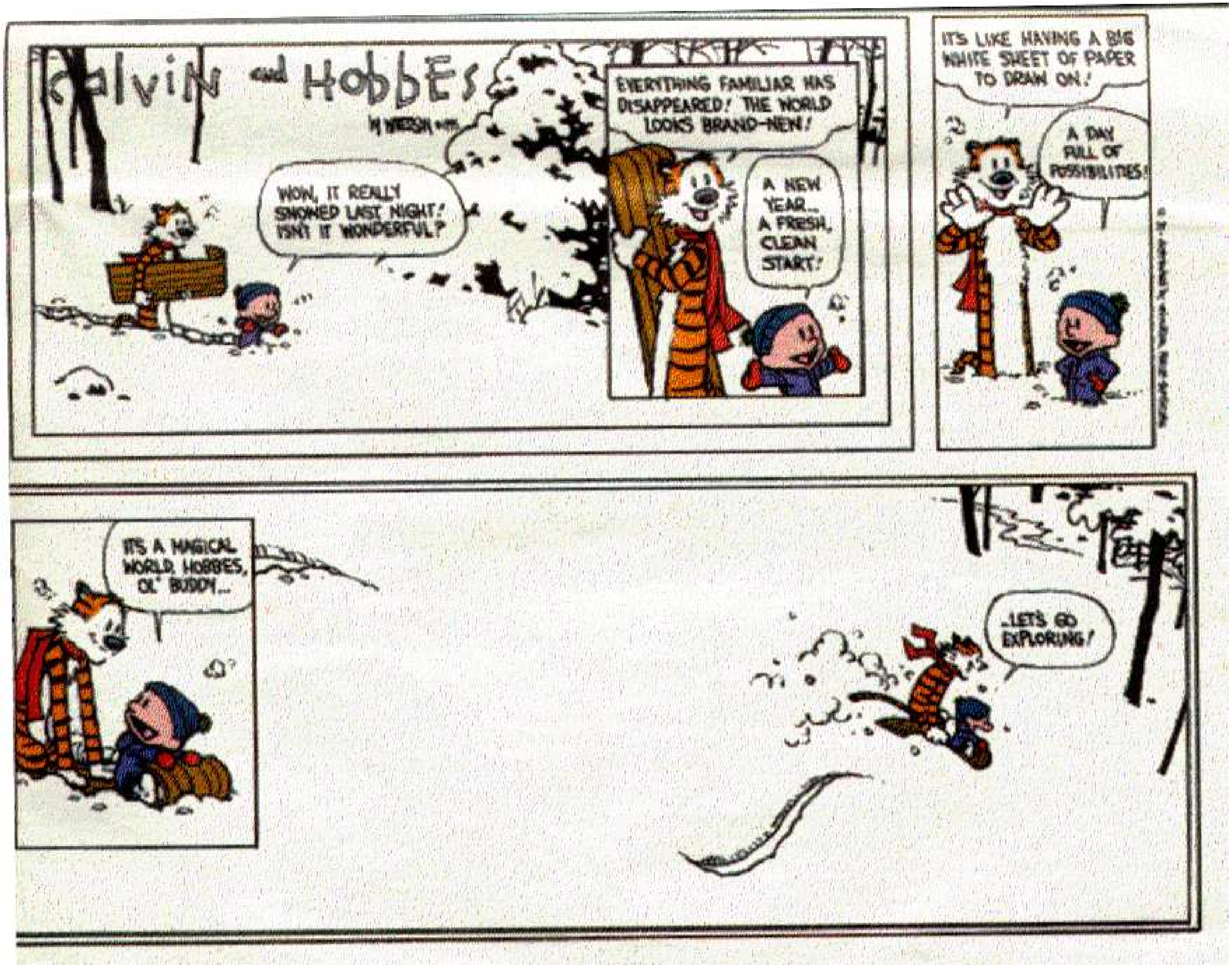
Calvin and Hobbes
WOW, IT REALLY SNOWED LAST NIGHT! ISN'T IT WONDERFUL?
EVERYTHING FAMILIAR HAS DISAPPEARED! THE WORLD LOOKS BRAND-NEW!
A NEW YEAR... A FRESH, CLEAN START!
IT'S LIKE HAVING A BIG WHITE SHEET OF PAPER TO DRAW ON!
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...LET'S GO EXPLORING!