COSMO05
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# EVOLUTION OF A SIMPLICIAL UNIVERSE

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## ***A. It is the time to bring the quantum gravity view in cosmology.***

* Observations of the CMB anisotropy show existence of

i) ***Inflations:***

The anisotropy requires the quantum correlation before the inflation,

and the persistence during the inflation

ii) ***Dark Energy:***

Inflation is the rapid expansion due to the vacuum pressure.

* They should be the consequences of quantized space-time.

## ***B. Quantum Gravity is expected to be to produce the universe.***

*Such a theory should not contain physical laws, since they appear after the creation of space-time.

* It contains the least assumptions.

An example: *the Peano axioms* for the natural numbers:

1. Existence of the element *1*.
2. Existence of the successor *S(a)* of a natural number *a*.

***C. Axiom for the simplicial universe.***

1. Existence of the element, a d-simplex.
2. Existence of the neighbor to form a simplicial complex.

*Remarks:*

Of course, these conditions are not enough. During the course of creating a universe we might choose additional conditions. In such cases we choose most general or least prejudiced one.

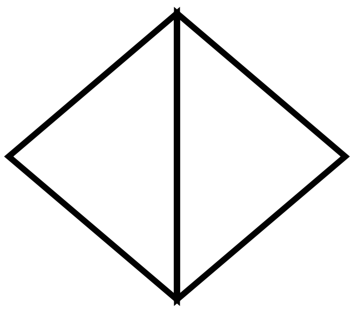
*We may think the world has no a priori rules. The rules emerge as a result of the evolution, which sustains large structure with long life.*

***D. Quick tour of the simplicial quantum gravity.***

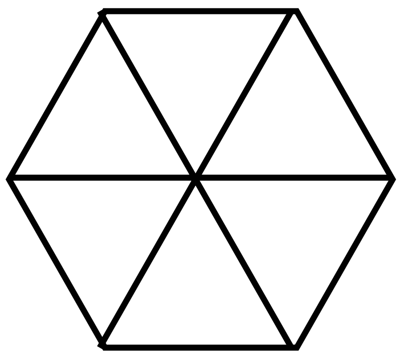
* Let me illustrate the 2d-simplicial quantum gravity as an example.

1. The element = an equilateral triangle
2. The neighbor = a 2d triangulated surface constructed under the manifold conditions:

i) Two triangles attach through one link.



ii) Triangles sharing one vertex form a disk.



* The partition function is a set of simplicial manifolds with distinct triangulations (configurations).

* Known facts:

It is equivalent to the matrix model,

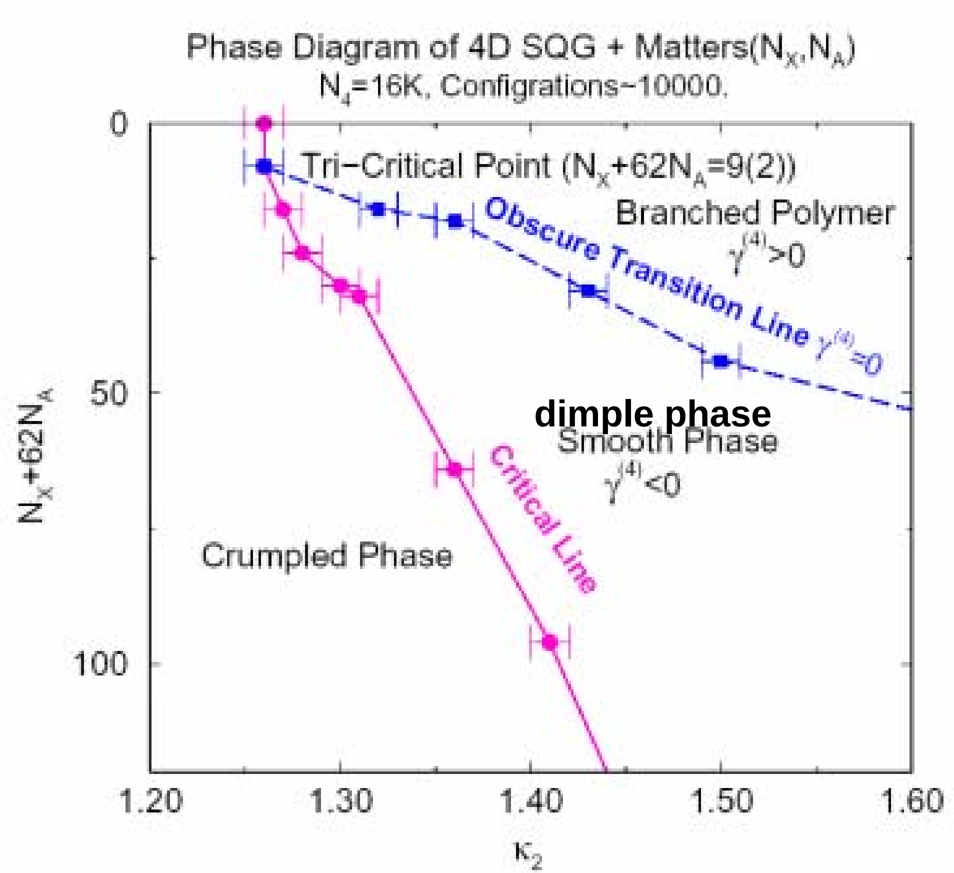
and in the same universality class as the quantum Liouville theory

## *E. What do we know about the 4d-simplicial quantum gravity?*

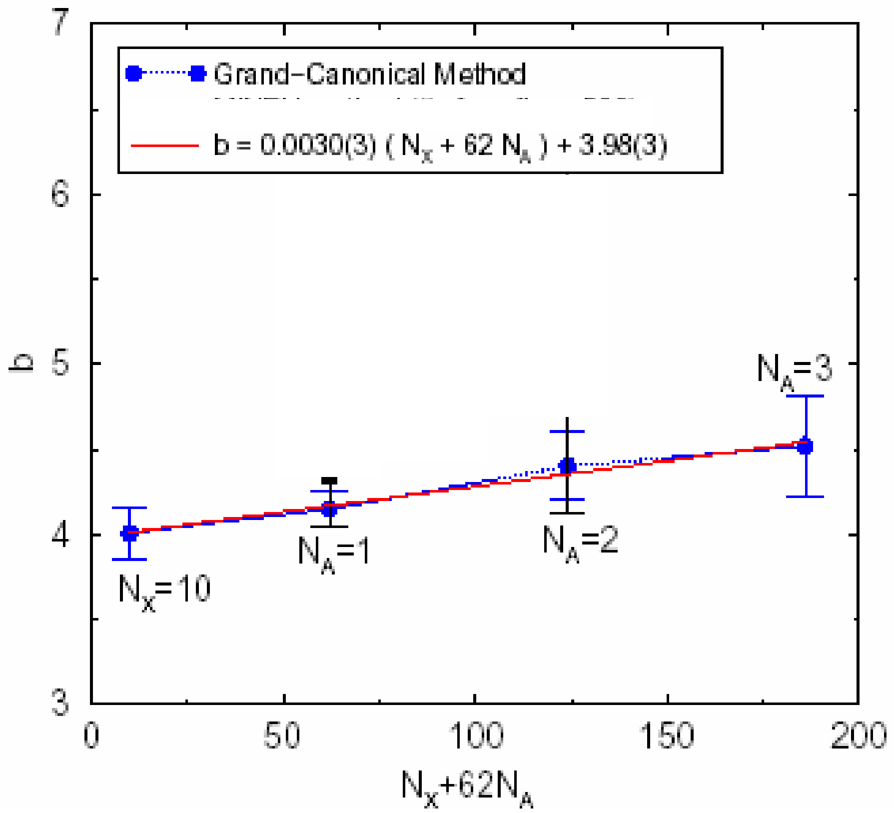
* We have shown numerically:

i) there exist three phases: *crumple-BP-dimple* , and

ii) between crumple and dimple phases the transition is continuous as the 2nd order phase transition.



iii)It has the string susceptibility similar to the conformal gravity, suggesting to be in the same universality class.



## *F. Introducing an extra dimension in space with an open boundary.*

* In the standard dynamical triangulation method we used to construct a space with a closed topology, and study its thermo-dynamical property.
* To describe the evolution we need to consider an ensemble of configurations with varying boundaries, and bring the time coordinate in them.
* Space with $R \times S^{d-1}$ topology is considered.

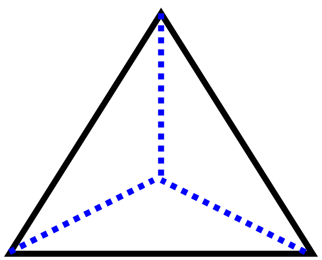
## *G. Let’s construct a d-dimensional space with $R \times S^{d-1}$ topology.*

* Again, we consider the 2d-tynamical triangulation as an example.

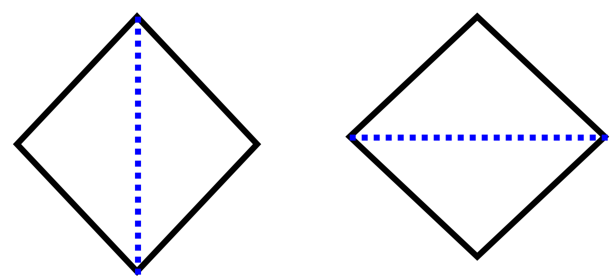
*i) The partition function* consists of a set of spaces with distinct triangulations (configurations) of a fixed $S^2$ topology weighted appropriately.

ii) Configurations are produced by the $(p,g)$-moves forming a Markov chain:

(1,3) (3,1)



(2,2)



with the move probability constrained by the detailed balance:

$$\frac{p_a}{n_a} w_{a \to b} = \frac{p_b}{n_b} w_{b \to a}$$

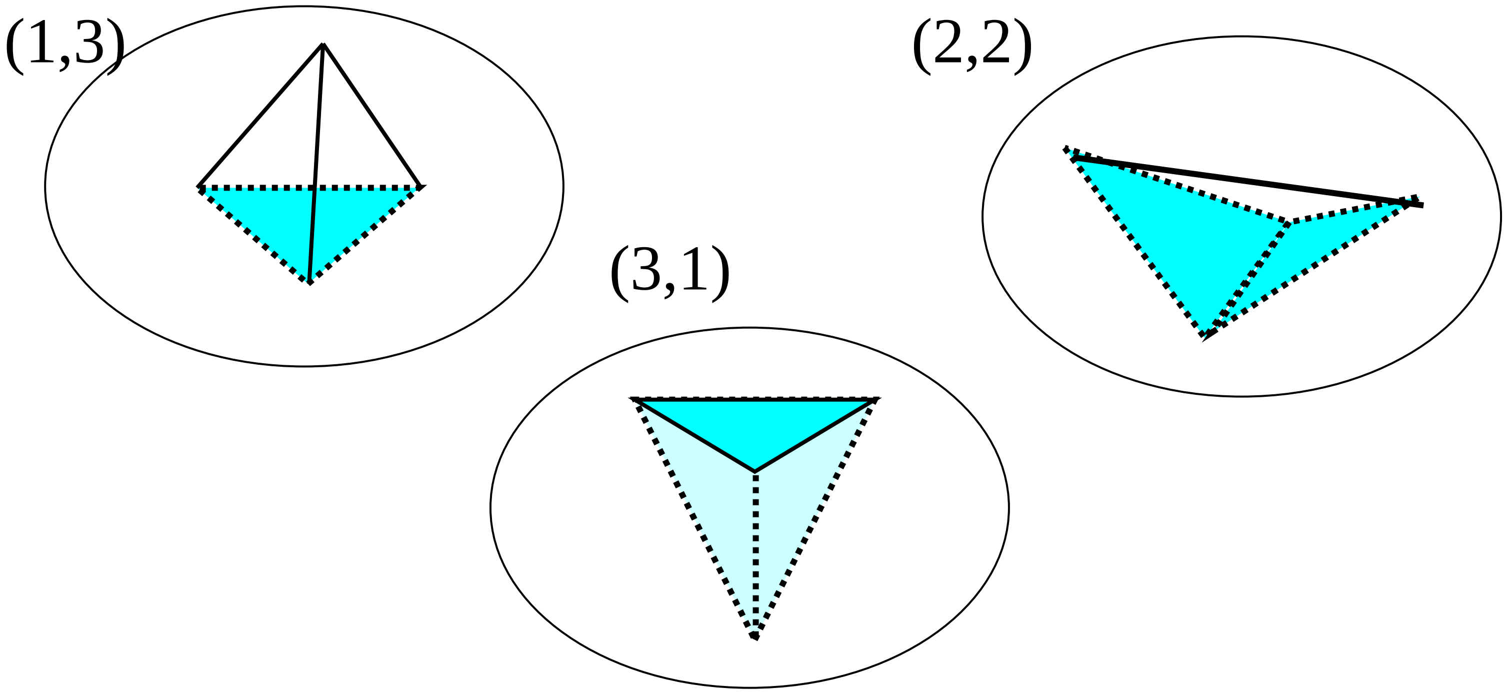
$p_a$: probability weight for the configuration $a$,

$n_a$: number of all possible moves starting from a configuration $a$

## *H. Adding one extra dimension.*

* We regard the $S^2$ surface as the boundary of a 3-ball($B^3$)

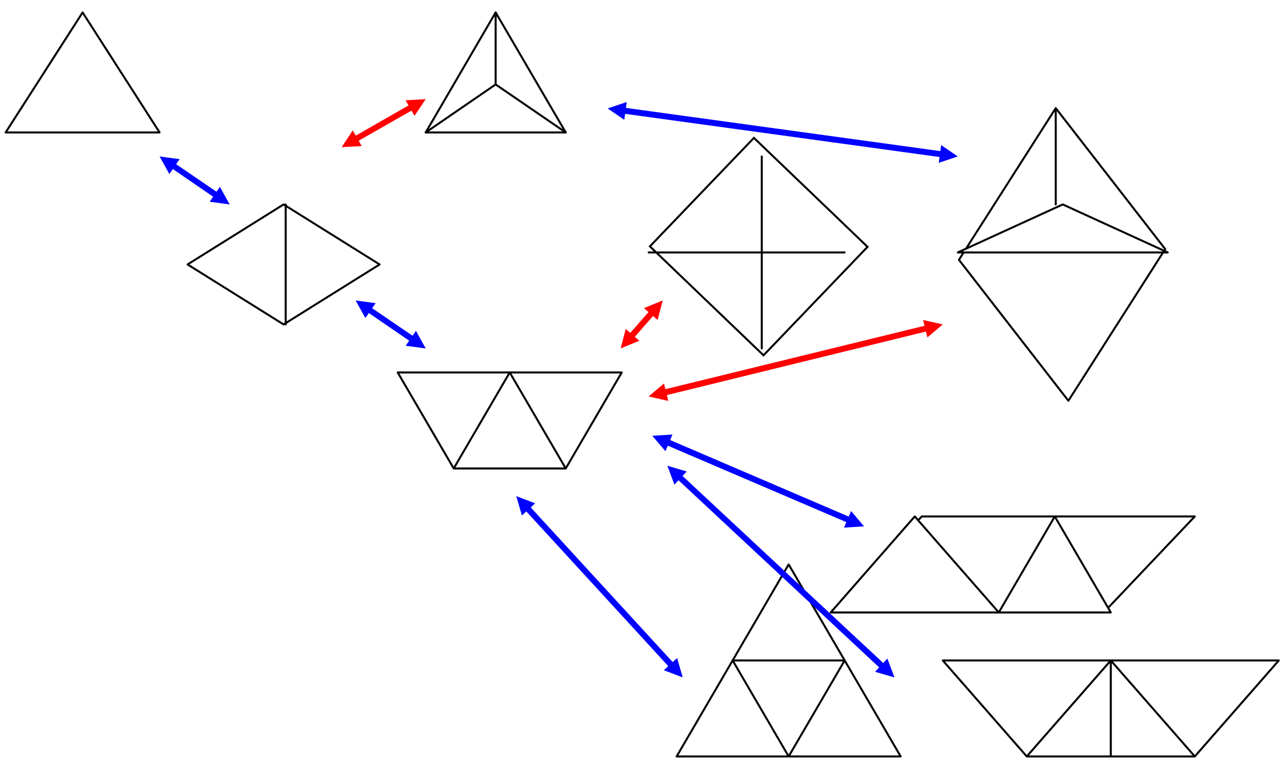
* Moves on the $S^2$ boundary are regarded by either attaching a tetrahedron on the boundary sharing one, two, or three triangle(s) in common, or to the contrary taking out one tetrahedron:




* We call them as $\{\Delta S, \Delta V\}$-move.
  In the 3-d case there are 6 moves: $\Delta S=\{-2,0,2\}$ and $\Delta V=\{-1,1\}$

## I. Creation of the quantum universe.

* The 2d-quantum universe is a set of all possible disks of distinct triangulations with appropriate probabilities.
* For example up to $N_2=4$:



* Four types of moves:
  {1,1},{-1,1} and the inverses {-1,-1},{1,-1}.

## *J. Types of the 2d-unverse*

* Number of distinct configurations with the volume *V* and the area *S* increases exponentially .
* There are *d* control parameters $\{u_i\}$ in *d*-dimension:

$$A = \sum_{i=0}^{d} u_i N_i$$

*Using the Eular relation, $N2-N1+N0= \chi$, which reduces independent parameter by one, for a 2-dimensional where $\{Ni\}$ is the number of *i*-simplex. universe, we write

$$A = \mu N_2 + \mu_B \tilde{N}_1$$

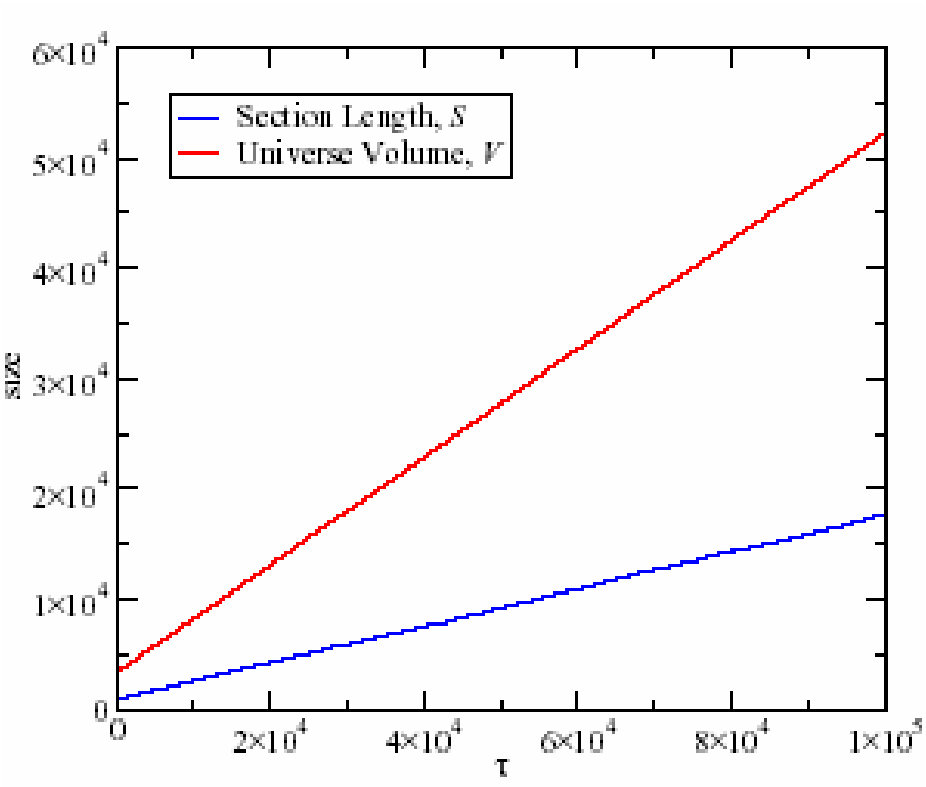
where $\tilde{N}_1$ is the number of triangles on the boundary.

* μ and $\mu_B$ are the cosmological constant and the boundary cosmological constant.

## *Introducing the Monte Carlo computer time $\tau$*

*Numbers of Metroplis checks in the Monte Carlo method for a Markov chain.

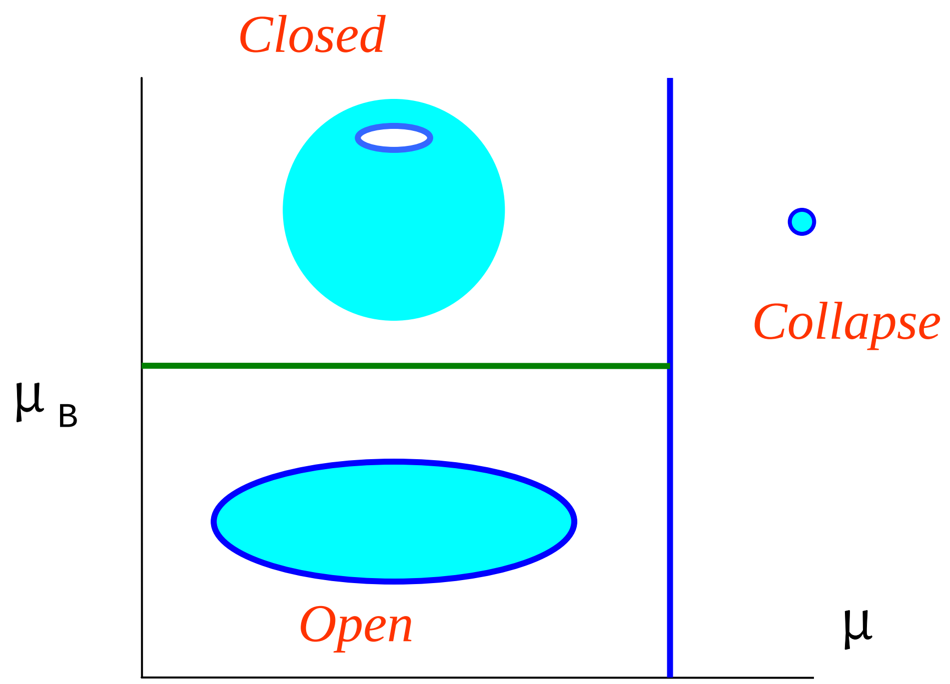
* The volume $V$ and the area $S$ grow almost linear in $\tau$:




$$V = V_0(\mu, \mu_B)\tau,$$
$$S = S_0(\mu, \mu_B)\tau$$

* $V_0$ and $S_0$ change sign at the critical line $\mu^c$ and $\mu^c_B$

* In the two dimensional parameter space we expect three types of universes: *open-closed-collapse*



## *L. Physical time is defined*

* Within a scale factor *the physical time t* is defined through

$$V(t) = \int^{t} S(t')dt'$$

* These two times are related as

$$t = \int^{\tau} d\tau' \frac{1}{S(\tau')} \frac{dV(\tau')}{d\tau'}$$

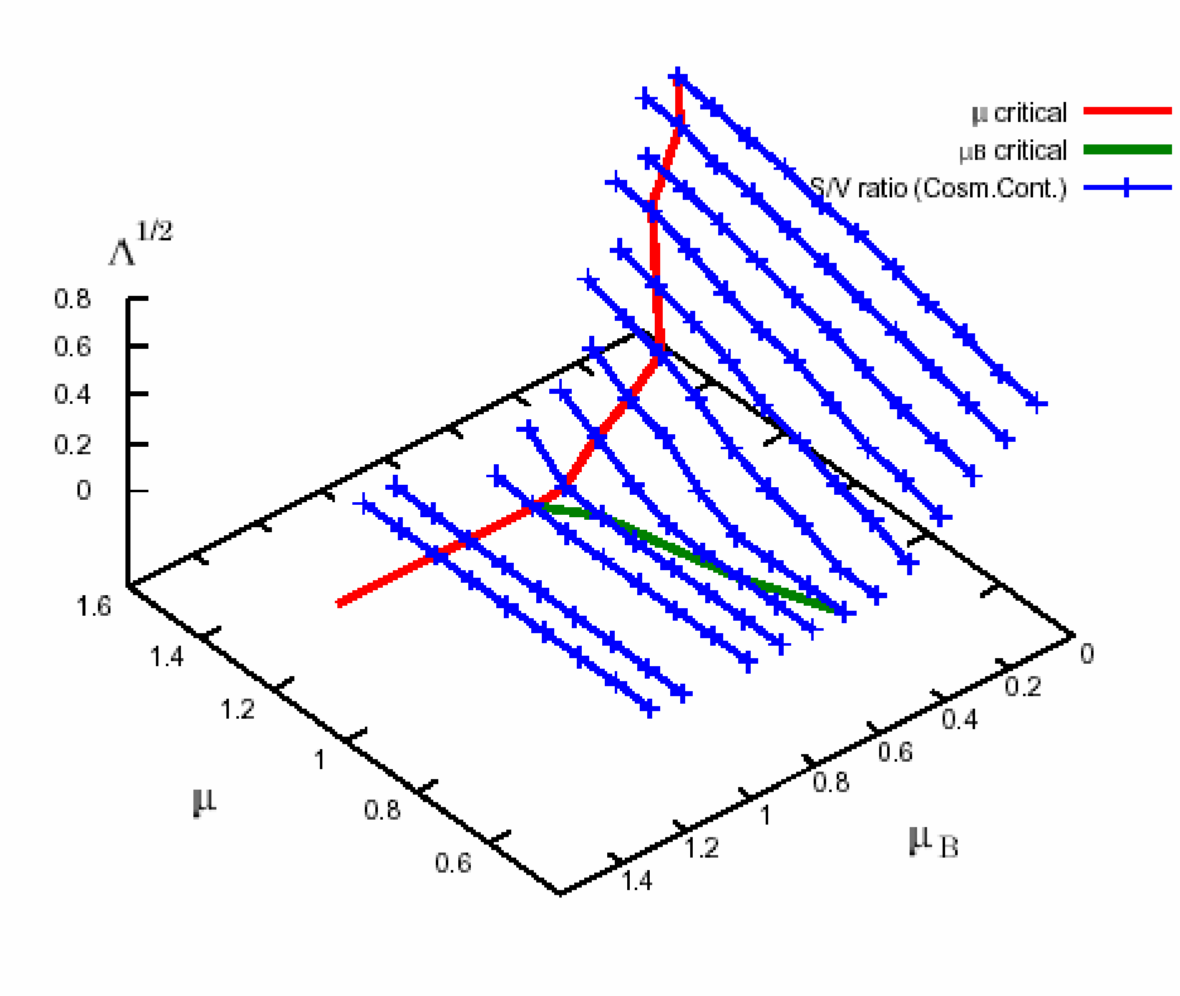
* From the computer simulation

$$\tau \approx \exp(\frac{S_0}{V_0} t)$$

which means the *inflation!* with the velocity $S_0/V_0$.

## *M. Numerical results*

* The inflation velocity $\Lambda^{1/2}=S_0/V_0$:




* The critical values are close to what we expect from the $M^3$-matrix model:
$\mu^c \sim 1.1$ (1/2log[256/27])
at $\mu_B=0$, (all diagrams)
$\mu^c \sim 1.5$ (1/4log[432])
at $\mu_B=0.8$ (1/2log[16/3])
(1PI without tadpole and self-energy diagrams

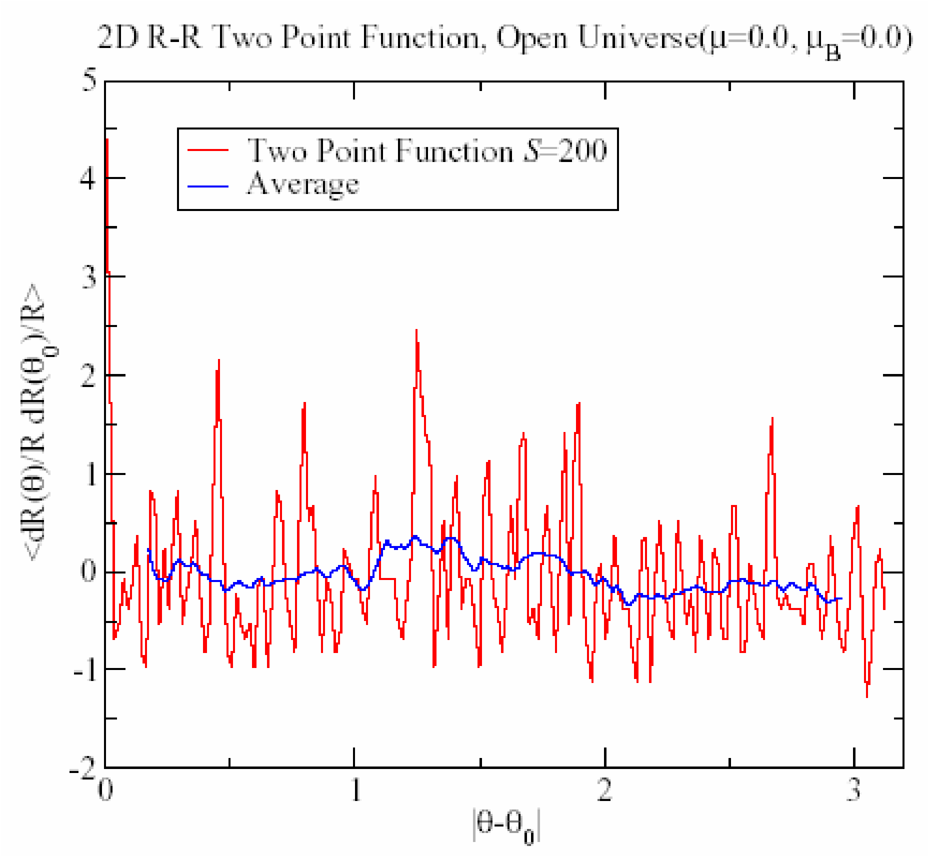
N.B. Beyond $\mu$-critical (the red line) the $\Lambda^{1/2}$ changes its sign,
*i.e.* collapse to a point.

# *N. Correlations*

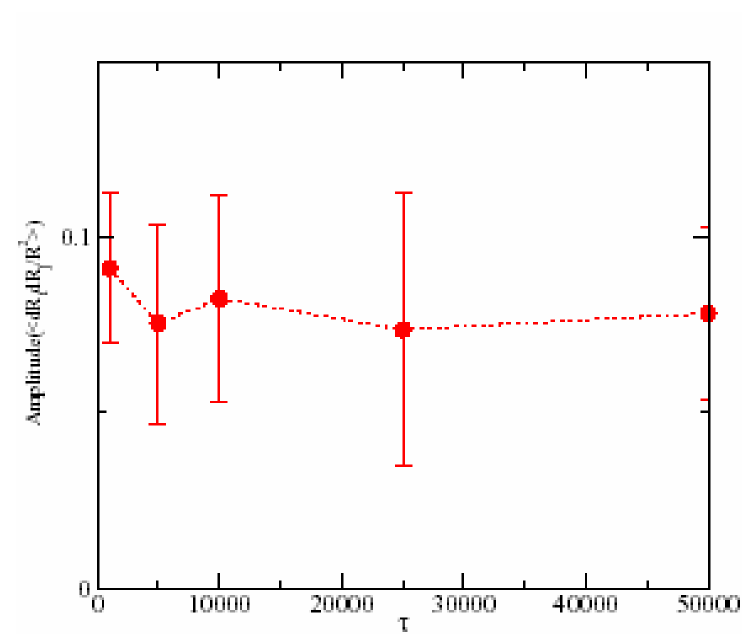
* Geodesic distance ~ $r\,\theta$ ($r$: radius of the universe: $r=S/2\pi$)
* The two-point correlation function is defined as

$$\left\langle \frac{(R(0)-\langle R\rangle)(R(\theta)-\langle R\rangle)}{\langle R\rangle^2} \right\rangle$$

* Strong quantum correlation at small $\theta$ (~1/r)

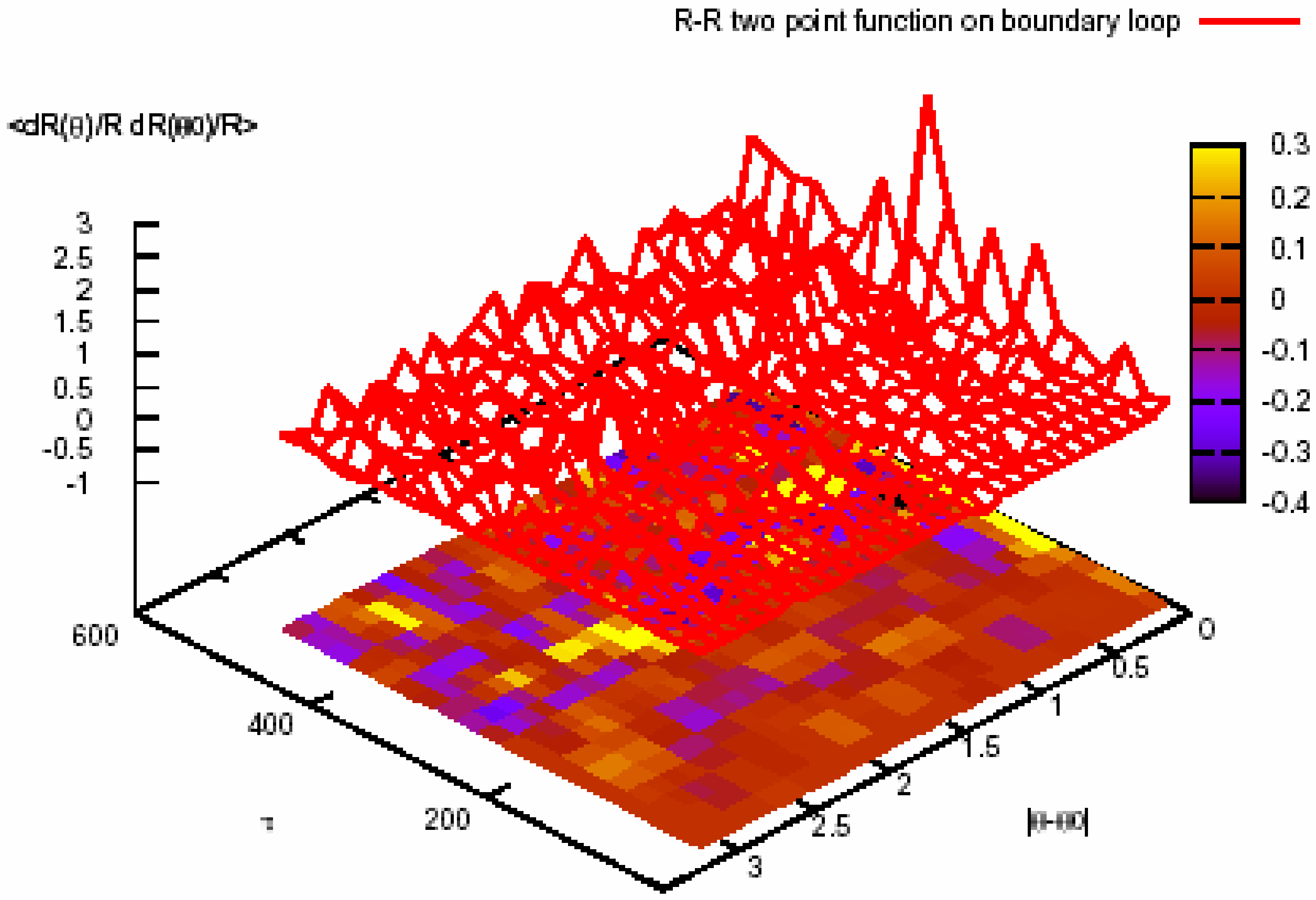



* Large angle correlation remains!

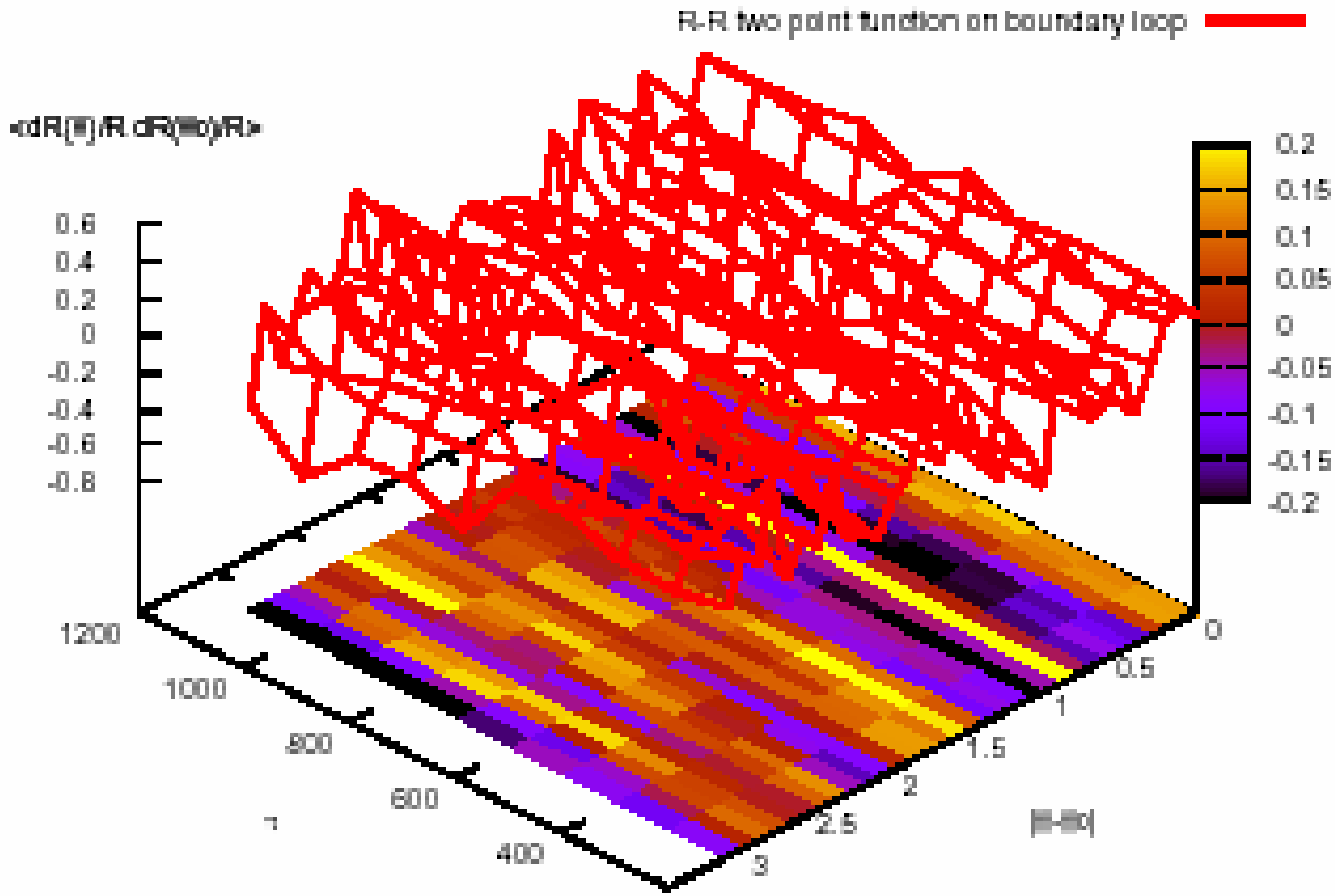


## *O. Where the large angle correlation is born?*

* Initial correlation ( $N_2$<500 )



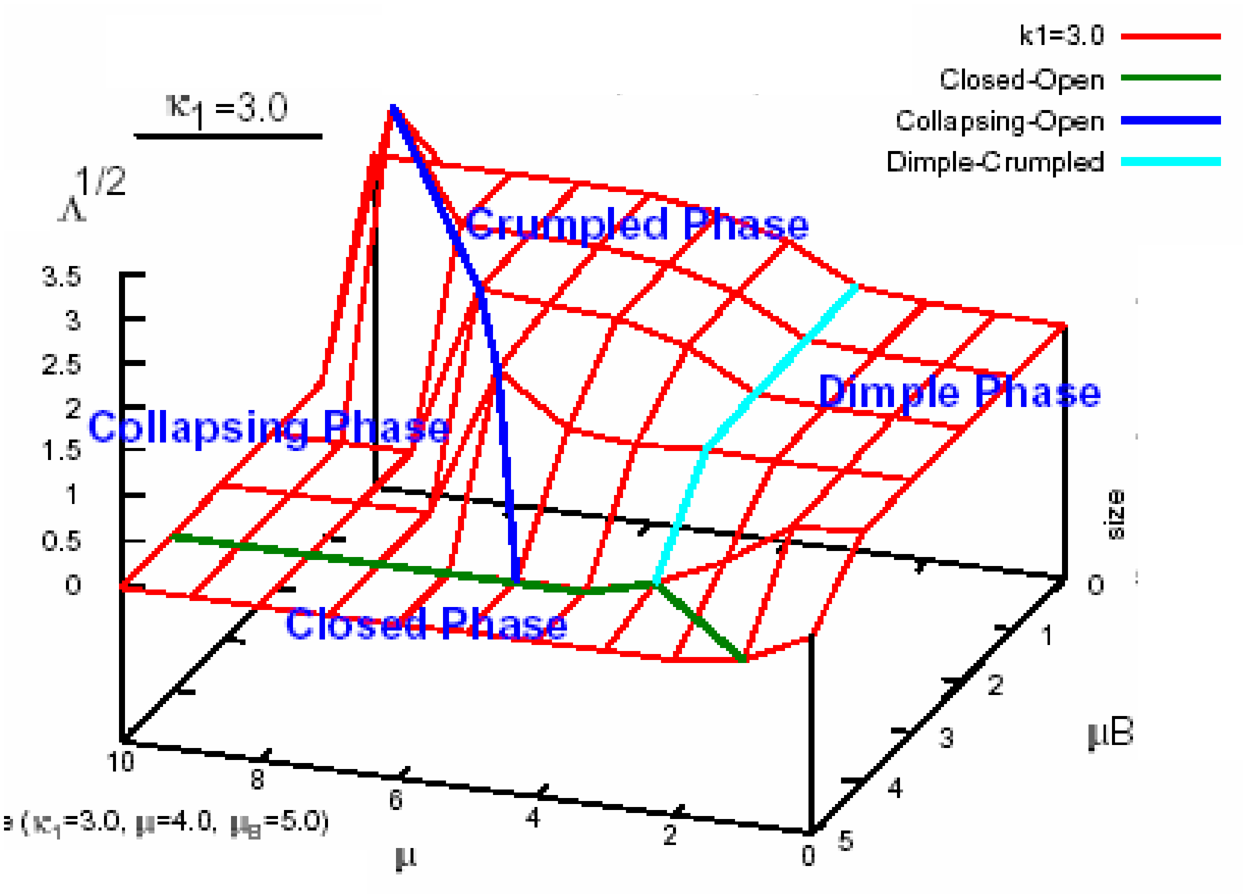
* During the inflation ($N_2$<1100)



# *P. What will come out in higher dimensions?*

* In 3d , there are 3 controlling parameters:

$$A = \mu N_3 + \mu_B \tilde{N}_2 - \kappa_1 N_1$$



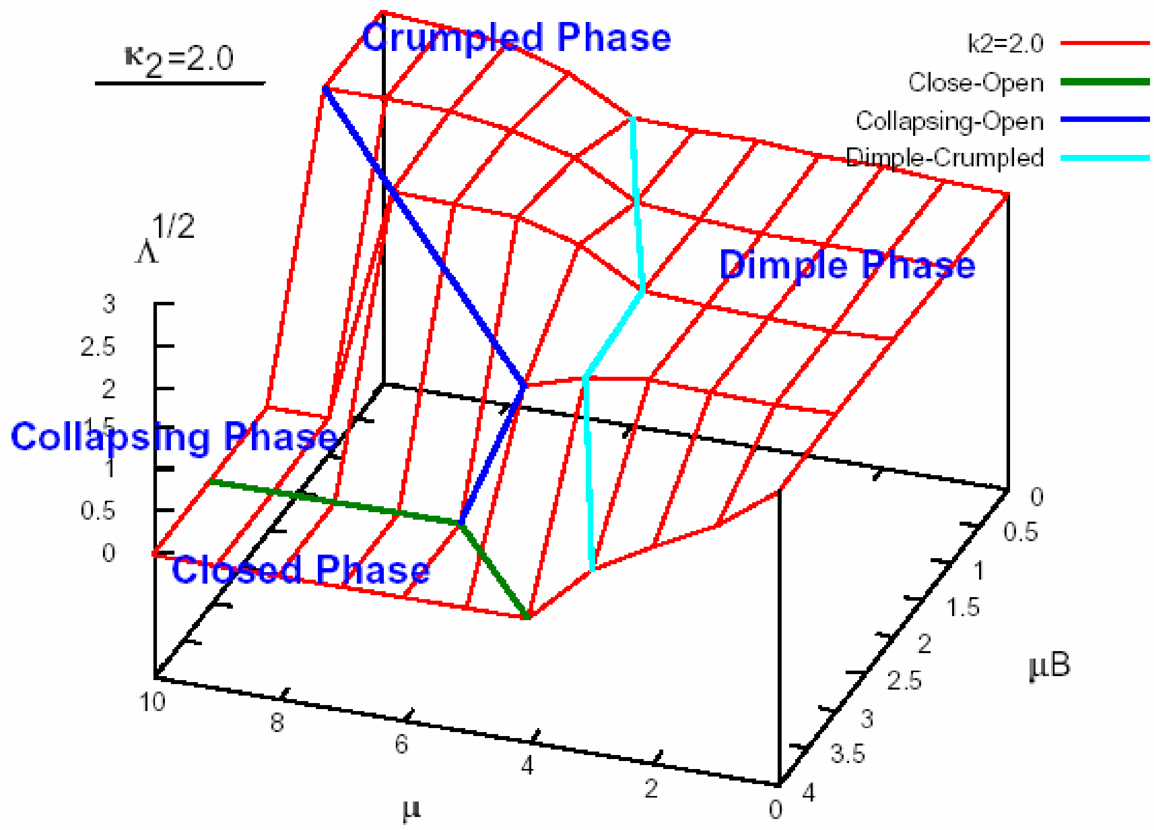

* $\kappa_1$ introduces two phases (*dimple-crumple*) in open universe

## *Q. In 4d*

* There are 4 controlling parameters:

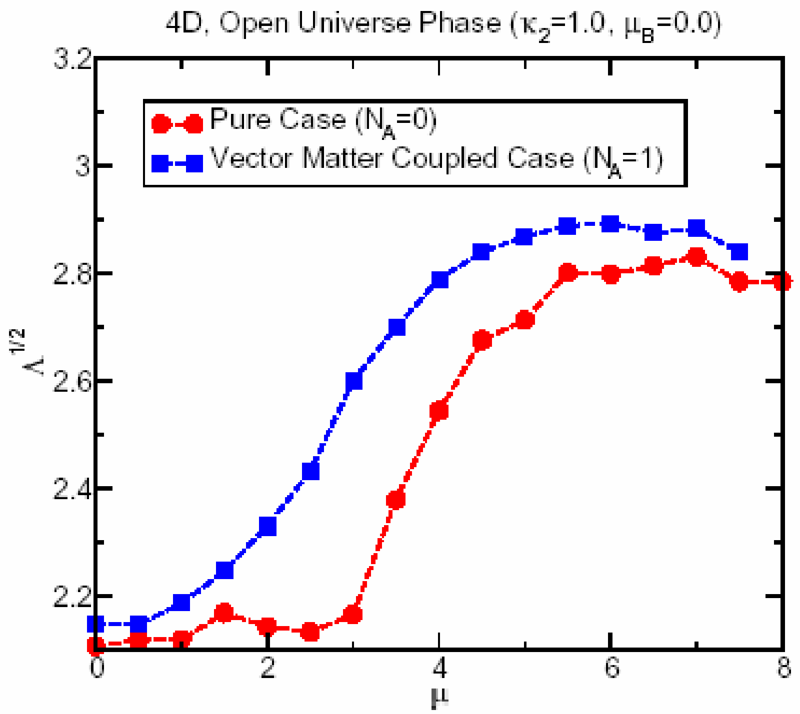
$$A = \mu N_4 + \mu_B \tilde{N}_3 - \kappa_2 N_2 + \kappa_1 N_1$$

* The physical meaning of the new term is not studied, yet.
(Take it to be 0.)



## *R. Effect of matter fields in 4d*

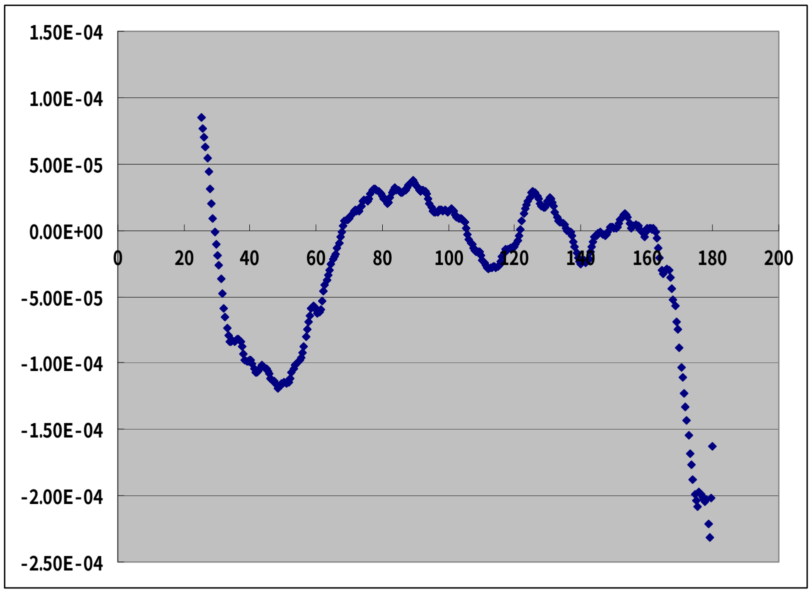
* Adding one vector field the transition between two phases become smooth.



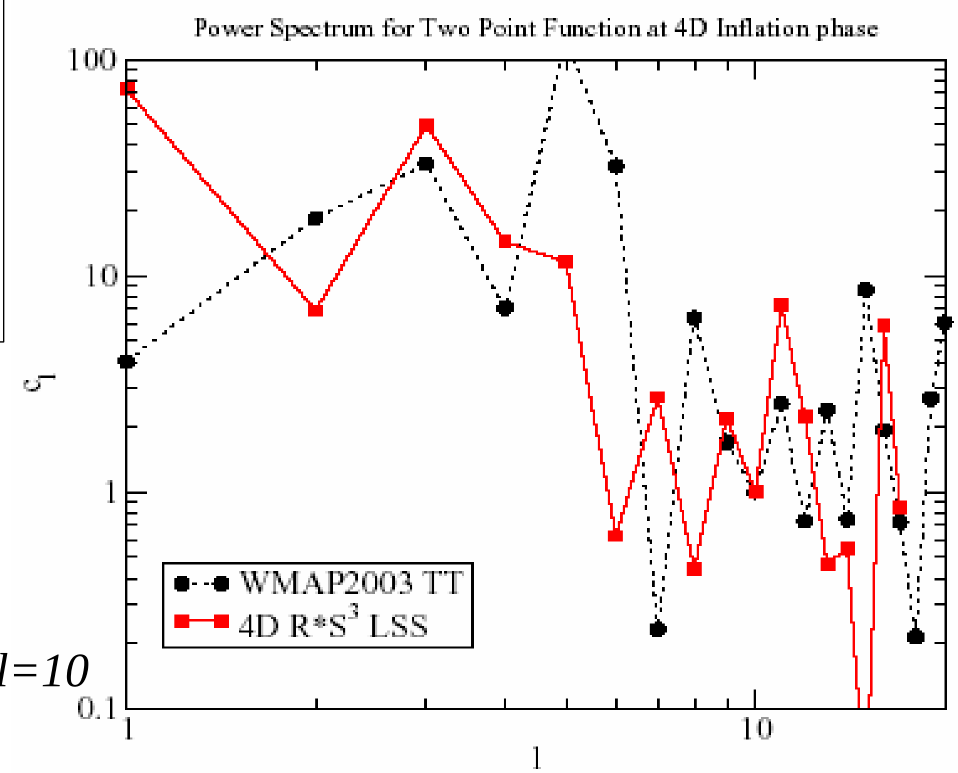
## *S. How about the correlation in 4d ?(Preliminary)*

* Two point correlation on the last scattering surface (*lss*)

Compared with the WMAP observation (TT-corr)




* The power spectrum in $P_l(\cos\theta)$




N.B. Normalized at *l=10*

*To be continued*

Thank you for your patience!