Instability of Dark Energy with Mass Varying Neutrinos

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My Collaborators

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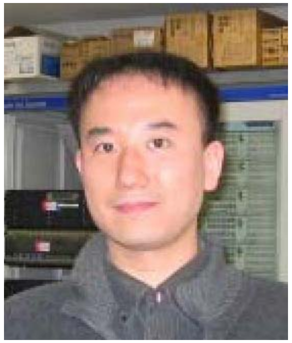
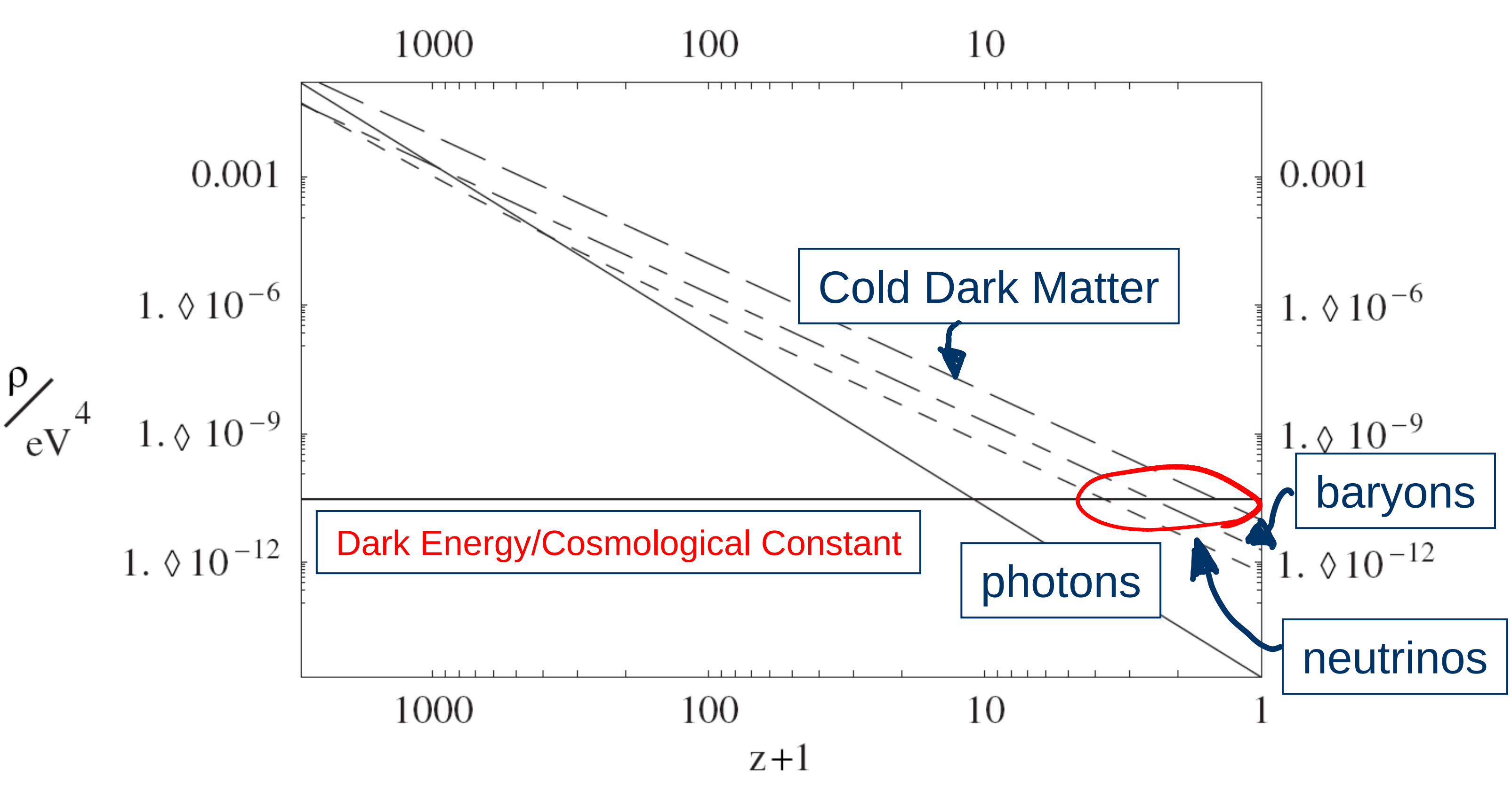


Kazunori Kohri

Outline

- Dark Energy and Cosmic Coincidences
- Mass Varying Neutrinos (MaVaNs) and Dark Energy
- MaVaNs Phenomenology and Interactions
- Instability of non-relativistic MaVaNs
- Trans-Relativistic Phase Transition
- Stability of Adiabatic Dark Energy Perturbations
- Conclusions

Cosmic Coincidences in ΛCDM



Coupling Neutrinos and Dark Energy:
Mass Varying Neutrinos (MaVaNs)

- A way to couple neutrinos with dark energy to explain their density coincidence

Fardon, Nelson, and Weiner 2004

Coupling Neutrinos and Dark Energy: ***Mass Varying Neutrinos (MaVaNs)***

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- Neutrino Mass becomes a function of neutrino density

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Coupling Neutrinos and Dark Energy: *Mass Varying Neutrinos (MaVaNs)*

- Equation of state for acceleron/neutrino fluid:

$$w \equiv \frac{\text{Pressure}}{\text{Density}} \simeq -\frac{V_0(\mathcal{A})}{V} = -1 + \frac{n_\nu m_\nu(\mathcal{A})}{V},$$

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- Assuming $M(\mathcal{A})/\mu \gg 1$ and integrating out ν_r

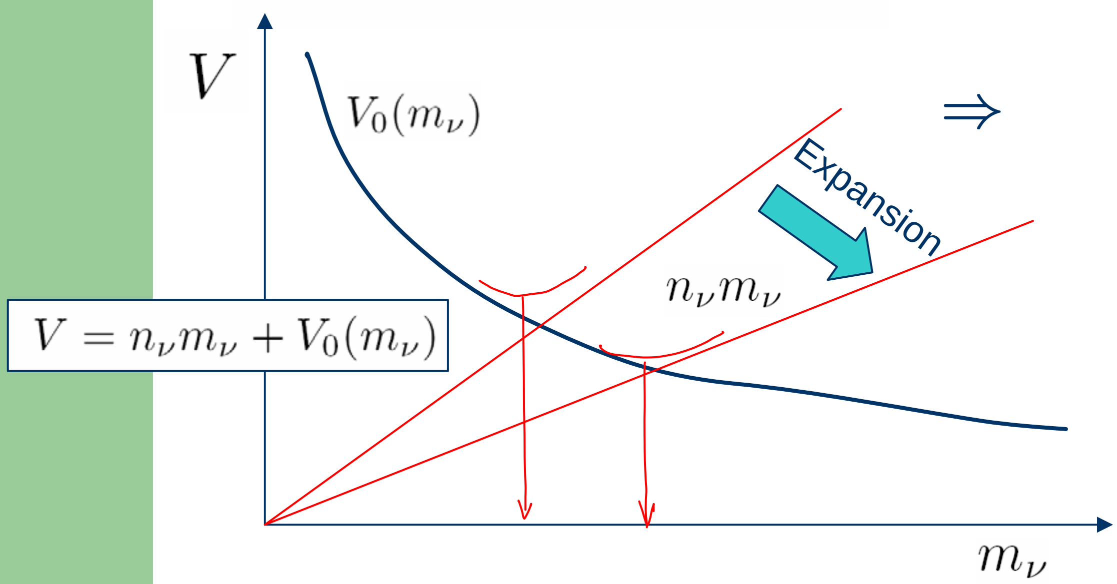
$$\mathcal{L} = \frac{m_{lr}^2}{M(\mathcal{A})}\nu_l\nu_l + \text{h.c.} + \Lambda^4 \log(M(\mathcal{A})/\mu).$$

Logarithmic Model of MaVaNs

- $$m_\nu(\mathcal{A}) = \frac{m_{lr}^2}{\mathcal{M}(\mathcal{A})}$$
- $$V_0(\mathcal{A}) = \Lambda^4 \ln\left[\mathcal{M}(\mathcal{A})/\mu\right]$$

$$\Rightarrow \quad V_0 = -\Lambda^4 \ln\left(\frac{m_\nu \mu}{m_{lr}^2}\right) \Rightarrow n_\nu = \frac{\Lambda^4}{m_\nu},$$

$$\Rightarrow \quad w = -1 + \left[1 + \ln\left(\frac{m_{lr}^2}{m_\nu \mu}\right)\right]^{-1}$$



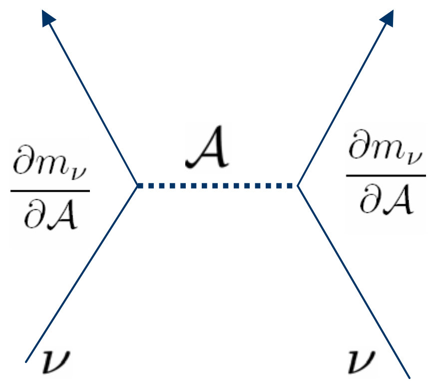
Why do we like MaVaNs?

- As $n_\nu \to 0$, $\rho_{DE} \to 0$, i.e. no cosmological constant is needed
- The only constraint on acceleron mass is $m_A \lesssim n_\nu^{1/3} \sim 10^{-4}$ eV, as opposed to Quintessence where $m_Q \lesssim 10^{-33}$ eV
- Both neutrino and dark energy densities are fixed by one parameter, Λ (one coincidence solved.. two to go)

Scalar Interaction of MaVaNs

- Yukawa Coupling:

$$\Delta\mathcal{L} = \left(\frac{\partial m_\nu}{\partial \mathcal{A}}\right) \nu_l \nu_l \delta\mathcal{A}$$



$$F = -\left(\frac{\partial m_\nu}{\partial \mathcal{A}}\right)^2 \frac{e^{-m_{\mathcal{A}} r}}{r^2}$$

Neutrinos attract each other

- However, the coupling can be small enough to avoid neutrino/acceleron thermalization in the early universe

The Negative Speed of Sound

- $T_\nu << m_\nu$ and $m_A >> H$

  → Adiabatic Perturbations

- If pressure follows density:

$$c_s^2 = \frac{\dot{P}}{\dot{\rho}} = \frac{\dot{w}\rho + w\dot{\rho}}{\dot{\rho}} = w - \frac{\dot{w}}{3H(1+w)} = \frac{\partial \ln m_\nu}{\partial \ln n_\nu},$$

- In the Logarithmic model:

$$\rho + P = m_\nu n_\nu = \Lambda^4 = \text{const.} \Rightarrow c_s^2 = \frac{\dot{P}}{\dot{\rho}} = -1.$$

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Finite Temperature Corrections

- When neutrinos dominate the pressure perturbations: $\dot{P} < 0, \dot{\rho} < 0 \Rightarrow c_s^2 = \frac{\dot{P}}{\dot{\rho}} > 0$
- For relativistic neutrinos, one has to solve Boltzmann equation: $\frac{\partial f}{\partial \eta} + \mathbf{u} \cdot \nabla f - \gamma^{-1} \nabla m_\nu \cdot \frac{\partial f}{\partial \mathbf{p}} = 0,$

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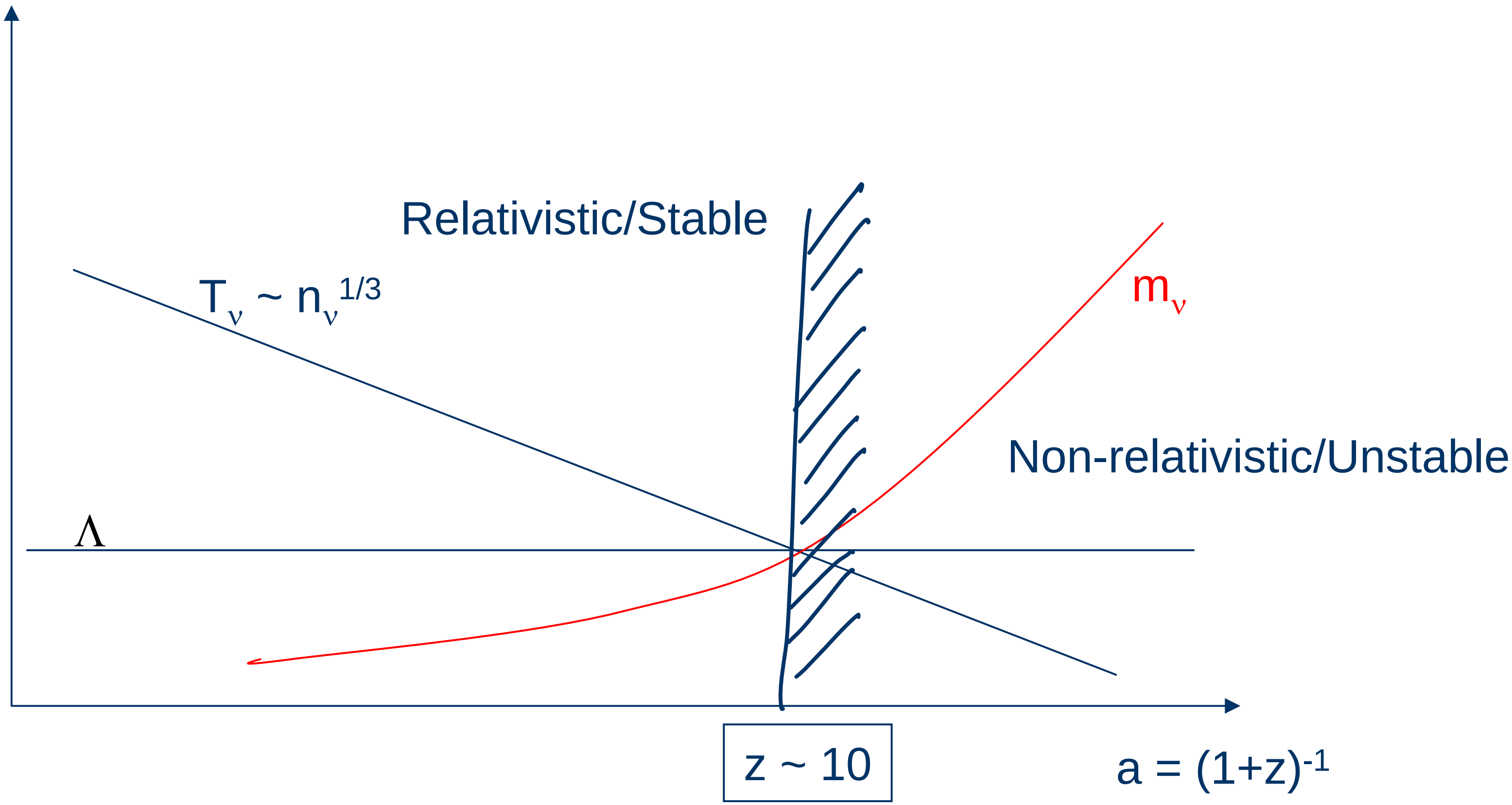
- Corrections to c^2_s for the Logarithmic model:

$$c_s^2 = -1 + 51.8 \left(\frac{T_\nu}{m_\nu}\right)^2 + O\left(\frac{T_\nu}{m_\nu}\right)^4$$

Thermal History of MaVaNs



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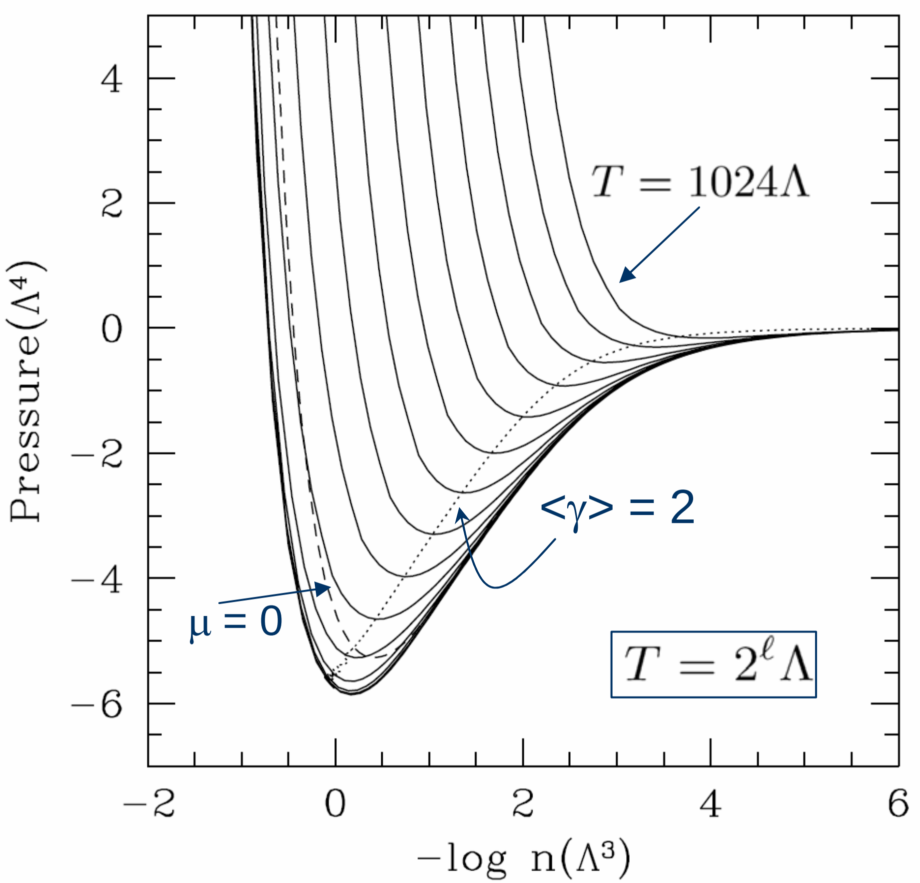
- Nuggets form when $T_\nu \lesssim m_\nu \lesssim \Lambda$

The Trans-Relativistic Phase Transition



c.f. Liquid-Gas Phase Transition

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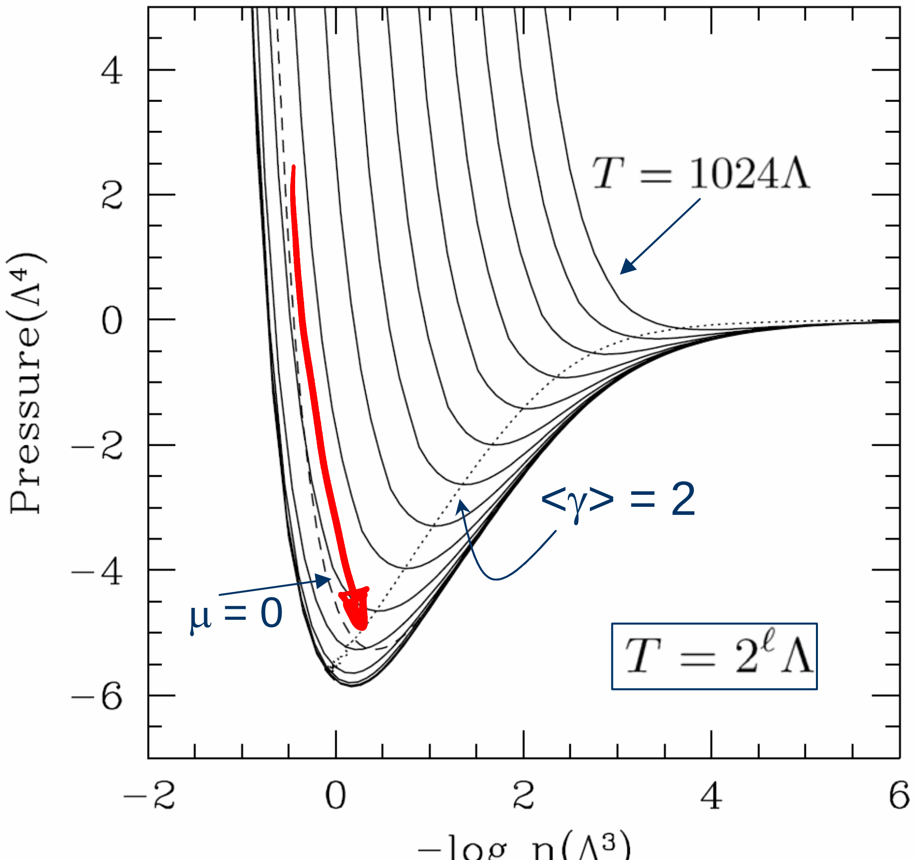
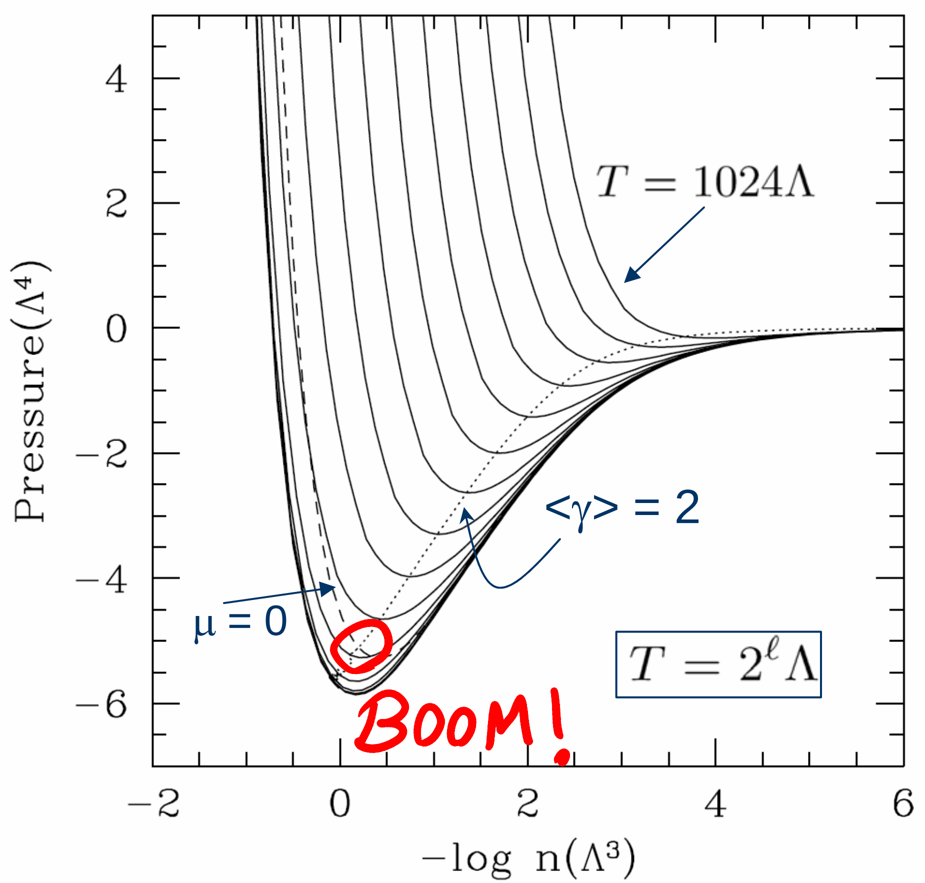
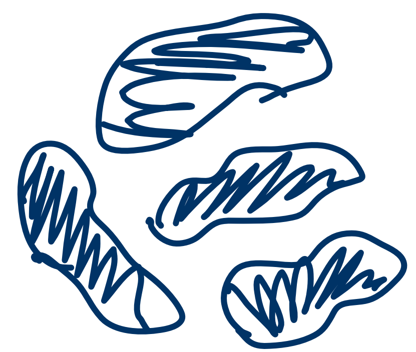


Homogeneous Expansion

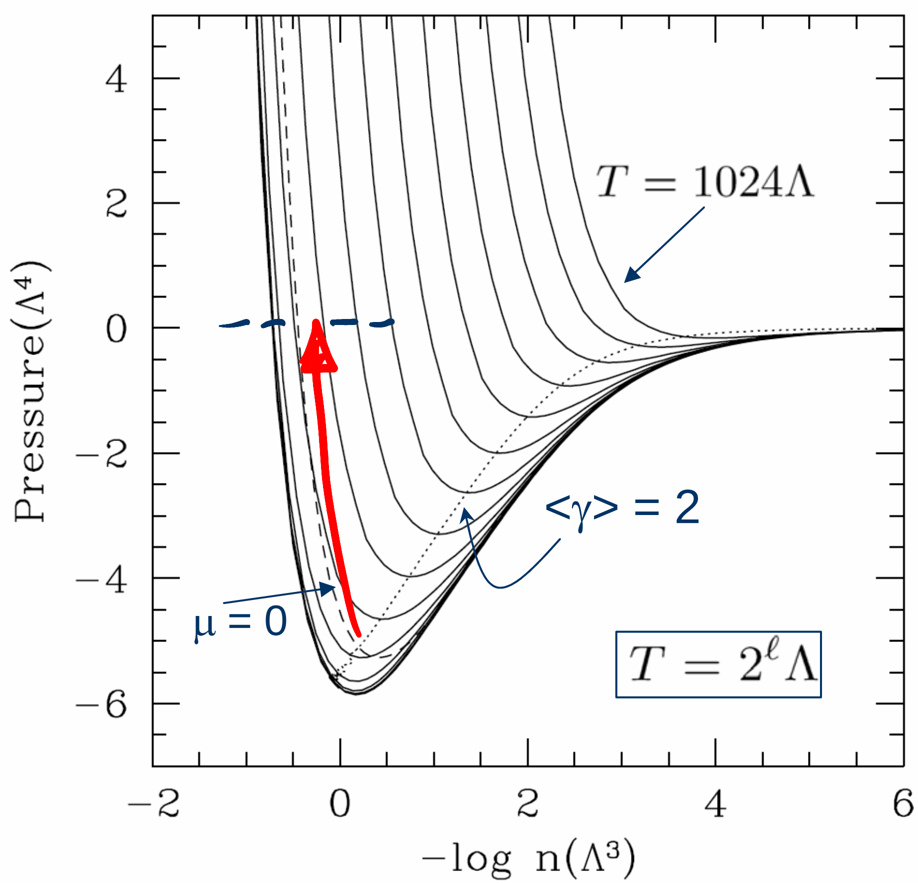
The Trans-Relativistic Phase Transition

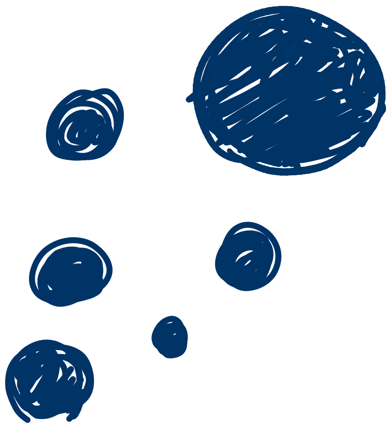




Onset of Instability

The Trans-Relativistic Phase Transition





Neutrinos condense into Nuggets with zero net pressure

Adiabatic Perturbations of Dark Energy: The General Case

- Assuming that
  1) $c_s^2 > 0$ (stable perturbations)
  2) Dark Energy perturbations are adiabatic, i.e. $P_{DE}=F(\rho_{DE})$

$\Rightarrow$ As $t \to \infty$; $\rho_{DE} \to$ const;

i.e. we need a cosmological constant

$$c_s^2 = \frac{\dot{P}_{DE}}{\dot{\rho}_{DE}} > 0 \;\Rightarrow\; P_{DE}(t_1) = P_{DE}(t_0) + \int_{t_0}^{t_1} c_s^2 \dot{\rho}_{DE} dt < P_{DE}(t_0) < 0$$

Is there any way to stabilize MaVaNs?

- Light Acceleron ($m_A \sim H$) → Quintessence
  - Bi, Feng, Li, Xin-min & Zhang 2004
  - Brookfield, van de Bruck, Mota & Tocchini-Valentini 2005
- Very Light Neutrinos ($T_\nu > m_\nu$) → ΛCDM

  → Only lightest neutrino couples to acceleron

  → Lightest neutrino is relativistic (atmospheric neutrinos)

  $m_\nu < 10^{-4}$ eV
  - Fardon, Nelson, & Weiner 2005
- Decoupled Neutrinos ($m_\nu \sim$ const) → ΛCDM
  - Takahashi & Tanimoto 2005

Conclusions

- Non-relativistic MaVaNs are in general unstable, unless there is a cosmological constant
- MaVaNs undergo a phase-transition as they become non-relativistic, and form neutrino nuggets
- Despite its rich phenomenology, the Logarithmic MaVaNs model is unlikely to act as Dark Energy
- Possible ways out give back ΛCDM or Quintessence