COSMIC STRING EVOLUTION IN HIGHER DIMENSIONS

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work with E.P.S. Shellard

Based on:

hep-ph/0410349 [PRD]

COSMO '05, Bonn

Summary

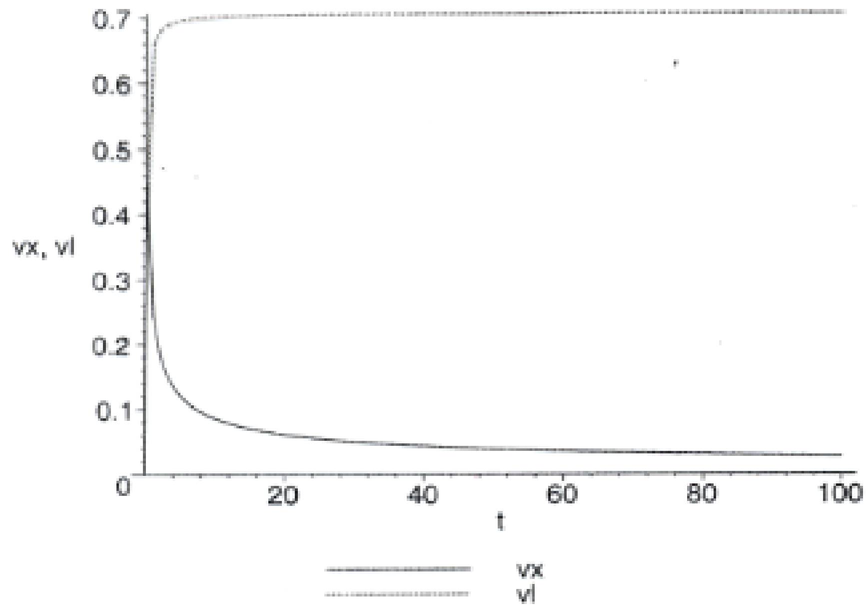
Have developed a model for quantitative string evolution in higher dimensions.

This verifies the existence of a scaling solution with a density enhanced by a factor $\sim P_{eff}^{-2}$

Further, quantitative corrections arise due to variable 3D velocity U_x

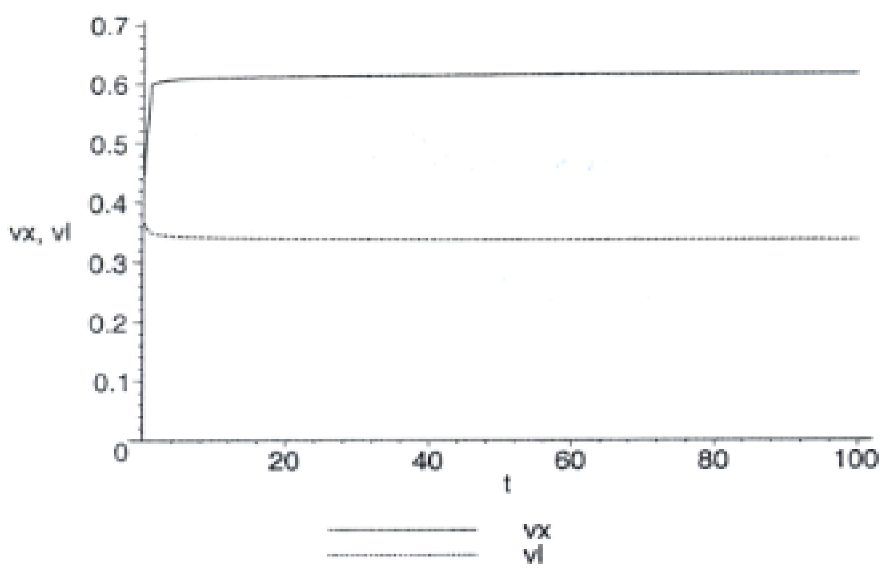
Effect of extra dimensional velocities U_ℓ is to slow down the strings in 3D.

> kl~kx



$v_x \to 0$
No strong motion
in 3D $\Rightarrow$
No interactions!

> kl<<kx



v_x slowed down

>

The EDVOS model 2. Macroscopic eqns

- Correlation length

$$E(t) \Longrightarrow \dot{E} \Longrightarrow \dot{\rho} \Longrightarrow \dot{L}\,, \quad \text{let} \quad L = \gamma(t)\,t$$

$$\gamma^{-1}\frac{d\gamma}{dt} = \frac{1}{2t}\Big\{ b\big[(2+w_1^2)+(2-w_1^2)v_x^2 + (1-w_1^2)v_1^2 - 2 + \frac{\tilde{c}\,v_x}{\gamma} \Big\}$$

where $w_1^2 = \left\langle \frac{\xi'^2}{a^2\underline{x}'^2+\underline{\xi}'^2} \right\rangle$

- String velocities

$$v_x \equiv \frac{\int \dot{\underline{x}}^2 \varepsilon\, d\sigma}{\int \varepsilon\, d\sigma} \Longrightarrow \dot{v}_x$$

$$v_x\frac{dv_x}{dt} = \frac{k_x v_x}{R}(1-v^2) - (2-w_1^2)H v_x^2(1-v^2) - H v_x^2 v_1^2$$

$$v_1\frac{dv_1}{dt} = \frac{k_1 v_1}{R}(1-v^2) - (1-w_1^2)H v_1^2(1-v^2) + H v_1^2 v_x^2$$

where $\frac{k_x v_x(1-v^2)}{R} = \dot{\underline{x}}\cdot\underline{u}\,\left(1-\dot{\underline{x}}-\frac{\dot{\xi}^2}{a^2}\right)$

sim. k_1

[AA & EPSS hep-ph/0410349 (PRD)]

The EDVOS model 1. Microscopic eqns

Nambu action $S = -\mu \int \sqrt{-\gamma}\, d^2\zeta$, $\gamma_{ab} = g_{\mu\nu} \partial_a x^\mu \partial_b x^\nu$

Metric : $ds^2 = N(t)^2 dt^2 - a(t)^2 d\underset{\sim}{x}^2 - b(t)^2 d\underset{\sim}{\ell}^2$

$\quad x^0 \qquad x^1, x^2, x^3 \qquad x^4, \ldots, x^D$

Choose gauge : $\zeta^0 = t$, $\dot{x} \cdot x' = 0$

• e.o.m. $\dfrac{\delta S}{\delta x^\mu} = 0$:

$$\dot{\varepsilon} = -N^{-2} \varepsilon \left\{ N\dot{N} + a\dot{a}\left[\dot{\underset{\sim}{x}}^2 - \left(\frac{\underset{\sim}{x}'}{\varepsilon}\right)^2\right] + b\dot{b}\left[\dot{\underset{\sim}{\ell}}^2 - \left(\frac{\underset{\sim}{\ell}'}{\varepsilon}\right)^2\right] \right\}$$

$$\ddot{\underset{\sim}{x}} + \left\{ 2\frac{\dot{a}}{a} - N^{-2} \left\{ N\dot{N} + a\dot{a}\left[\dot{\underset{\sim}{x}}^2 - \left(\frac{\underset{\sim}{x}'}{\varepsilon}\right)^2\right] + b\dot{b}\left[\dot{\underset{\sim}{\ell}}^2 - \left(\frac{\underset{\sim}{\ell}'}{\varepsilon}\right)^2\right] \right\} \right\} \dot{\underset{\sim}{x}} = \left(\frac{\underset{\sim}{x}'}{\varepsilon}\right)' \varepsilon^{-1}$$

$$\ddot{\underset{\sim}{\ell}} + \left\{ 2\frac{\dot{b}}{b} - N^{-2} \left\{ N\dot{N} + a\dot{a}\left[\dot{\underset{\sim}{x}}^2 - \left(\frac{\underset{\sim}{x}'}{\varepsilon}\right)^2\right] + b\dot{b}\left[\dot{\underset{\sim}{\ell}}^2 - \left(\frac{\underset{\sim}{\ell}'}{\varepsilon}\right)^2\right] \right\} \right\} \dot{\underset{\sim}{\ell}} = \left(\frac{\underset{\sim}{\ell}'}{\varepsilon}\right)' \varepsilon^{-1}$$

where $\varepsilon = \dfrac{-x'^2}{\sqrt{-\gamma}}$

• E-M tensor from $\dfrac{\delta S}{\delta g_{\mu\nu}}$:

$$T^{\mu\nu} = \frac{1}{N a^3 b^{D-3}} \mu \int d\zeta \left(\varepsilon \dot{x}^\mu \dot{x}^\nu - \varepsilon^{-1} x'^\mu x'^\nu \right) \delta^{(D)}\left(\underset{\sim}{x} - \underset{\sim}{x}(\zeta,t), \underset{\sim}{\ell} - \underset{\sim}{\ell}(\zeta,t)\right)$$

Energy : $E = \displaystyle\int_{t=\mathrm{const}} \sqrt{h}\, n_\mu n_\nu T^{\mu\nu} d^3\underset{\sim}{x}\, d^{D-3}\underset{\sim}{\ell}$

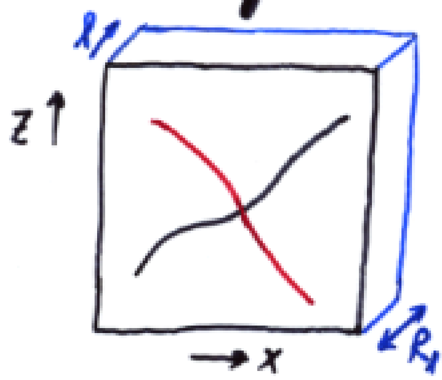
$$\Rightarrow E(t) = N(t) \mu \int \varepsilon\, d\zeta$$

Strings in Higher Dimensions

Motivation from Brane Inflation [Dvali & Tye, 1999; BMQRZ, 2001; DSS, 2001]
where string-like defects can be produced at the end of inflation. (Sarangi & Tye, 2002; Dvali & Vilenkin, 2004)

The strings propagate in (FRW) × I space where I a compact manifold.

Consider string collision:



Apparent point of intersection:

Strings may miss each other in extra dimension.

Probability of reconnection $P \sim \frac{\delta}{R_1}$ ← string width

Introduce P in one-scale model

$$2\frac{dL}{dt} \approx 2HL + 1 \cdot P$$ (Jones, Stoica & Tye, 2003)

Scaling density $\frac{\rho}{\rho_{stand}} \sim P^{-2}$

Cycloops

[AA & EPSS hep-ph/0504049 (JHEP)]

Recall string intercommutings produce loops which radiate and shrink to zero radius.

If compact space admits non-trivial 1-cycles then loops can be trapped.

For loops $\rho(t) \propto t^{-2}$ (like radiation)

For cycloops $\rho(t) \propto t^{-3/2}$ (like matter)

To avoid cycloop domination for typical choice of parameters in Brane Inflation need $G\mu < 10^{-18}$ too strong

Small regions in parameter space exist for which $G\mu \sim 10^{-14}$ Dark Matter

The VOS model (Martins & Shellard 1996)

Starting from Nambu-Goto action, derive evolution equations for the macroscopic quantities:

$v = \sqrt{\langle \dot{\underline{x}}^2 \rangle}$, L

From e.o.m. $\ddot{\underline{x}} = \ldots$

From $T^{\mu\nu} \to E, \dot{E}$

$\Rightarrow \dot{\rho} \to \dot{L}$ $\left[\rho = \mu L^{-2}\right]$

We find:

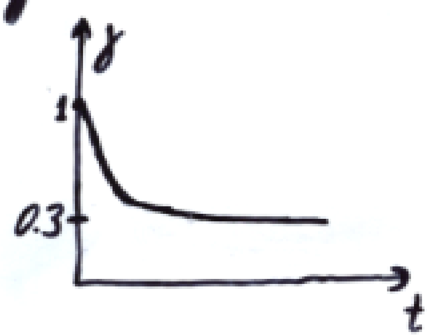
$$\gamma^{-1}\frac{d\gamma}{dt} = \frac{1}{2t}\left[2b(1+v^2) - 2 + \frac{\tilde{c}v}{\gamma}\right] \quad , L = \gamma(t)\, t$$

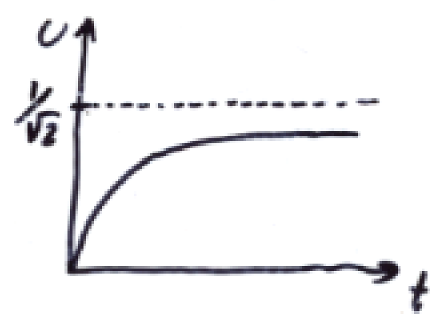
$$\frac{dv}{dt} = (1-v^2)\left(\frac{k}{R} - 2Hv\right)$$

where k - momentum parameter

b - expansion exponent $a(t) \propto t^b$

Scaling Solution:





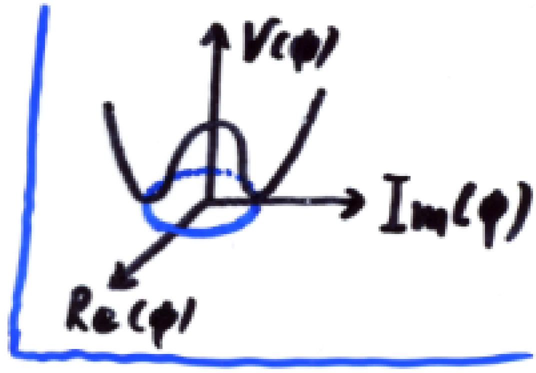
ATTRACTOR

Constraint: $v^2 \le 1/2$, saturated in flat space

Q. In higher dimensions $v_x^2 + v_1^2 \le 1/2$ $\quad v_1^2 \to 1/2$, $v_x^2 \to 0$?

Cosmic Strings & Evolution

Cosmic Strings: Line-like concentrations of energy arising as topological defects in cosmological phase transitions.



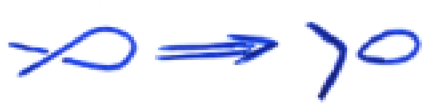
Evolution

- Field Theory Simulations
- Nambu Strings
  - simulations
  - 'thermodynamic' approach

String network is Brownian
Characterised by correlation length L

Energy density $\rho = \frac{\mu}{L^2}$

$$\dot{\rho} \approx \overbrace{-2\frac{\dot{a}}{a}\rho}^{\text{Hubble expansion}} - \underbrace{\frac{\mu}{L}}_{\text{Loop production}}$$



Find Scaling solution $L \sim t \sim H^{-1}$
$\rho \propto t^{-2}$ (Kibble 1985)

Plan

- Cosmic strings & evolution in 3D
- Strings in higher dimensions
- The EDVOS model
- Results & Summary