Behind the CMB: From interpretation of blackbox computations to predictive analytical description

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COSMO 05

CMB Anisotropy

- **Theory**

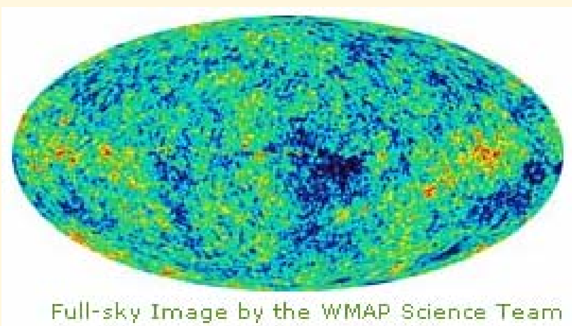
  + Kinetic Theory

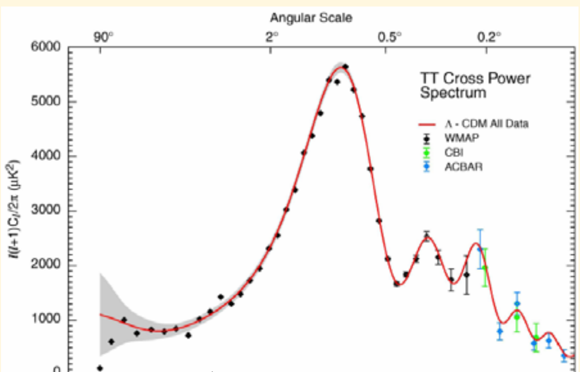
$$\dot{f}_a(\tau,\mathbf{x},\mathbf{p}) + \dot{x}^i \frac{\partial f_a}{\partial x^i} + \dot{p}^i \frac{\partial f_a}{\partial p^i} = [\,\dot{f}_a\,]_{\text{collisions}}$$

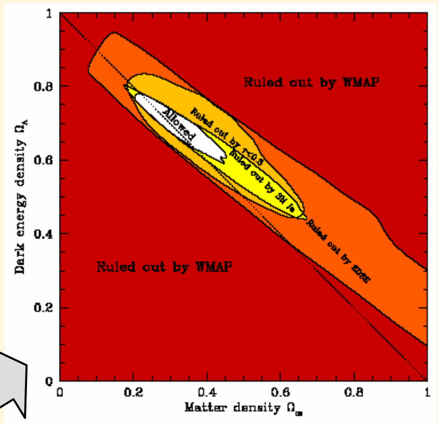
  + General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

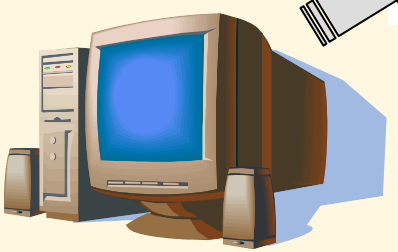
- **Analysis**







Cosmological Parameters:
$P_\zeta(k)$, $P_h(k)$, $\Omega_b h^2$, $\Omega_c h^2$, $\Omega_{dark} h^2$,
Ω_{curv}, $w_{dark}(z)$, N_ν, m_ν, Y_{He}, …

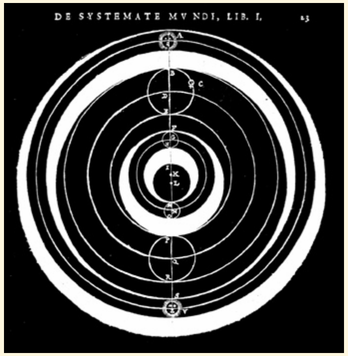


- **What is to be improved and why?**

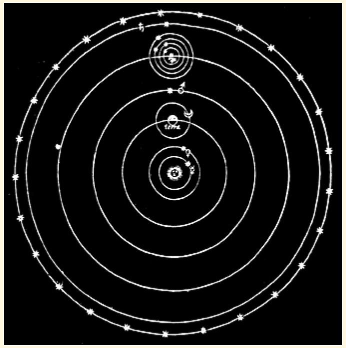
Motivations for an efficient formalism

- Direct connection of leading or subleading (unknown) interactions to observable features in the perturbation spectra
  in view of
  - **gauge** freedom
  - freedom of defining general-relativistic **variables**

Example: Equivalent descriptions in different gauges

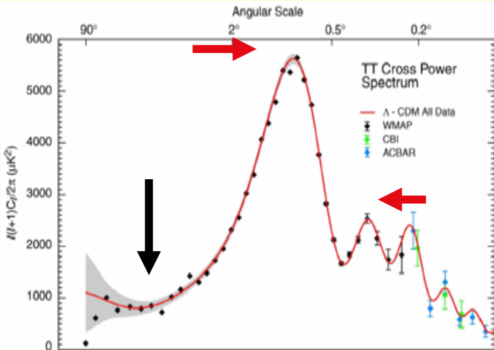


Claudius Ptolemy



Nicolas Copernicus

- Relative simplicity
- Explicit reflection of causal dependencies



Formalism: Measure of Density Perturbations

Our requirements to a measure of density perturbations:

1. Explicit Freezing of its **superhorizon** evolution.

Linear evolution of superhorizon perturbations of densities is insensitive to changes of the metric

   - perturbations of densities evolve manifestly causally despite generally non-causal changes of the metric
   - species decouple gravitationally in the linear order

2. Reality of its gravitational **driving**.

All the terms which describe gravitational driving of species by metric inhomogeneities vanish in physically homogeneous geometry

Formalism: Measure of Density Perturbations

1. Freezing of species density perturbations on superhorizon scales.

- Various measures with this property: Bardeen 80; Lyth 85

$$\zeta = \frac{1}{6}\delta \ln\left[{}^{(3)}g^{(\text{unif. }\rho)}\right] = -\Psi + \frac{\delta\rho}{3(\rho+p)}$$

$$\mathcal{R} = \frac{1}{6}\delta \ln\left[{}^{(3)}g^{(\text{comov.})}\right] = -\Psi - \frac{2\rho}{3(\rho+p)}\left(\Phi + \frac{\dot\Psi}{\mathcal{H}}\right)$$

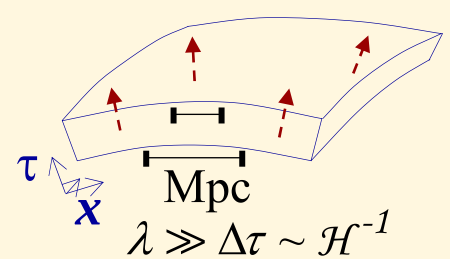
(We use the Newtonian gauge, $ds^2 = a^2[-(1+2\Phi)d\tau^2 + (1-2\Psi)d\mathbf{x}^2]$)

- On superhorizon scales, all such measures are mutually proportional

  (by their freezing & phase space squeezing)

- Their time-independence is a consequence of energy conservation and causality:

$$3\zeta = \frac{\delta\rho}{\rho+p} - 3\Psi = \frac{\delta n_{\text{coo}}}{n_{\text{coo}}} \equiv d$$



- **Ambiguities**:
  - normalization
  - continuation to small scales

Formalism: Measure of Density Perturbations

2. Reality of gravitational driving.

- Requirements 1 and 2 select

$$d_a \equiv \frac{\delta n_{a\ \mathrm{coo}}}{n_{a\ \mathrm{coo}}} = \frac{\delta \rho_a}{\rho_a + p_a} - 3\Psi$$

as a **unique** generalization of the Newtonian density perturbation $\delta n_a / n_a$ to superhorizon scales

Proof:

- Consider an artificially fine-tuned scenario where the geometry becomes homogeneous while the perturbations are superhorizon. (E.g., add artificial homogeneous species which first have $p = -\rho/3$. After these species become dominant, set them to generate FRW metric with any desired $\mathcal{H}(z)$.)
- In the homogeneous geometry, by condition 2, the Newtonian $\delta n_a / n_a$ matches to $\delta\rho_a/(\rho_a + p_a)$
- Beyond the horizon, by condition 1, $\delta\rho_a/(\rho_a + p_a)$ matches to d_a

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Formalism: View of the evolution of multiple species

- Superhorizon

vs.

Frozen density perturbations d_a of individual **uncoupled** species

This view of superhorizon evolution was first proposed by Wands, Malik, Lyth, and Liddle 2000 in terms of the uniform-density curvature ζ_a.

Note that $d_a = 3\zeta_a$ can also be viewed as perturbations of (coordinate) densities of the various species in a **single** Newtonian slicing.

$$d_{\text{total}}(\tau) = \sum_a x_a(\tau)\; d_a\,,$$

$$x_a(\tau) \equiv \frac{\rho_a + p_a}{\rho + p}$$

Isocurvature Modes

A disadvantage of this picture:

The condition $\rho_{\text{total}} = 0$ is not preserved by superhorizon evolution

Formalism: Evolution of multiple species

- Horizon entry

  (Then the species couple gravitationally through the dependence of v_a evolution on Φ and Ψ)

The variables

d_a, v_a, + possible anisotropy multipoles

(not Φ and Ψ, which follow from non-dynamical elliptic equations)

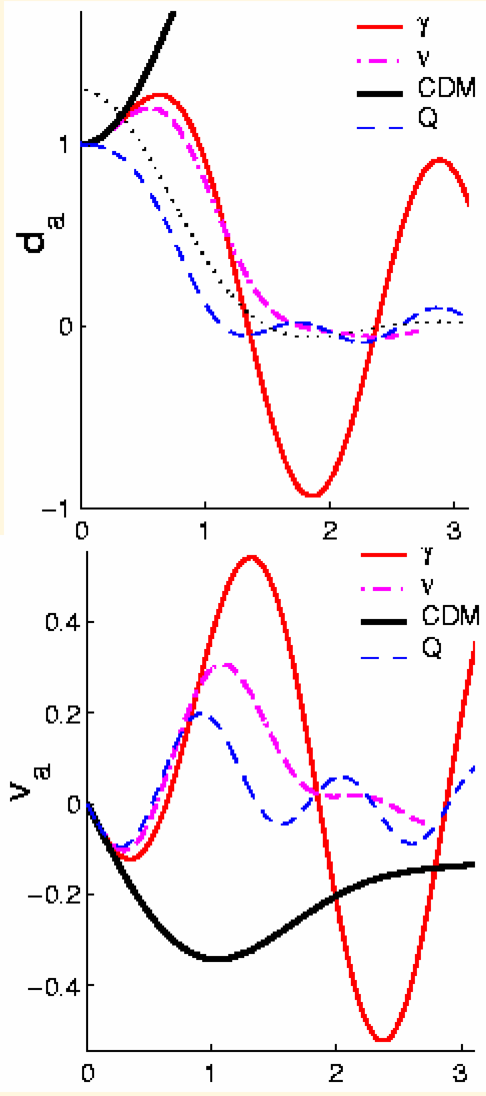
are unconstrained and provide

A. One-to-one match to physically distinct initial conditions

B. Cauchy structure of the evolution equations

The variables D_a (Durrer 2001; Doran CMBEASY), for which A.& B. also hold, are simply related to d_a, which in addition are conserved on superhorizon scales:

$$d_a = \frac{D_a}{1+w_a}$$



Formalism: Simplicity of Equations

Suggested | **Traditional**

Photon-baryon fluid

$$\ddot{d}_\gamma + \mathcal{H}\frac{R_b}{1+R_b}\dot{d}_\gamma - \frac{1}{3(1+R_b)}\nabla^2 d_\gamma = \nabla^2\left(\Phi + \frac{1}{1+R_b}\Psi\right)$$

$$\ddot{\Theta}_\gamma + \mathcal{H}\frac{R_b}{1+R_b}\dot{\Theta}_\gamma - \frac{1}{3(1+R_b)}\nabla^2\Theta_\gamma = \frac{1}{3}\nabla^2\Phi + \mathcal{H}\frac{1}{1+R_b}\dot{\Psi} + \ddot{\Psi})$$

CDM

$$\ddot{d}_c + \mathcal{H}\dot{d}_c = \nabla^2\Phi$$

$$\ddot{\delta}_c + \mathcal{H}\dot{\delta}_c = \nabla^2\Phi + 3\mathcal{H}\dot{\Psi} + 3\ddot{\Psi}$$

Neutrinos (here, massless)

$$\dot{I}_\nu + n_i\nabla_i I_\nu = -4\,n_i\nabla_i(\Phi+\Psi)$$

$$\dot{\Delta}_\nu + n_i\nabla_i\Delta_\nu = 4\,(-n_i\nabla_i\Phi + \dot{\Psi})$$

- Ψ is a non-local functional of δ_a and $\dot{\delta}_a$
- The contribution of $\dot{\Psi}$ and $\ddot{\Psi}$ is dominant during the horizon entry

- γ scattering and polarization can be included, SB astro/0405157
- The last eq. has a fully nonlinear generalization, SB astro/0505502

Formalism: Simplicity of Solutions

Suggested | **Traditional**

Radiation era, ignoring neutrinos

$$\varphi \equiv k\int_0^\tau c_s d\tau$$

Suggested

$$d_\gamma = 3\zeta_{\text{in}}\left(-\cos\varphi + \frac{2\sin\varphi}{\varphi}\right)$$

$$d_c = 6\zeta_{\text{in}}\left(\ln\varphi + \gamma - \frac{1}{2} + \frac{\sin\varphi}{\varphi} - \text{ci}\,\varphi\right)$$

Traditional

$$\delta_\gamma = 4\zeta_{\text{in}}\left(-\cos\varphi + \frac{2\sin\varphi}{\varphi} + \frac{2\cos\varphi}{\varphi^2} - \frac{2\sin\varphi}{\varphi^3}\right)$$

$$\delta_c = 6\zeta_{\text{in}}\left(\ln\varphi + \gamma - \frac{1}{2} + \frac{\sin\varphi}{\varphi} - \text{ci}\,\varphi + \frac{\cos\varphi}{\varphi^2} - \frac{\sin\varphi}{\varphi^3}\right)$$

including neutrinos

Solved analytically

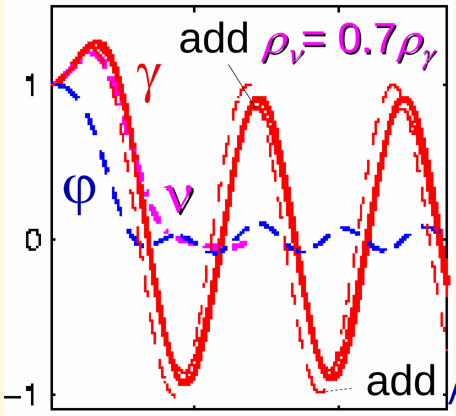
SB, Seljak 03

Analytic solution not found

Implications: “Radiation driving” is a gauge artifact

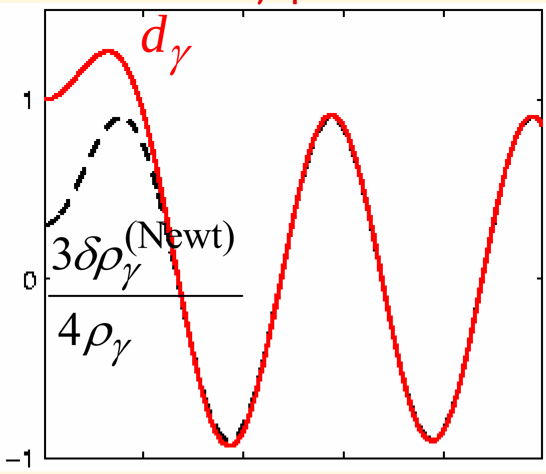
Horizon entry in the radiation era:

- Without neutrinos, the amplitude of $\delta\rho_\gamma/\rho_\gamma$ rises 3-fold in the Newtonian gauge.
- A resonant boost of $\delta\rho_\gamma/\rho_\gamma$ by specially timed decay of Φ?
  - $\delta\rho_\gamma/\rho_\gamma$ is affected little by adding species with different perturbations:
  - The subhorizon amplitude of $\delta\rho_\gamma/\rho_\gamma$ is **unaffected** if the metric is flattened on superhorizon scales:



$$\frac{3\delta\rho_\gamma^{\text{(undriven)}}}{4\rho_\gamma} = 3\zeta_{\text{in}}\cos\varphi$$

Measures of γ perturbations



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  - The rise is a large-scale **artifact** of the Newtonian gauge
  - In appropriate variables it is absent:

$$d_\gamma^{(\gamma\ \text{dom.})} = 3\zeta_{\text{in}}\left(\frac{2\sin\varphi}{\varphi} - \cos\varphi\right)$$

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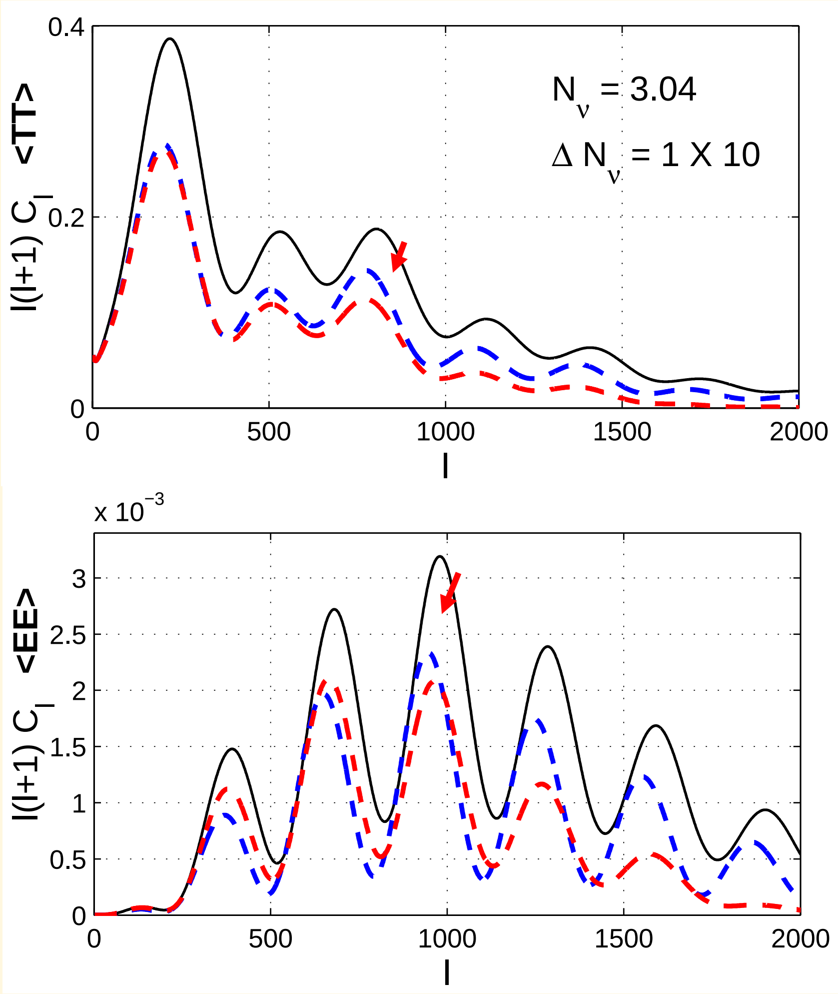
Implications: Acoustic phase is a non-degenerate sensitive probe of the radiation era

- Calculating analytically the Green's function of d_γ in the radiation era we find
  - After the horizon entry

$$d_\gamma(\tau,k) \rightarrow A_\gamma \cos\left(\frac{k\tau}{\sqrt{3}} + \Delta\varphi\right)$$

- amplitude reduction $\quad \dfrac{\Delta A_\gamma}{A_\gamma} \approx -0.27\dfrac{\rho_\nu}{\rho},$
- phase shift $\quad \Delta\varphi \approx 0.19\,\pi\,\dfrac{\rho_\nu}{\rho}$

  - With the adiabatic i.c., $\Delta\varphi \neq 0$ if only perturbations of some species (ν or quintessence) propagate faster than c_s



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Implications: Describing the CMB Anisotropy

$$\frac{\Delta T(\hat{\boldsymbol{n}})}{T} = \int d\tau \left[\left(e^{-\kappa}\right)^{\bullet} \left(\Theta_{\gamma\,\mathrm{eff}} - \mathrm{v}_b^i n_i + Q^{ij} n_i n_j \right) + e^{-\kappa} \left(\dot{\Phi} + \dot{\Psi} \right) \right]$$

- Sachs-Wolfe term:

vs.

$$\Theta_{\gamma\,\mathrm{eff}} \equiv \frac{\delta T_\gamma^{(\mathrm{Newt})}}{T_\gamma} + \Phi$$

Suited for small scales, on which δT_γ is measurable (GPS)

$$\Theta_{\gamma\,\mathrm{eff}} \equiv \frac{1}{3} d_\gamma + \Phi + \Psi$$

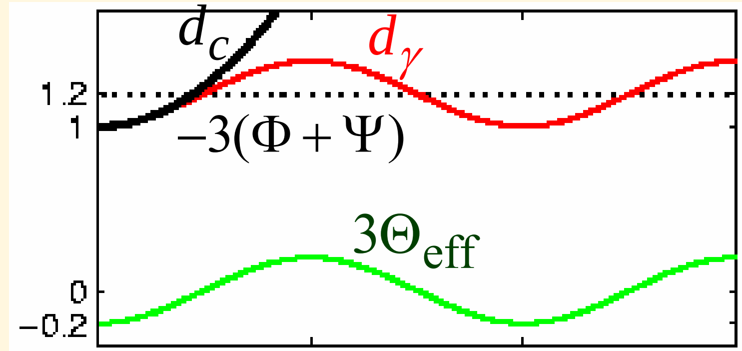
Better suited for self-gravitating systems (cosmology, CMB)

$$\ddot{d}_\gamma + \mathcal{H} \frac{R_b}{1+R_b} \dot{d}_\gamma + c_s^2 \boldsymbol{k}^2 \left[d_\gamma + 3\left(\Phi + \Psi + R_b \Phi\right) \right] = 0$$

Implications: 5-fold Sachs-Wolfe suppression by CDM

$$\Theta_{\rm eff} \equiv \frac{\delta T_\gamma}{T_\gamma} + \Phi = \frac{1}{3} d_\gamma + \Phi + \Psi$$

Matter domination



$\Theta_{\rm eff}(k\tau << 1)$, and so large-angle $\Delta T_{\rm CMB}$ receive contribution from both $\delta\rho_{\gamma\,\rm primordial}$ and the potential Φ due to $\delta\rho_{\rm m}$.

In what proportions?

– **Sachs-Wolfe**, naive:

$$\Theta_{\rm eff} : \frac{\delta T_\gamma}{T_\gamma} : \Phi = -\frac{1}{2} : 1 : -\frac{3}{2}$$

revised:

$$\boxed{3\Theta_{\rm eff} : d_\gamma : 3(\Phi + \Psi) = -\frac{1}{5} : 1 : -\frac{6}{5}}$$

– **Switch off** $\delta\rho_{\gamma\,\rm prim}/\rho_\gamma$
(set isocurvature i.c. $\delta\rho_\gamma + \delta\rho_c = 0$, hence $d_\gamma << d_m$, set same $\zeta_{\rm m}$):

$$3\Theta_{\rm eff} : d_\gamma : 3(\Phi + \Psi) = -\frac{6}{5} : 0 : -\frac{6}{5}$$

– **Switch off** Φ due to $\delta\rho_{\rm m}$:
(set homogeneous matter, $d_m = 0$, and same $\zeta_{\rm prim}$ in the rad. era):

$$3\Theta_{\rm eff} : d_\gamma : 3(\Phi + \Psi) = 1 : 1 : 0$$

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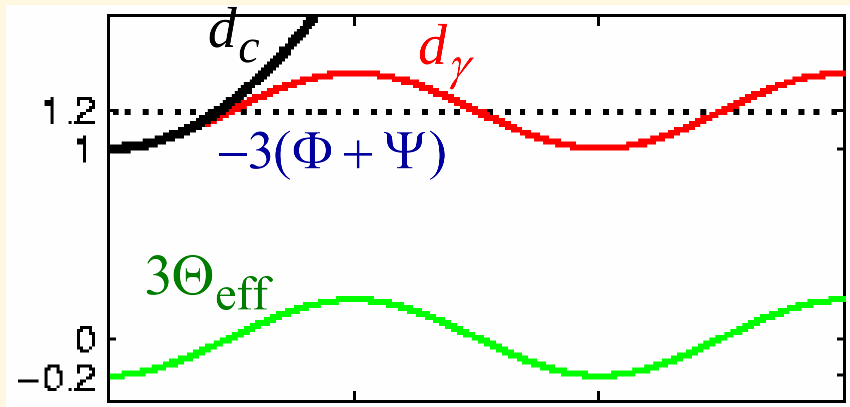
$3\Theta_{\rm eff} : d_\gamma : 3(\Phi + \Psi)$

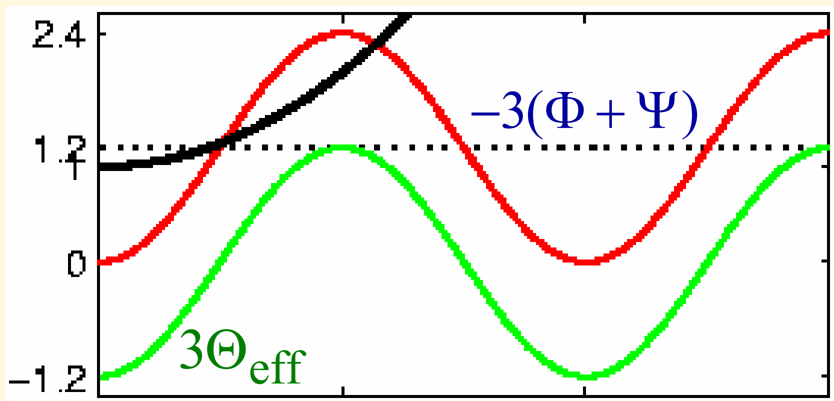
Matter domination

Sachs-Wolfe (adiabatic)



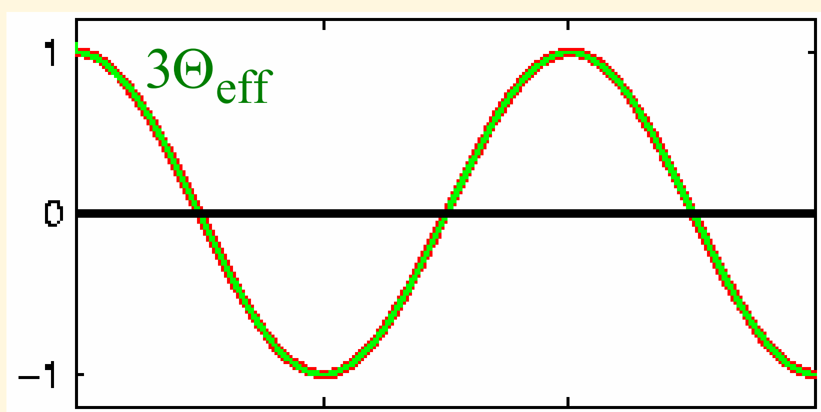
$$-\frac{1}{5} : 1 : -\frac{6}{5}$$

Isocurvature (negligible $\delta\rho_{\gamma\,\rm prim}/\rho_\gamma$)



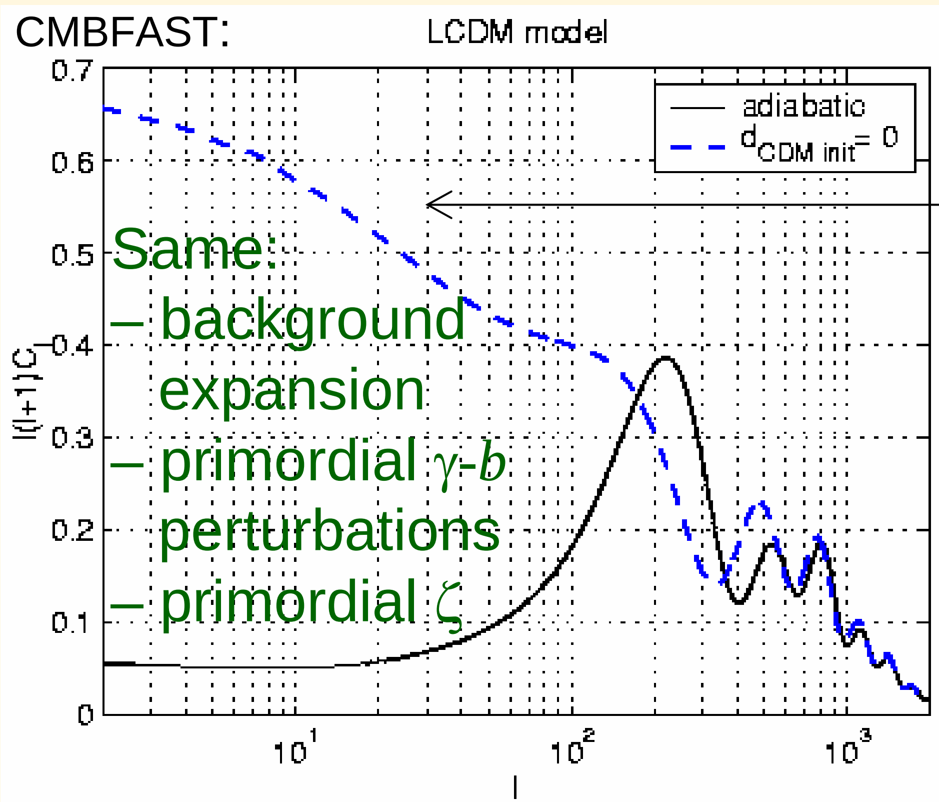
$$-\frac{6}{5} : 0 : -\frac{6}{5}$$

Homogeneous (zero $\delta\rho_{\rm m}$)



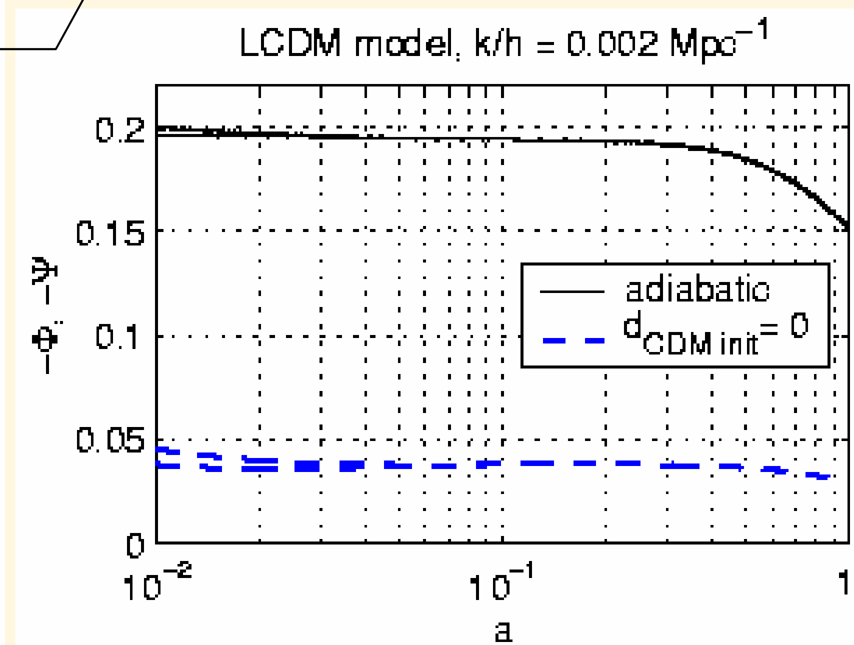
$$1 : 1 : 0$$

Implications: The magnitude of the CMB fluctuations for $l < 200$ is a sensitive probe of the universe composition in the matter era



Same:
– background expansion
– primordial γ-b perturbations
– primordial ζ

This is NOT enhancement due to the ISW effect:



The **low-*l* suppression** of C_l by CDM potential is

– an order of magnitude effect

– statistically significant, $1/\sqrt{\sum_{l=2}^{200}(2l+1)} \simeq 5\times 10^{-3}$

Conclusions:

- The measure of density perturbations $d_a = 3\zeta_a$ which manifests explicit superhorizon **freezing and reality** of gravitational driving offers a more mathematically simple and dynamically oriented formalism for linear CMB dynamics than the known alternatives.

- The CMB fluctuations for $l \geq 200$ do not experience a resonant self-gravitational boost during the horizon reentry. Nevertheless, the low cosmic variance and the existence of non-degenerate signatures at small scales allow precision CMB studies of the radiation era.

- The CMB temperature auto-correlation C_l is suppressed by non-decaying matter potential by $5^2 = 25$ times in the Sachs-Wolfe limit. The large magnitude of this suppression makes the effect a valuable probe of the matter era.