Bouncing Universes in Stringy-Gravity

at

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by

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Why Non-perturbative Gravity?

- Stringy Motivation:
  - ∗ Higher Derivative Corrections appear as a series in α'
  - ∗ In String Field Theory HD modification is Gaussian $(e^{\Box}\phi)$ in vertex factors, redefinition: $\phi \to e^{-\Box}\phi$
- Freedom from Ghosts
  - ∗ Ghosts violate Unitarity &/or carry Negative Energy
  - ∗ Higher Derivative Theories generically have Ghosts

$$S = \int d^4x\ \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

  - ∗ No extra states, Non-perturbative Ex: $\Box e^{-\Box}\phi = 0$

- Asymptotic Freedom
  - $*$ Improved UV behaviour: 4th Order Gravity

    $S = \int d^4x \sqrt{-g}(R + c_0 R^2 + b_0 C^2)$

    even Renormalizable [Stelle, 1978]

    (Ghost+Asymptotically) Free Gravity $\Rightarrow$ NP Gravity
  - $*$ Address Singularities: Big Bang, Black Holes...

    At Short distance Gravity Weakens

    $\Rightarrow$ Pressure can prevent Collapse $\Rightarrow$ Bounce

    Newtonian Intuition

    $H^2 \equiv \frac{\dot{r}^2}{r^2} = \frac{M_p^2}{3}\left(-\rho r V(r) + \frac{6E}{M_p^2 m r^2}\right)$

Model

- Action: $S = \int d^4x \, \sqrt{-g} F \, , F = R + \sum_{n=0}^{\infty} c_n R \Box^n R$
- "Generalized" Einstein's Field Equations [Schmidt]:

  $\widetilde{G}_{\mu\nu} \equiv G_{\mu\nu} + \sum_{n=0}^{\infty} G_{\mu\nu}^n = T_{\mu\nu}$

  $G_{\mu\nu}^n \sim c_n (\Box^{n+1} R + \Box^p R \Box^m R)$
- Conservation Equation: $\nabla^\mu \widetilde{G}_{\mu\nu} = 0$

  $\Rightarrow$ For Cosmology $\widetilde{G}_{00}$ equation suffices

Propagator

- Trace Equation (Lorentz Gauge): $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
  $\Rightarrow \tilde{G} = -\frac{1}{2}\Box(1 - 6\sum_0^\infty c_i \Box^{i+1})h = -\frac{1}{2}\Box\Gamma(\Box)h$
  $\Rightarrow \Delta(p^2) = \frac{1}{p^2\Gamma(-p^2)}$
- Ghost Free Theory $\Leftrightarrow \Gamma(\Box)$ has single/no zero
- Potential for h (scale factor):
  $\tilde{G} \sim -m\delta(\vec{r}) \Rightarrow h(r) \sim \frac{1}{r}\int_{-\infty}^{\infty} dp \frac{p}{p^2\Gamma(-p^2)} e^{ipr}$
- Asymptotic Freedom: Integrand falls faster than $\frac{1}{p}$
  $\Rightarrow h(r) \stackrel{r\to 0}{\to}$ constant
- Newtonian Limit: $\Gamma(-p^2) \stackrel{p\to 0}{\to} 1 \Rightarrow h(r) \stackrel{r\to\infty}{\to} \frac{1}{r}$
- Example: $\Gamma(\Box) = e^{-\Box} \Rightarrow h(r) \sim \frac{\mathtt{erf}(r)}{r}$

Non-singular Bounce

- Prescription: Find $a = a(t)$ such that $\Box R(t) \sim R(t)$
  $(\ldots)R(t) + (\ldots)R^2(t) \sim$ matter sources
  Entails only solving Algebraic Equations

- Hyperbolic Bounce: $a(t) = \cosh \lambda t$ works!
  $R(t) \sim 2 - \mathtt{sech}^2 \lambda t \ , \ \Box^n R \sim \mathtt{sech}^2 \lambda t$

- $\widetilde{G}_{00}$ Equation:
  $\widetilde{G}_{00} = T_{00} = \frac{1}{3}\left[\Lambda + \rho_0 \mathtt{sech}^4 \lambda t\right]$

- Solutions Exist to
  Ghost free condition, $\rho_0 > 0$ & "Bounce Constraints"
  $\Lambda \neq 0$

Transition to FRW, $\Lambda = 0$

- Late times: $a(t) \to \frac{a_0}{2} e^{\lambda t}$ & HD Terms $\sim \mathtt{sech}^2 \to 0$
  $\Rightarrow$ Einstein Gravity & deSitter Universe $\Rightarrow \Lambda \neq 0$

- Near Bounce: $G_{00} \to 0$ while HD Terms are Finite

- Approximate Bounce:

  * Small times: HD Terms $=$ Radiation
    We found examples of Ghost-free bounces

  * Transition: $G_{00} \sim$ HD Terms

  * Large times: FRW cosmology, HD Terms $<< G_{00}$
    $a(t) \sim t^{1/2}$, $G_{00} \sim \frac{1}{t^2}$, $\widetilde{G}^n_{00} \sim \frac{1}{t^{2(n+2)}}$

Conclusions

- Non-perturbative Gravity can be Ghost and asymptotically free
- Small times/distances: HD terms important (bounce)
- Large times/distances: Newtonian dynamics & FRW
- More exact Solutions, Better understanding
- Adressing Black hole Singularity: (Ricci & Weyl)