# WMAP and LSS constraints on the Running Mass Model

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based on work in collaboration with

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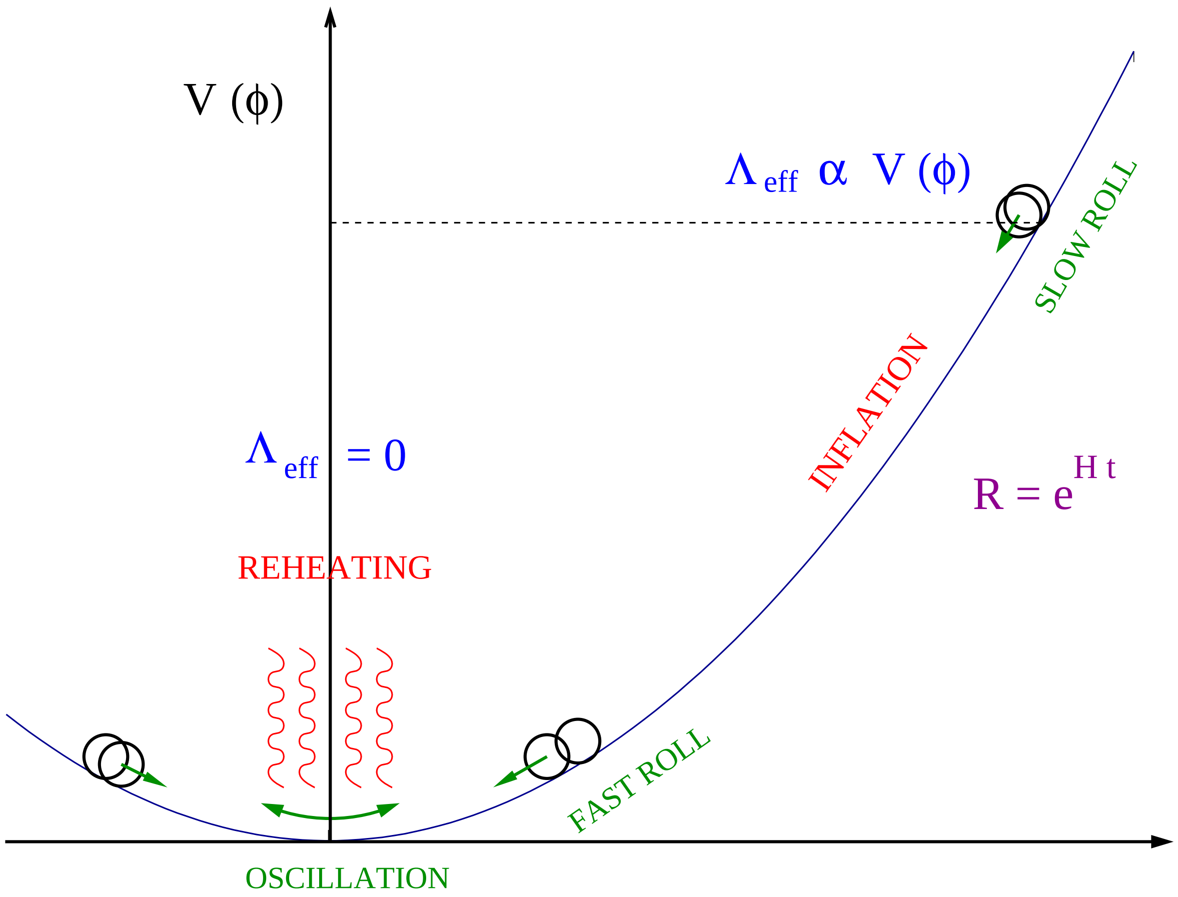
# OUTLINE

1. Introduction:
   – single field inflationary models, the primordial spectrum and its running
2. The running mass model(s):
   – theoretical motivation
   – characteristic predictions
3. Comparison with recent data:
   – CMBR & LSS & Ly-$\alpha$
   – reionization constraints
4. Conclusions

INFLATION: period of quasi-exponential expansion, explaining the FLATNESS, ISOTROPY, HOMOGENEITY of the Universe, the absence of UNWANTED RELICS and producing the initial SMALL PERTURBATIONS.

How to sustain inflation ???

$\Rightarrow$ USE THE POTENTIAL ENERGY OF A SCALAR FIELD $\phi$ AS AN EFFECTIVE COSMOLOGICAL CONSTANT




$\Rightarrow$ The scalar field has to slow roll in an ALMOST FLAT POTENTIAL such that

$$\ddot{\phi} \ll 3H\dot{\phi} \;\Rightarrow\; 3H\dot{\phi} = -V'$$

$\Rightarrow$ slow roll expansion

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by
$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\bigg|_{k=aH} \qquad \propto k^{n-1}$$

and its spectral index is:
$$n(k) - 1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\bigg|_{k=aH} = 2\eta - 6\epsilon + \ldots$$

$n'$ arises only at $2^{nd}$ order in SLOW ROLL:
$$n'(k) = \frac{2}{3}\left((n-1)^2 - 4\eta^2\right) + 2\xi$$

where the slow roll parameters are
$$\epsilon = \frac{M_P^2}{16\pi}\frac{(V')^2}{V^2} \qquad \eta = \frac{M_P^2}{8\pi}\frac{V''}{V} \qquad \xi = \frac{M_P^4}{64\pi^2}\frac{V'V'''}{V^2}$$

so we expect $n' \propto (n-1)^2 < |n-1|$!!!

Surprisingly WMAP seemed to require a large running...; possible for large $\xi$, but are there “natural” models giving it ? $\Rightarrow$ RUNNING MASS model !

Another motivation for the running mass...: SUSY broken in inflation $\Longrightarrow$ SUGRA !

A model is defined by superpotential $W(\Phi)$ & Kähler potential $K(\Phi, \bar{\Phi})$

$$\mathcal{L} = K_{n^* m} \partial_\mu \bar{\Phi}^n \partial^\mu \Phi^m - V(\Phi, \bar{\Phi})$$

$$V(\Phi, \bar{\Phi}) = e^{K(\Phi, \bar{\Phi})} \left( \mathcal{F}_m K^{mn^*} \mathcal{F}_n^* - 3|W|^2 \right) + V_D$$

where $\mathcal{F}_m = \frac{\partial W}{\partial \Phi_m} + \frac{\partial K}{\partial \Phi_m} W(\phi)$, $\qquad K_{n^* m} = \frac{\partial^2 K}{\partial \Phi_m \partial \bar{\Phi}_{n^*}}$ $\qquad K^{mn^*} = \left(K^{-1}\right)^{mn^*}$

Take a canonical Kähler $K = \Phi_n \bar{\Phi}_n$ and we have

$$V = e^{|\Phi_n|^2} \left( \left| \frac{\partial W}{\partial \Phi_n} + \bar{\Phi}_n W \right|^2 - 3|W|^2 \right) + V_D$$

so that from the exponential one obtains

$$V'' = V + .... \quad \to \eta \simeq 1$$

$\eta$ problem

NO SLOW ROLL POSSIBLE IN SUGRA ?!

There are a couple of ways out... ... one of them: the running mass !

[Stewart '96, '97]

# The running mass model:

$\phi \longrightarrow$ flat direction of the SUSY potential $\quad V'_{SUSY}(\phi) = 0$

SUSY breaking generates a soft mass for $\phi$:

$$V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 + \ldots \quad \text{for } \phi < M_P.$$

At tree level, for a generic scalar field one has naturally $|m^2| \simeq V_0/M_P^2$ $\quad \eta$ problem !

$\rightarrow V(\phi)$ is NOT flat at high scale

But if the inflaton field interacts not so weakly, the one loop corrections to the potential give

$$m^2 \rightarrow m^2(Q = \phi) \quad \text{running mass}$$

The running of the mass can flatten the potential somewhere in the region $\phi < M_P$.

$\Downarrow$

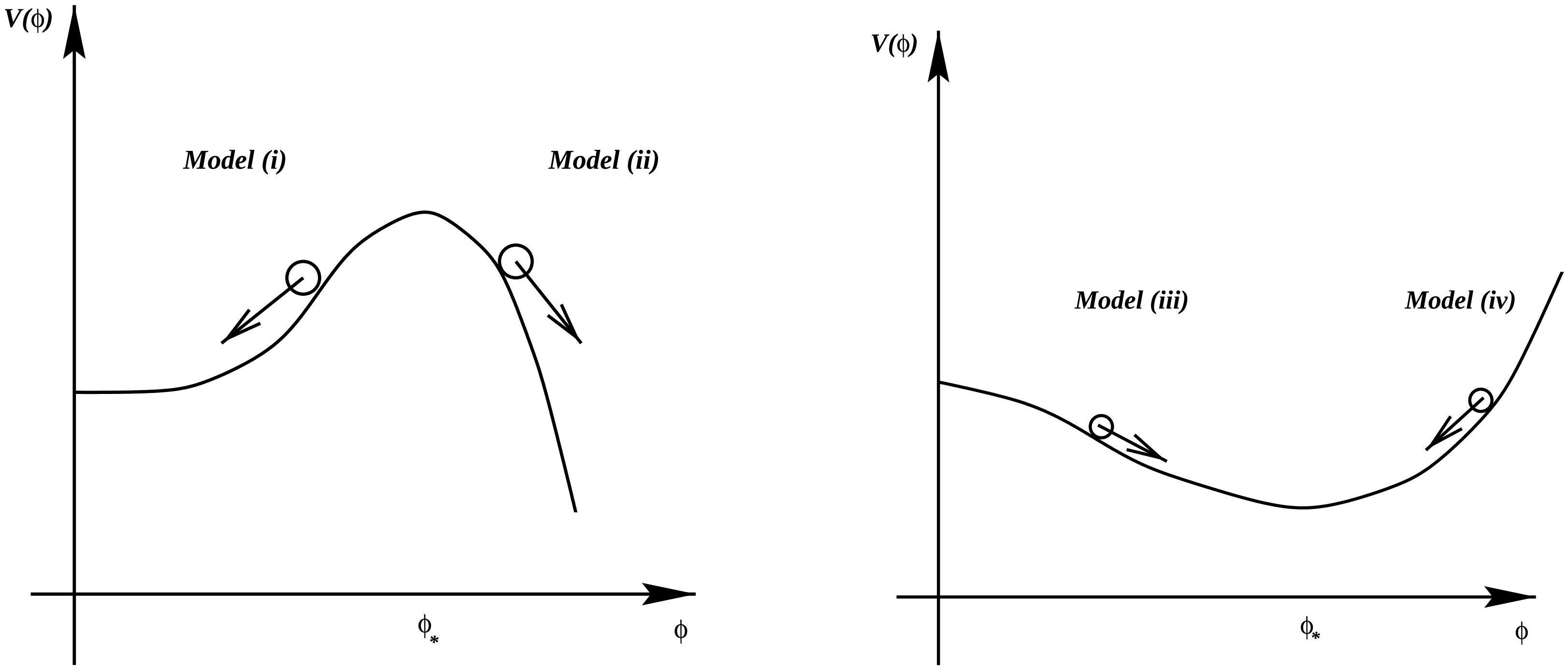
Slow roll inflation

In general any type of coupling can be responsible for the inflaton's mass running, we have in fact

$$\frac{dm^2}{d\log(Q)} = -\frac{2C}{\pi}\alpha\tilde{m}^2 + \frac{D}{16\pi^2}|\lambda|^2 m_s^2$$

gauge Yukawa

For the mechanism to work, the inflaton has to couple sufficiently strongly, but still in the perturbative regime... Different realizations exist depending on the sign of the running and the initial conditions:




What are the observable consequences of non-weakly coupled inflaton ???

A "strongly" scale-dependent $n(k)$ and $\mathcal{P}_{\mathcal{R}}(k)$ !

$n(k) - 1 \ll 1$ on cosmological scales $\Rightarrow$ linear expansion around pivot $\phi_0$ $(\leftrightarrow k_0)$

So take the running mass as $m^2(\phi) \simeq m^2(\phi_0) + c * \log\left(\frac{\phi}{\phi_0}\right)$ where $c \propto \frac{dm^2}{d\log(Q)}(\phi_0)$

Then defining $\phi_*$ by $V'_{lin}(\phi_*) = 0$ and introducing the parameter $s = c\log(\phi_*/\phi_0)$, we have

$$\frac{n(k)-1}{2} = s\left(\frac{k}{k_0}\right)^c - c \qquad \text{and} \qquad n'(k) = 2sc\left(\frac{k}{k_0}\right)^c \to \xi!$$

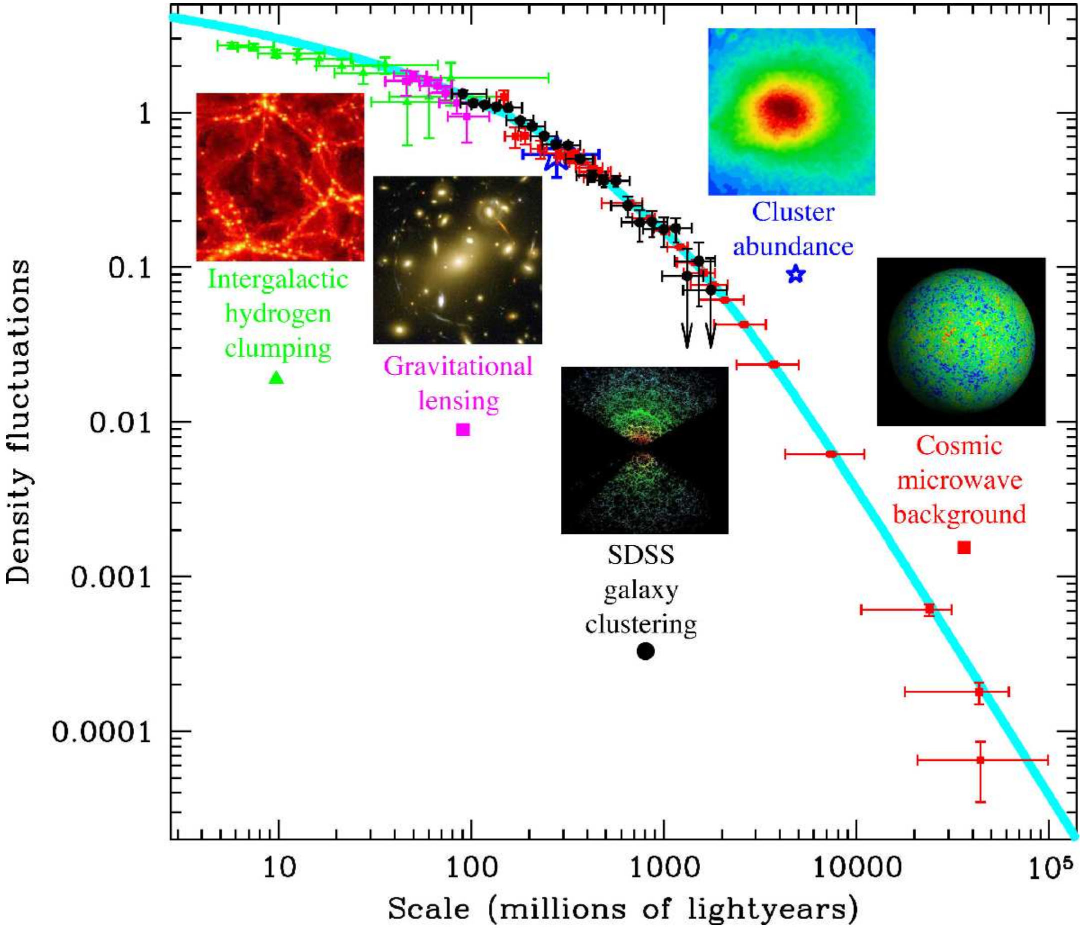
"Strong (exponential !)" scale dependence !!

NOTE: $s$, $c$ are related to physical parameters rescaled by the inflationary Hubble scale $H_I^2$:

$$c \equiv -\frac{\beta_m(\phi_0)}{3H_I^2} \qquad\qquad s + \frac{1}{2}c \equiv \frac{m^2(\phi_0)}{3H_I^2}$$

$c$ suppressed by a coupling, $s$ also to have slow roll...

Look for such strong scale dependence in the data, trying to extend the lever arm as far as possible:




[Figure by M. Tegmark]

- CMB: first year WMAP data

  astro-ph/0302207

- LSS: Sloan Digital Sky Survey results for the galaxy power spectrum

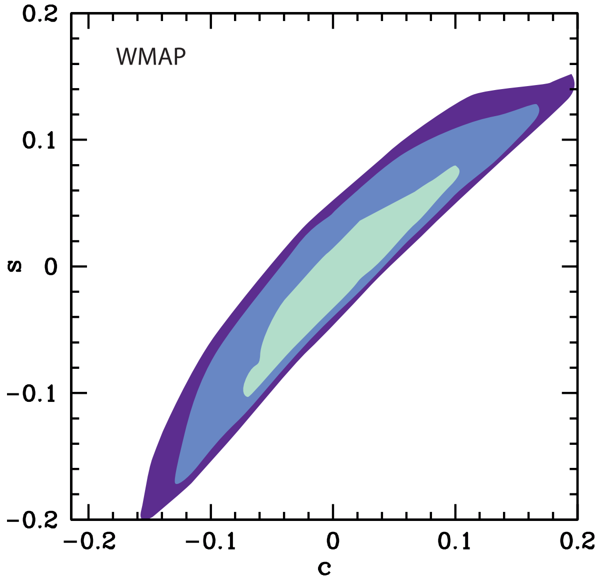
  astro-ph/0310723

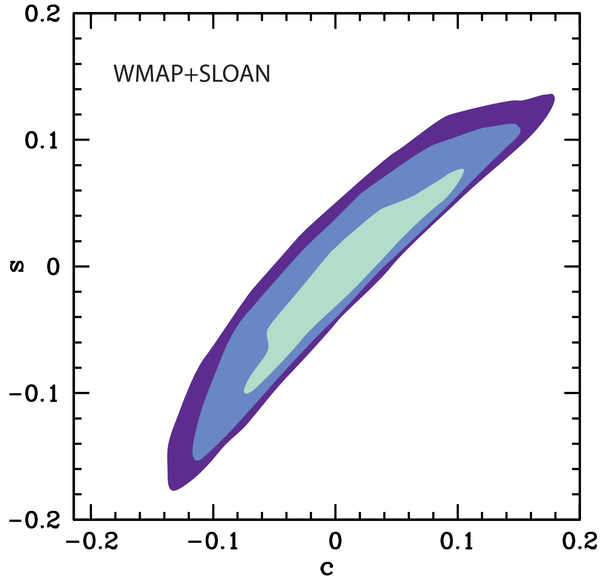
- LSS: Sloan Digital Sky Survey results on Lyman-$\alpha$

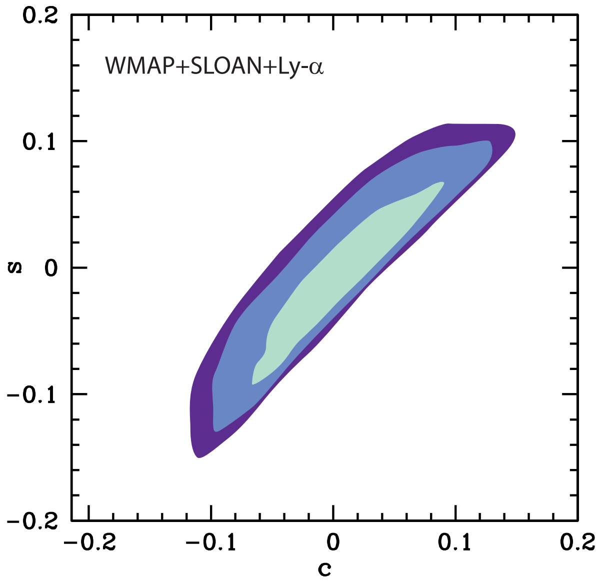
  astro-ph/0405013 & 0407372

What are the constraint from the new data for $s, c$ in such models ?

[LC, Lyth, Melchiorri & Odman astro-ph/0408129]

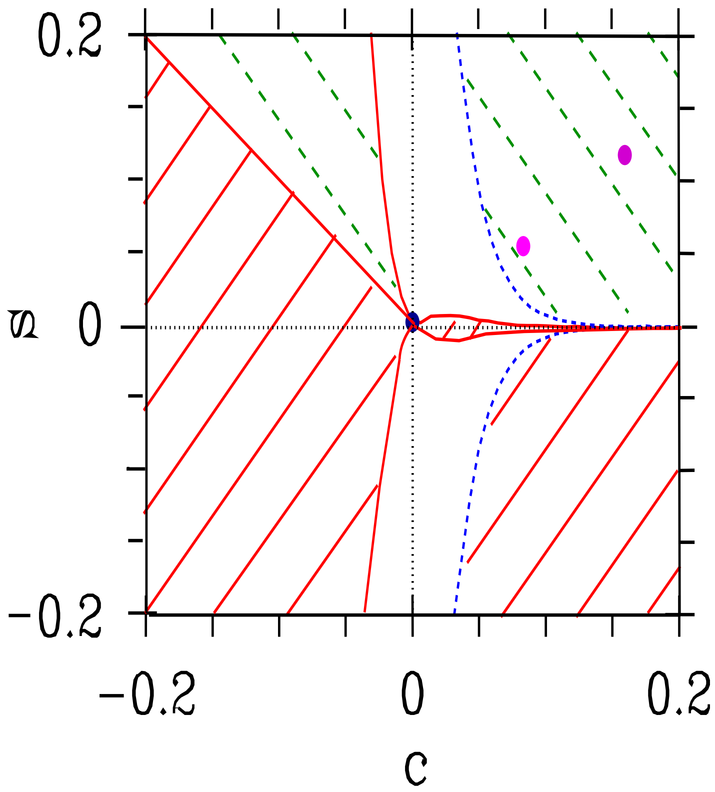



WMAP strongly constrains along the direction $s = c$, i.e. $n(k_0) - 1 = 0$

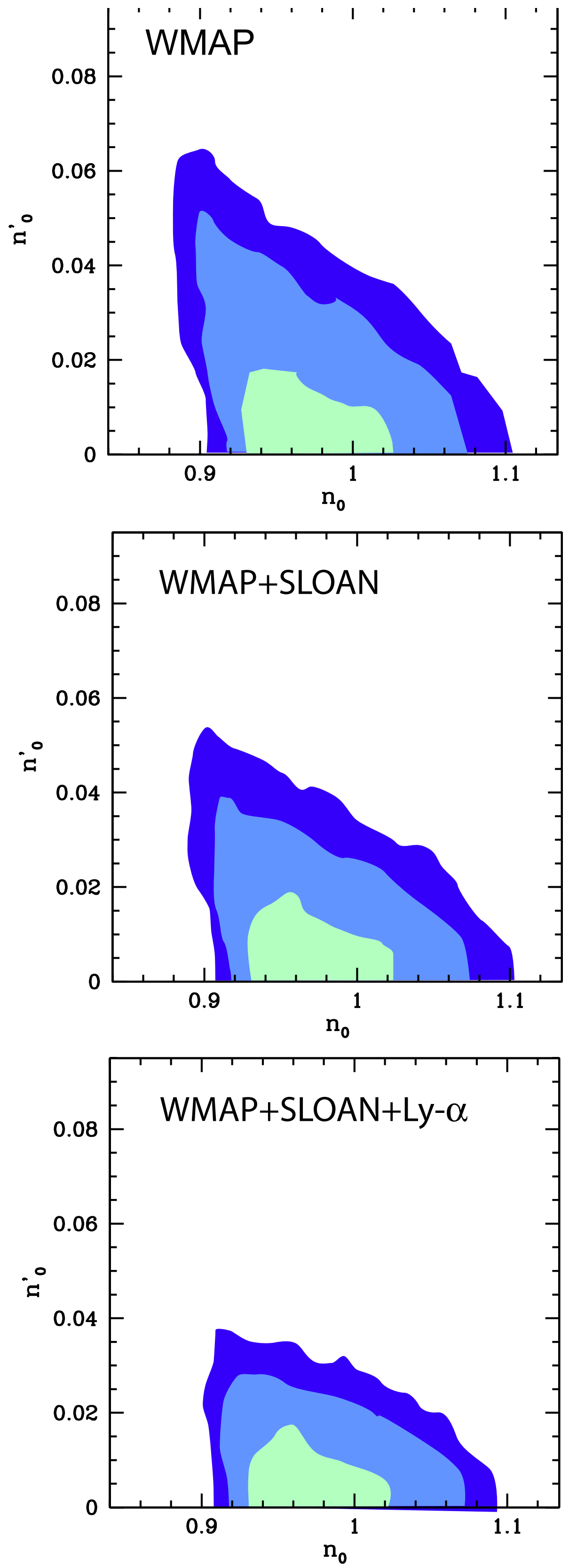
Theoretically expected region




Ly–$\alpha$ data tighten the bound on scale dependence and require

$$|c| \leq 0.12$$

Look at the constraints in the $n_0'$ vs $n_0$ plane instead




NOTE: negative $n_0'$ is allowed by the model only for

$$n_0' \leq -\frac{(n_0 - 1)^2}{4}$$

due to the dependence on $s, c$. The rest of the parameter space is unphysical !

Fitting for arbitrary $n_0, n_0'$ is not equivalent as fitting for the running mass model !

Again the most stringent bound on $n_0'$ comes from Ly–$\alpha$ data giving

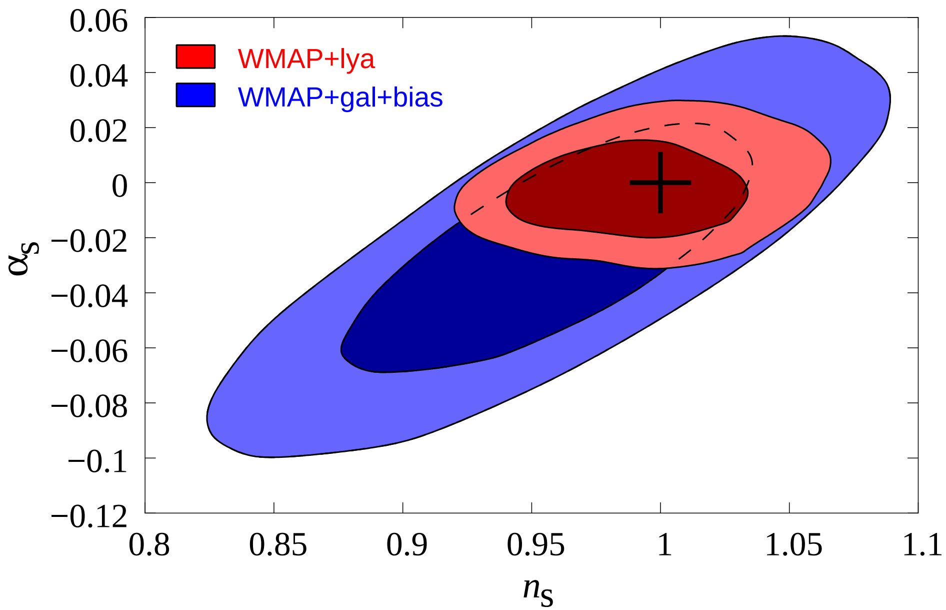
$$n_0' \leq 0.2$$

Compare the result with the fit for a general Taylor expansion: $n(k) = n_s + \alpha_s \log\left(\frac{k}{k_0}\right)$.

Using the same data Seljak et al. (astro-ph/0407372) find, contrary to WMAP,

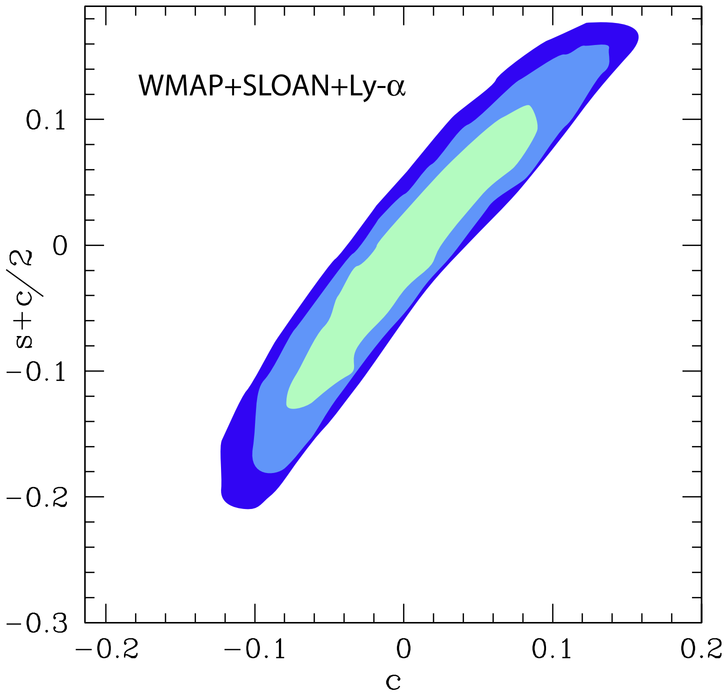
$$n_s = n_0 = 0.977^{+0.025}_{-0.021}$$
$$\alpha_s = n'_0 = -0.003 \pm 0.010$$

NO RUNNING !




That is fully compatible with our result; note that the data cannot yet distinguish between the different parameterizations ! In fact $\alpha_s \leq 0.2$...

What are the bounds on the "physical parameters" ?




$$\frac{m^2(\phi_0)}{3H_I^2}$$

$$c \propto \frac{m'^2(\phi_0)}{3H_I^2}$$

Strong running and therefore large inflationary scale is disfavoured...

From the WMAP normalization

$$H_I = 2\pi \mathcal{P}_{\mathcal{R}}^{1/2} |\phi_0||s| \sim 3 \times 10^{-4} |\phi_0||s|$$

and assuming linear running from $M_P$

$$m^2(\phi_0) \simeq 0 \rightarrow \frac{\phi_0}{M_P} \sim \exp\left(-\frac{1}{|c|} \frac{|m^2(M_P)|}{3H_I^2}\right)$$

Small $|c|$ implies $\phi_0 \ll M_P$ and therefore also $H_I \ll \phi_0 \ll M_P$ ...

... $H_I$ highly sensitive to $c$ !

Connect to simple examples:

$\longrightarrow$ gauge coupling $\alpha$ dominance for $\phi$ in the adjoint representation of $SU(N)$

$$c = \frac{2N\alpha(M_P)}{\pi} \, \frac{\tilde{m}^2(M_P)}{3H_I^2} \, \frac{\alpha^3(\phi_0)}{\alpha^3(M_P)} \qquad \tilde{m} \quad \text{gaugino mass}$$

$$s = -\frac{c}{2} + \frac{m^2(M_P) - 2\tilde{m}^2(M_P)}{H_I^2} + \frac{2\tilde{m}^2(M_P)}{3H_I^2} \, \frac{\alpha^2(\phi_0)}{\alpha^2(M_P)} \qquad m \quad \text{inflaton mass}$$

$\longrightarrow$ Yukawa coupling $\lambda$ dominance

$$c = -\frac{\lambda^2(M_P)}{12\pi^2} \left[ \frac{1}{1 - \frac{3}{8\pi^2} \lambda^2(M_P) \log\left(\frac{\phi_0}{M_P}\right)} \right]^2 \qquad \text{for} \quad m_{scalars} \simeq H_I$$

$$s = -\frac{c}{2} + \frac{2}{3} \left[ \frac{1}{1 - \frac{3}{8\pi^2} \lambda^2(M_P) \log\left(\frac{\phi_0}{M_P}\right)} - \frac{1}{2} \right]$$

## Another hint for a running index: REIONIZATION..... ???

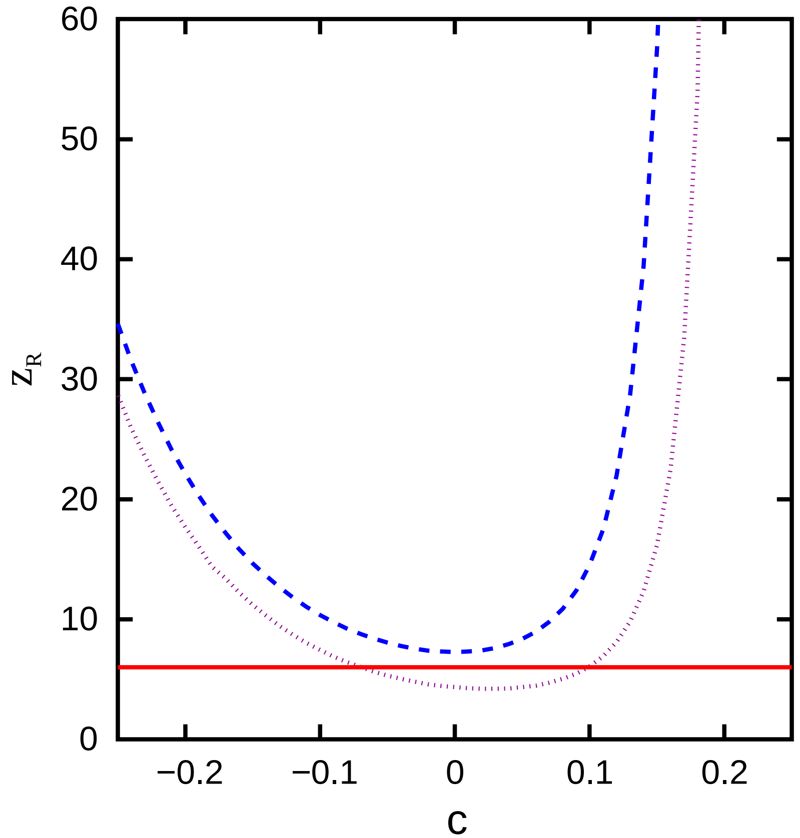
Estimate the reionization epoch $z_R$ using the Press-Schechter formula as the epoch of collapse of a fraction $f$ of matter into objects of mass $10^6 M_\odot$:

$$1 + z_R \simeq \frac{\sqrt{2}\sigma(10^6 M_\odot)}{1.7 g(\Omega_M)} \, \mathrm{erfc}^{-1}(f)$$

where $\sigma$ is the present linear rms density contrast computed from the primordial spectrum and the CDM transfer function and $g(\Omega_M)$ accounts for the suppression of the growth when $\Omega_M < 1$.

There is a strong correlation between $c$ and $z_R$:
$z_R$ grows cery quickly for large $c$

$s$ has been fixed to $c$ and $c - 0.05$.
The red line corresponds to $z_R = 6$.



# Conclusions

- The simple (single field) inflationary paradigm is very successful in describing present observations. Unfortunately it is still not clear which model of the many proposed is favoured...
- The running mass model is very well motivated from the particle physics point of view and has a characteristic observational signature

  $\longrightarrow$ scale-dependence of the spectral index !
- Present data allow still a relatively strong scale-dependence and cannot yet exclude this type of models.
- MORE DATA are expected soon (WMAP...?!?) and then the scale-dependence will be better constrained or detected.