STRING MODULI FIELDS AND INFLATION

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The case of Inflation

Inflation: A period of accelerated expansion in the Early Universe

Friedmann equation : $H^2 \equiv (\dot{a}/a)^2 = \frac{1}{3}\rho/M_P^2$

$$\rho \simeq M_P^2 \Lambda_{\text{eff}} \simeq \text{const.} \Rightarrow \underline{\underline{a \propto e^{Ht}}}$$

Inflation = the only compelling solution to:
Horizon & Flatness problems of Hot Big Bang

- Horizon: superluminal expansion rate $\Rightarrow$ superhorizon causal correlations
- Flatness: Curvature = inflated away

Precision cosmology $\Rightarrow$ Inflation $\surd$ (e.g. $\Omega, n_s \approx 1$)

Inflation is typically achieved through the domination of the Universe by the potential density of a scalar field ϕ (inflaton field)

Who is the inflaton?

V-domination: $V(\phi) \gg \rho_{\text{kin}} \equiv \frac{1}{2}\dot{\phi}^2$

$$\Rightarrow V_{\text{inf}} \simeq \text{const.} \Rightarrow \boxed{\Lambda_{\text{eff}} \equiv V_{\text{inf}}/M_P^2}$$

$$\rho_{\text{kin}} \ll V \Rightarrow \text{Slow Roll} \Leftrightarrow \boxed{H_{\text{inf}} \gg m_{\text{eff}}} \equiv \sqrt{|V''|}$$

Slow Roll Inflation lasts long enough to solve the Horizon & Flatness problems

Flat Direction required for Slow Roll Inflation

The problem with Flat Directions

Global SUSY

$\exists$ flat directions in Global SUSY due to the non-renormalisation theorem:

Superpotential W: non-renormalised to all orders of perturbation theory

Possible to tune parameters without danger from non-renormalisable terms

Supergravity

$$\boxed{\begin{aligned} V_F &\simeq e^{K/M_P^2}\,|W|^2 \times \\ &\times \left[\sum_{nm}\left(\frac{\partial_n K}{M_P^2}+\frac{\partial_n W}{W}\right)\partial^n\partial^{\bar m}K\left(\frac{\partial_m K}{M_P^2}+\frac{\partial_m W}{W}\right)^* - 3M_P^{-2}\right]\end{aligned}}$$

K = Kähler potential parameterises the geometry of field space

$$\boxed{\mathcal{L}_{\rm kin} = \sum_{nm}(\partial_n\partial_m K)\;\partial_\mu\phi_n\;\partial^\mu\bar\phi_m}$$

$\Rightarrow$ Canonical: $\partial_n\partial_m K = \delta_{nm}$

Supergravity corrections lift flatness of V

$$\left.\begin{array}{c} V_F'' = K''(V_F/M_P^2)+\cdots \\ K:\ \text{Canonical}\end{array}\right\} \Rightarrow \underline{\underline{m_{\rm eff}\sim H_{\rm inf}}}$$

Kähler moduli: $K \neq K(\phi) \Rightarrow m_{\rm eff}$ under control

Compelling candidates: String Axions

String Axions

$K = -\Sigma_i \ln[(\Phi_i + \bar{\Phi}_i)/M_P]$ (Tree level)

Hidden sector gaugino condensates

$\Rightarrow \quad W \simeq \Lambda^3 \exp[-\Sigma_i \beta_i \Phi_i / M_P]$

$\Rightarrow V_F =$ independent of phase of W

$\boxed{V_F = \text{Flat w.r.t. } \mathrm{Im}(\Phi_i)}$: String Axions

Obtained by duality transformations
from the Antisymmetric Tensor B

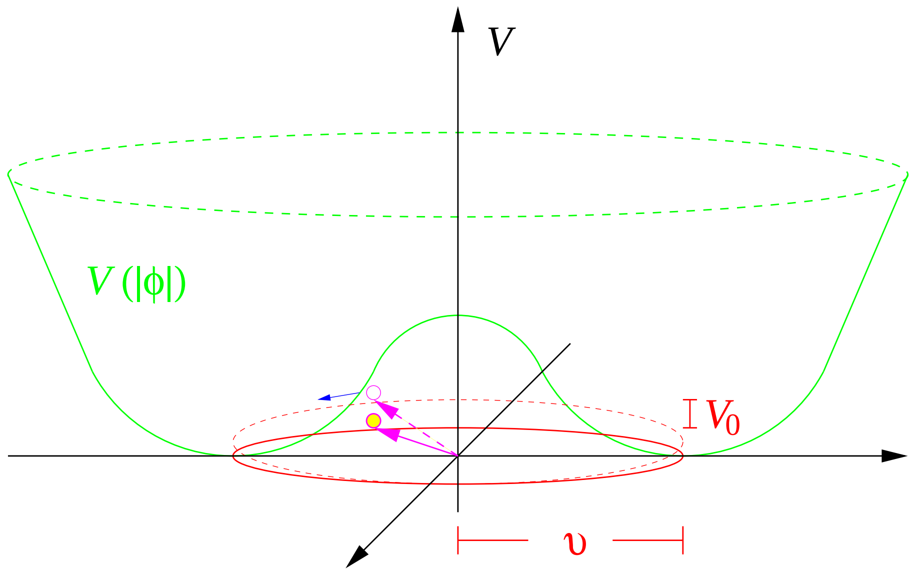
Modular Invariance : $\mathrm{Im}(\Phi_i) = \mathrm{Im}(\Phi_i) + 2\pi f_P \quad (f_P \sim M_P)$

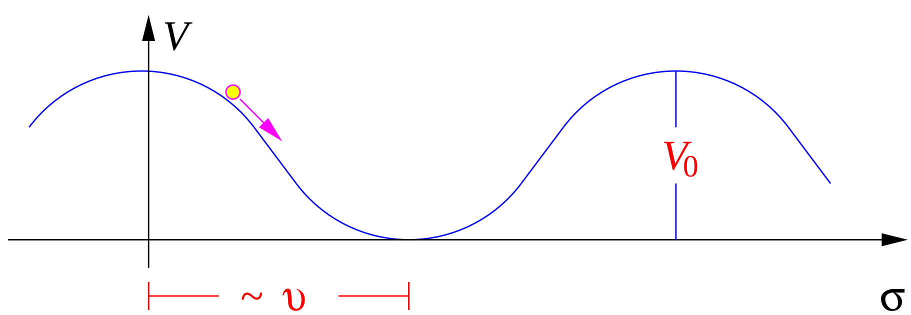
Effective mass from supersymmetry breaking

PNGB potential : $V(\phi) = (m_\phi f_P)^2[1 - \cos(\phi/f_P)]$

$$\Rightarrow \quad \boxed{V(\phi \ll M_P) \simeq \frac{1}{2} m_\phi^2 \phi^2}$$

where $\phi \equiv \mathrm{Im}(\Phi_\mathrm{i})$ and $\underline{m_\phi \sim M_S^2/f_P}$
with M_S : SUSY breaking scale


V
V (|φ|)
V0
υ


V
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Modular Hybrid Inflation

$$\boxed{V(\Phi,\phi) = \tfrac{1}{2}m_\Phi^2\Phi^2 + \tfrac{1}{2}\lambda\Phi^2\phi^2 + \tfrac{1}{4}\alpha(\phi^2 - M^2)^2}$$

Φ, ϕ : Kähler moduli (string axions)

$\delta V/V \sim \mathcal{O}(1)$ when $\Delta\Phi, \Delta\phi \sim M_P \Rightarrow$

$$M \sim M_P \qquad \alpha \sim \left(\frac{M_S}{M_P}\right)^4$$

$$m_\Phi \sim \frac{M_S^2}{M_P} \qquad m_\phi \sim \sqrt{\alpha}\, M$$

Gravity mediated SUSY breaking:

$$M_S \sim \sqrt{m_{3/2} M_P} \sim 10^{10.5}\ \text{GeV} \Rightarrow$$

$$\boxed{m_\Phi \sim m_\phi \sim m_{3/2} \sim \text{TeV}}$$

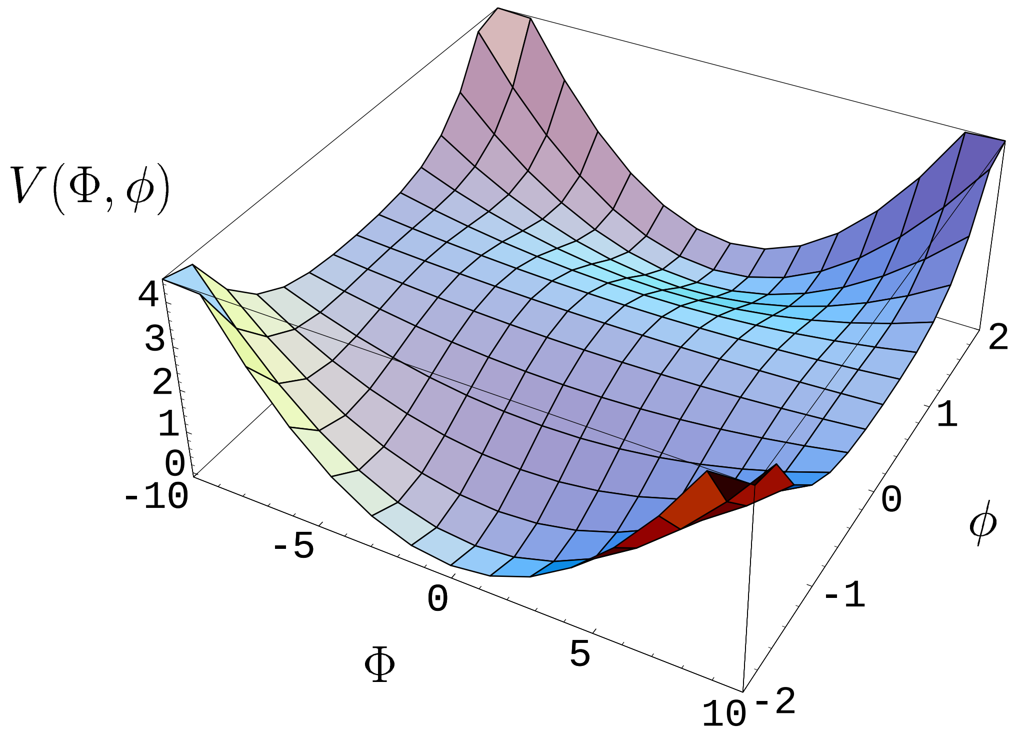
Vacuum density: $V_{\text{inf}} \sim \alpha M^4 \sim M_S^4 \ \Rightarrow\ \underline{\underline{H_{\text{inf}} \sim m_{3/2}}}$

Both flat directions are lifted $\Rightarrow$ No Slow Roll!

Can we have enough inflation to solve
the Horizon and Flatness problems?

Two-stage Inflation: before & after phase transition:

$$(m_\phi^{\text{eff}})^2 = \lambda\Phi^2 - \alpha M^2 \ \Rightarrow\ \boxed{\Phi_c \equiv \sqrt{\frac{\alpha}{\lambda}}\, M \sim \frac{m_{3/2}}{\sqrt{\lambda}}}$$



Saddle point: $|\boldsymbol{\Phi}| \leq \boldsymbol{\Phi}_c$ **and** $|\phi| \leq M$

First stage of inflation

Assume $\phi \to 0 \Rightarrow \boxed{V(\Phi) \simeq M_S^4 + \frac{1}{2}m_\Phi^2\Phi^2} \approx V_{\rm inf}$

Klein-Gordon: $\ddot{\Phi} + 3H_{\rm inf}\dot{\Phi} + m_\Phi^2\Phi = 0 \;\;\Rightarrow$

$$\Phi \propto e^{\omega t} \;\; \text{with} \;\; \omega = -\frac{3}{2}H_{\rm inf}\left[1 \pm \sqrt{1 - \frac{4}{9}\left(\frac{m_\Phi}{H_{\rm inf}}\right)^2}\,\right]$$

A: Fast-Roll Inflation $\;(m_\Phi \leq \frac{3}{2}H_{\rm inf})\;$ [Linde, 2001]

$$\Phi = \Phi_0 \exp(-F_\Phi \Delta N) \;\; \text{with} \;\; F_\Phi \equiv \frac{3}{2}\left[1 - \sqrt{1 - \frac{4}{9}\left(\frac{m_\Phi}{H_{\rm inf}}\right)^2}\,\right]$$

B: Locked Inflation $\;(m_\Phi > \frac{3}{2}H_{\rm inf})\;$ [Dvali & Kachru, 2003]

$$\Phi = \Phi_0 \exp\left(-\frac{3}{2}\Delta N\right)\,\cos(\omega_\Phi \Delta t) \;\; \text{with}$$

$$\omega_\Phi = H_{\rm inf}\sqrt{\left(\frac{m_\Phi}{H_{\rm inf}}\right)^2 - \frac{9}{4}} \approx m_\Phi$$

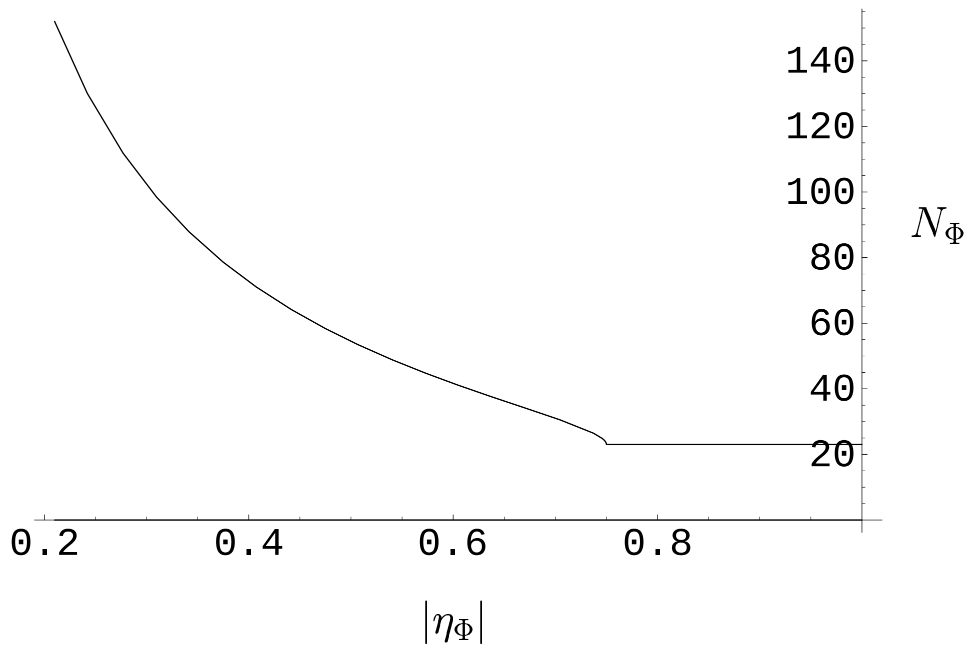
If $\omega_\Phi =$ large enough $\Rightarrow$ Φ spends too little time on saddle point $\Rightarrow$ cannot escape oscillatory trajectory

$$\omega_\Phi \Delta t \sim \frac{\Delta\Phi}{\bar{\Phi}} \;\Rightarrow\; (\Delta t)_{\rm saddle} \sim \frac{\Phi_c}{m_\Phi \bar{\Phi}} \sim \frac{1}{\sqrt{\lambda}}\frac{1}{\bar{\Phi}}$$

$$\bar{\Phi} = \Phi_0 e^{-\frac{3}{2}\Delta N} = \text{amplitude} \Rightarrow \boxed{\bar{\Phi}_{\rm end} \sim \Phi_c}$$

$$\boxed{N_\Phi = \frac{1}{\bar{F}_\Phi}\ln\left(\frac{\Phi_0}{\Phi_c}\right) \simeq \frac{1}{\bar{F}_\Phi}\ln\left(\frac{M_P}{m_{3/2}}\right) + \frac{1}{\bar{F}_\Phi}\ln\sqrt{\lambda}}$$

where $\bar{F}_\Phi \equiv \min\left\{\frac{3}{2},\, F_\Phi\right\}$



where $|\eta_\Phi| \simeq \frac{1}{3}\left(\frac{m_\Phi}{H_{\rm inf}}\right)^2$ **and** $\lambda \sim 1$

There is a minimum number of inflationary e-foldings ($N_\Phi \leq 24$) while ϕ remains at (or near) the origin

Second stage of inflation

$$\Phi < \Phi_c \Rightarrow \boxed{V(\phi) \simeq V_{\rm inf} - \tfrac{1}{2}m_\phi^2\phi^2 + \tfrac{1}{4}\alpha\phi^4} \approx V_{\rm inf}$$

Klein-Gordon: $\ddot{\phi} + 3H_{\rm inf}\dot{\phi} - m_\phi^2\phi \simeq 0 \;\Rightarrow$

$$\phi \propto \exp(\omega_\phi \Delta t) \;\text{ with }\; \omega_\phi = -\frac{3}{2}\,H_{\rm inf}\left[1 \pm \sqrt{1 + \frac{4}{9}\left(\frac{m_\phi}{H_{\rm inf}}\right)^2}\,\right]$$

(+)solution disappears, (−)solution = growing mode:

$$\phi = \phi_0 \exp(F_\phi \Delta N) \;\text{ with }\; F_\phi \equiv \frac{3}{2}\left[\sqrt{1 + \frac{4}{9}\left(\frac{m_\phi}{H_{\rm inf}}\right)^2} - 1\right]$$

$$\boxed{N_\phi = \frac{1}{F_\phi}\ln\left(\frac{\phi_{\rm end}}{\phi_0}\right) \simeq \frac{1}{F_\phi}\ln\left(\frac{M}{m_\phi}\right) \simeq \frac{1}{F_\phi}\ln\left(\frac{M_P}{m_{3/2}}\right)} \;\text{ with }\; \phi_0 \simeq \frac{m_\phi}{2\pi}$$

Total number of inflationary e-foldings:

$$\boxed{N_{\rm tot} = N_\Phi + N_\phi \simeq \left(\frac{1}{\bar{F}_\Phi} + \frac{1}{F_\phi}\right)\ln\left(\frac{M_P}{m_{3/2}}\right) + \frac{1}{\bar{F}_\Phi}\ln\sqrt{\lambda}}$$

The necessary e-foldings

$$\boxed{N_{\rm cosmo} \simeq 72 - \ln\left(\frac{M_P}{V_{\rm inf}^{1/4}}\right) - \frac{1}{3}\ln\left(\frac{V_{\rm inf}^{1/4}}{T_{\rm reh}}\right)}$$

$$T_{\rm reh} \sim \sqrt{\Gamma M_P} \;\; \& \;\; \Gamma \simeq g^2 m_\phi \;\; \text{with} \;\; \frac{m_{3/2}}{m_P} \leq g \leq 1$$

Horizon & Flatness problems: $N_{\rm tot} > N_{\rm cosmo} \Rightarrow$

$$\boxed{\frac{m_\phi}{H_{\rm inf}} < \frac{3}{2}\left\{\left[\left(\frac{108+\ln\sqrt{g}-\frac{3}{2\bar{F}_\Phi}\ln\sqrt{\lambda}}{\ln(M_P/m_{3/2})} - \frac{3}{4}\left(1+\frac{2}{\bar{F}_\Phi}\right)\right)^{-1} + 1\right]^2 - 1\right\}^{1/2}}$$

Tightest bound corresponds to : $\bar{F}_\Phi = 3/2 \Rightarrow$

$$\boxed{\frac{m_\phi}{H_{\rm inf}} < \frac{3}{2}\left\{\left[\left(\frac{108+\ln\sqrt{g/\lambda}}{\ln(M_P/m_{3/2})} - \frac{7}{4}\right)^{-1} + 1\right]^2 - 1\right\}^{1/2}}$$

For $\lambda \sim 1$ the bound interpolates between 2 and 3

It seems that enough inflation can be achieved with
$m_\Phi \sim m_\phi \sim H_{\rm inf}$

BUT: Grave danger from Primordial Black Holes

Primordial Black Hole production

Outburst of tachyonic fluctuations at phase transition
$\Rightarrow$ mountain of density perturbations $\delta\rho/\rho \sim 1$
$\Rightarrow$ copious PBH formation at horizon re-entry

PBH Mass = mass of horizon volume at re-entry

$$M_{\rm PBH} \sim \left.\frac{\rho}{H^3}\right|_{\rm PBH} \sim \frac{M_P^2}{H_{\rm PBH}}$$

$\Rightarrow$ PBHs dominate the Universe $\Rightarrow$
PBHs must evaporate before BBN

$$\Delta t_{\rm ev} \sim \frac{M_{\rm PBH}^3}{M_P^4} \Rightarrow M_{\rm PBH}^3 < \frac{M_P^5}{T_{\rm BBN}^2} \Rightarrow M_{\rm PBH} < 10^8 \text{ g}$$

corresponding bound : $\boxed{\left(\frac{M_P}{m_{3/2}}\right)^{\frac{9}{2F_\phi}+1} < \left(\frac{m_{3/2}}{T_{\rm BBN}}\right)^2}$

$\boxed{\text{Bound = impossible to satisfy for } F_\phi > 0}$

Is catastrophic PBH formation inevitable?

Solution to PBH problem

The only option is to avoid producing PBH at all

$\boxed{\phi \text{ must be significantly displaced from the origin at the phase transition: } \phi_0 \gg m_\phi}$

$\Rightarrow$ System out of diffusion zone (classical trajectory)

$\boxed{\phi_0 \gg m_\phi \text{ : is possible because of the limited number of e-foldings of the first stage of inflation}}$

$$\phi\text{–evolution:} \quad \boxed{\phi_0 \sim \phi_{\rm in}\, \exp\left(-\tfrac{3}{4} N_\Phi\right)}$$

$$\lambda\, \phi_{\rm in}^2 \Phi_0^2 \lesssim V_{\rm inf} \;\Rightarrow\; \boxed{\phi_{\rm in} \sim \frac{m_{3/2}}{\sqrt{\lambda}}} \;\Rightarrow$$

$$\ln\left(\frac{\phi_0}{m_\phi}\right) \simeq -\frac{3}{4} N_\Phi - \ln\sqrt{\lambda}$$

$$\simeq -\left(1 + \frac{3}{4\bar{F}_\Phi}\right)\ln\sqrt{\lambda} - \frac{3}{4\bar{F}_\Phi}\ln\left(\frac{M_P}{m_{3/2}}\right)$$

$$\phi_0 \gg m_\phi \;\Rightarrow\; \ln\sqrt{\lambda} < -\frac{\ln(M_P/m_{3/2})}{1 + \frac{4}{3}\bar{F}_\Phi} \leq -\frac{1}{3}\ln\left(\frac{M_P}{m_{3/2}}\right) \;\Rightarrow$$

$$\Rightarrow \boxed{\frac{m_\Phi}{H_{\rm inf}} > \frac{3}{2}\left[1 - \frac{1}{4}\left(3 + \frac{\ln(M_P/m_{3/2})}{\ln\sqrt{\lambda}}\right)^2\right]^{1/2}} \;\&\; \boxed{\lambda < 10^{-10}}$$

Smaller $m_\Phi \Rightarrow N_\Phi =$ large enough for $\phi_0 < m_\phi$

Cosmological bound: $N_{\rm tot} > N_{\rm cosmo}$ gives:

$$\boxed{\begin{aligned}\frac{m_\phi}{H_{\rm inf}} < \frac{3}{2}\Bigg\{\Bigg[&\left(\frac{108 + \ln\sqrt{g}}{\ln(M_P/m_{3/2})} - \frac{3}{4} - \frac{3}{2\bar{F}_\Phi}\left(1 + \frac{\ln\sqrt{\lambda}}{\ln(M_P/m_{3/2})}\right)\right)^{-1} \times \\ &\times \left(1 + \frac{3}{4\bar{F}_\Phi}\right)\left(1 + \frac{\ln\sqrt{\lambda}}{\ln(M_P/m_{3/2})}\right) + 1\Bigg]^2 - 1\Bigg\}^{1/2}\end{aligned}}$$

Example case $\lambda \sim \sqrt{\alpha} \sim 10^{-15}$

$$\boxed{\frac{m_\phi}{H_{\rm inf}} < \frac{3}{2}\left\{\left[\left(\frac{108+\ln\sqrt{g}}{\ln(M_P/m_{3/2})} - \frac{3}{4}\left(1+\frac{1}{\bar{F}_\Phi}\right)\right)^{-1}\frac{1}{2}\left(1+\frac{3}{4\bar{F}_\Phi}\right)+1\right]^2 - 1\right\}^{1/2}}$$

$$\boxed{\frac{m_\Phi}{H_{\rm inf}} > \frac{3\sqrt{3}}{4} \approx 1.3} \quad \& \quad \boxed{\ln\left(\frac{\phi_0}{m_\phi}\right) \simeq \frac{1}{2}\left(1-\frac{3}{4\bar{F}_\Phi}\right)\ln\left(\frac{M_P}{m_{3/2}}\right)}$$

If $m_\Phi > \frac{3}{2}H_{\rm inf} \Rightarrow \phi_0 \sim 10^4\, m_\phi\checkmark$ and also

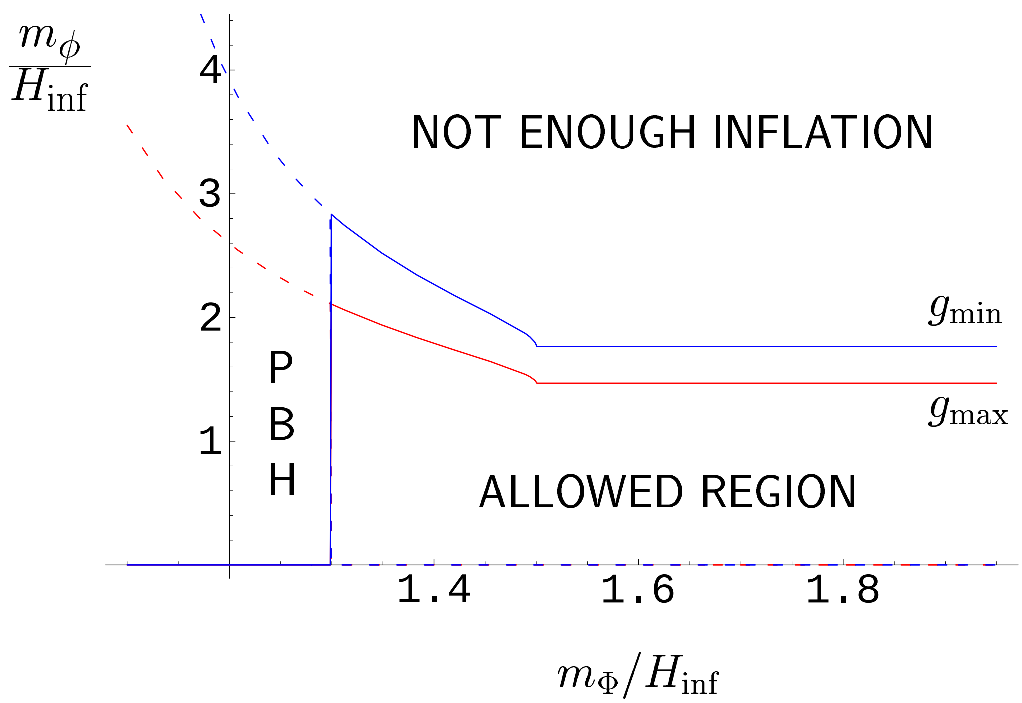
$$\boxed{\frac{m_\phi}{H_{\rm inf}} < \frac{3}{2}\left\{\left[\left(\frac{72+\frac{2}{3}\ln\sqrt{g}}{\ln(M_P/m_{3/2})} - \frac{5}{4}\right)^{-1}\frac{1}{2}+1\right]^2 - 1\right\}^{1/2}}$$

The bound interpolates between 1.5 and 1.8

The combination of fast-roll and locked inflation is capable of providing enough e-foldings of inflation to encompass the cosmological scales without flat directions

i.e. with $\boxed{m_\Phi \sim m_\phi \sim H_{\rm inf}}$

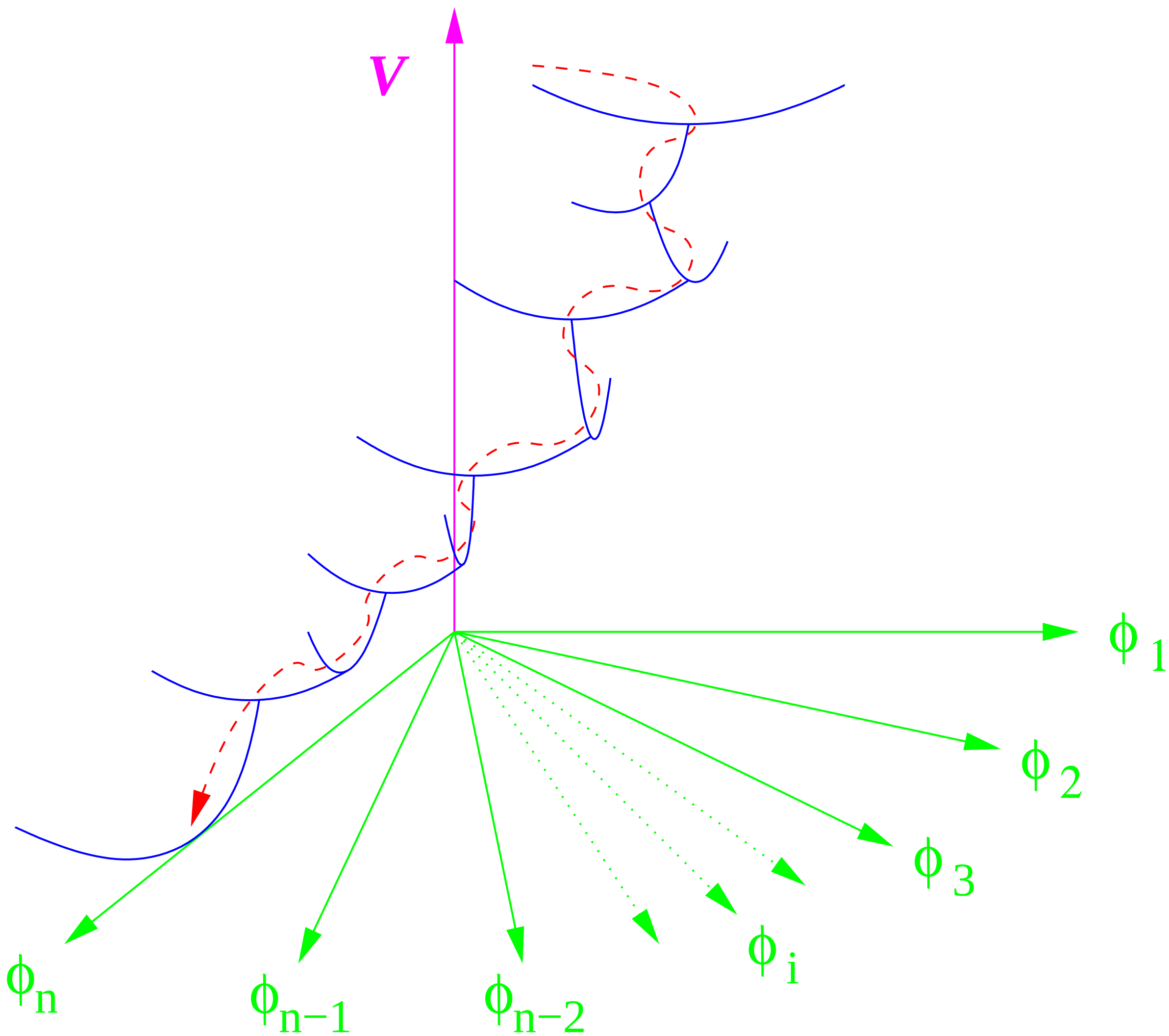
In String Theory the potential Landscape for the moduli may allow a cascade of periods of locked inflation, with additional periods of fast-roll inflation inbetween, when the system is rolling from one saddle point to another



$\frac{m_\phi}{H_{\rm inf}}$
4
3
2
1
NOT ENOUGH INFLATION
$g_{\rm min}$
$g_{\rm max}$
P
B
H
ALLOWED REGION
1.4
1.6
1.8
$m_\Phi/H_{\rm inf}$

Summary

- Inflation is strongly supported by the latest precise cosmological observations
- Typically inflation requires one (or more) flat directions in field space so that the slow roll can last long enough for the solution of the flatness and horizon problems
- However, supergravity corrections typically lift the flatness of the scalar potential
- Using Kähler moduli fields in a hybrid-type inflationary model we have demonstrated that, even with lifted flat directions, enough inflation is quite possible in the context of the string theory landscape, by the combination of periods of fast–roll and locked inflation
- The possible formation of disastrous primordial black holes can be avoided by considering more realistic initial conditions, because each of the inflation stages is brief
- Structure formation in this model is due to some curvaton field



V
ϕ_1
ϕ_2
ϕ_3
ϕ_i
ϕ_n
ϕ_{n-1}
ϕ_{n-2}