A MULTI-PRONGED APPROACH TO THE DARK ENERGY PROBLEM

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The dark energy problem: is dark energy dynamical or due to a cosmological constant?

- Why is the cosmological constant so small? (Anthropic explanation? A dynamical explanation?)
- Dynamical dark energy can solve the cosmic coincidence problem (quintessence trackers, *k*-essence)

Measuring the equation of state

- A detection of $w + 1$ is unambiguous evidence of dynamical dark energy, however
- $w + 1$ is constrained indirectly by supernovae

$$d_L = \frac{1+z}{H_0}(1 + \Omega_m/\Omega_Q)^{1/2} \int_1^{1+z} \frac{dx}{x^{3/2}} \left[\frac{\Omega_m}{\Omega_Q} + \exp\left(3 \int_1^x \frac{dy}{y} w_Q(y) \right) \right]^{-1/2}$$

  and there may be limits to how well it can be measured (Upadhye *et al.* 2004)
- Some models (*e.g.* quintessence trackers) predict w approaching -1 in the late universe

Alternative: look for model-dependent signatures of dark energy

1. Dark energy perturbations
2. In this talk: look for equivalence principle violating interactions
   - The minimally coupled quintessence model of dark energy is an idealization
   - Equivalence principle violations are predicted by compactification, string/M theory moduli etc…

What kinds of effects arise?

Violations of:

- the universality of free fall
- variation of fundamental constants
- deviations from general relativity

Universality of free-fall

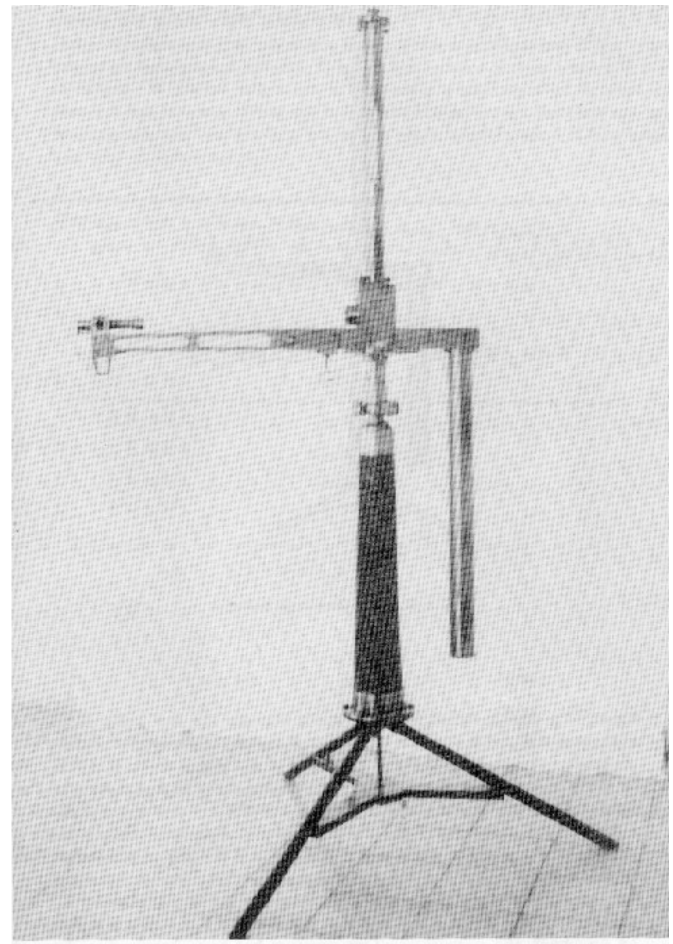
- Test of differential acceleration

$$\eta = 2\frac{|a_1 - a_2|}{|a_1 + a_2|}$$

- Eötvös torsion-balance experiment

$$\eta < 10^{-12}$$

(Braginsky & Panov, `72)



Eötvös's experiment

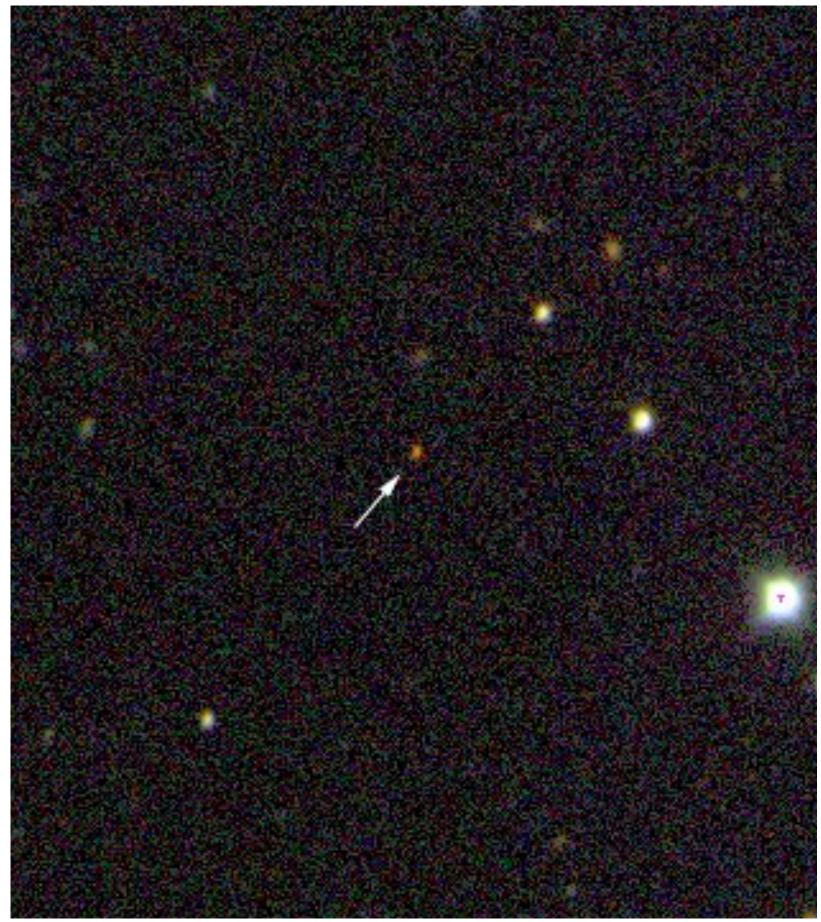
Variation of fundamental constants

- Quasar spectra / Oklo

  $$|H_0^{-1}\,\dot{\alpha}/\alpha| < 8 \times 10^{-7}$$

- Constraints on Newton's constant:

  ΔG no more than 40% since nucleosynthesis



SDSS quasar at z=5

Metric tests of gravity

Cassini

Precision tests of general relativity

- Laser lunar rangefinding
- Deflection of distant radio sources by the sun
- Time-delay experiments

Constrains Brans-Dicke parameter $\omega > 40{,}000$

or PPN parameter $\gamma = \frac{1+\omega}{2+\omega}$

$\gamma - 1 < 5 \times 10^{-5}$

- “Fifth force”

$$S_1 = \int d^4x\sqrt{-g}\{\tfrac{1}{2}R[g] - \tfrac{1}{2}(\partial\psi)^2 - V(\psi) + e^{4\beta\psi}\mathcal{L}_{\mathrm{NG}}[\Psi_i; e^{2\beta\psi}g_{\mu\nu}]\}$$

- Brans-Dicke with a potential

$$S_2 = \int d^4x\sqrt{-h}\{\phi R - \omega\phi^{-1}(\partial\phi)^2 + 2\phi\lambda(\phi) + \mathcal{L}_{\mathrm{NG}}[\Psi_i; h_{\mu\nu}]\}$$

$$\omega = \frac{1-6\beta^2}{4\beta^2}$$

The problem: equivalence principle violations are tightly constrained

- Couplings should generically be gravitational strength (*i.e.* suppressed by the Planck constant).
- The generic prediction is that $\Delta\alpha/\alpha$, $\Delta G/G$, γ, ω, η are of order unity which is in conflict with observation.

Local and cosmological tests of EP

- Local tests
  - universality of free fall
  - solar system tests of gravity
  - gravitational redshift experiments

  These tests *directly* measure couplings

- Cosmological tests
  - variation of fundamental constants

  Cosmological tests measure the *combined* couplings and rate of variation of the field

Dark energy *is* a model for the variation of the scalar field

$$w + 1 = \frac{1}{3}\left(\frac{\partial \phi}{\partial \log a}\right)^2$$

This implies

$$w + 1 = \frac{\omega}{3\Omega_Q}\left(\frac{d\log G}{d\log a}\right)^2 \sim \omega\left(\frac{\Delta G}{G}\right)^2$$

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which constrains the evolution of G: constraints on $w + 1$ and ω impose

$$H_0^{-1}\dot{G}/G < 4 \times 10^{-3}$$

for dark energy models, which is an order of magnitude or so stronger than observational constraints

Likewise,

$$w + 1 = \frac{1}{3\Omega_Q} \frac{C}{\eta} \left(\frac{d \log \alpha}{d \log a} \right)^2 \sim \frac{10^{-3}}{\eta} \left(\frac{\Delta\alpha}{\alpha} \right)^2$$

holds generally for dark energy models with a single light field. No new constraint is imposed on the variation of the fine-structure constant (since it is already so small).

$$w+1=\frac{\omega}{3}\left(\frac{d\log G}{d\log a}\right)^2 \qquad w+1=\frac{1}{3}\frac{C}{\eta}\left(\frac{d\log \alpha}{d\log a}\right)^2$$

These equations establish a relationship between the equivalence principle and the equation of state of dark energy.

Tests of the equivalence principle are a way of constraining some models of dark energy.

Dynamical compactifications violate the EP (at some level)

- Kaluza-Klein theory is a mess (Fierz 1956)
  - includes variation of the coupling constant of the Kaluza-Klein one-form

$$\frac{1}{2}\int d^4x\sqrt{-h}\left(R_4[h]-(\partial\phi)^2+\tfrac{1}{4}e^{\sqrt{6}\phi}F^2\right)$$

  - couplings of the radion and one-form all throughout the matter sector

Dynamical compactifications violate the EP (at some level)

- The simplest S^1/Z_2 compactification (*i.e.* compactification on an interval) is better.

$$\int d^5x\sqrt{-h}\Big(\tfrac{1}{2}R_4[h]-\tfrac{1}{2}(\partial\phi)^2+e^{-\sqrt{8/3}\phi}\mathcal{L}_0[\Phi_{i,0};e^{-\sqrt{2/3}\phi}h_{\mu\nu}]$$
$$+e^{-\sqrt{8/3}\phi}\mathcal{L}_1[\Phi_{i,1};e^{-\sqrt{2/3}\phi}h_{\mu\nu}]\Big),$$

Matter fields on orbifold planes all couple to the same conformally rescaled metric (this is a *universal* coupling). This is Brans-Dicke theory.

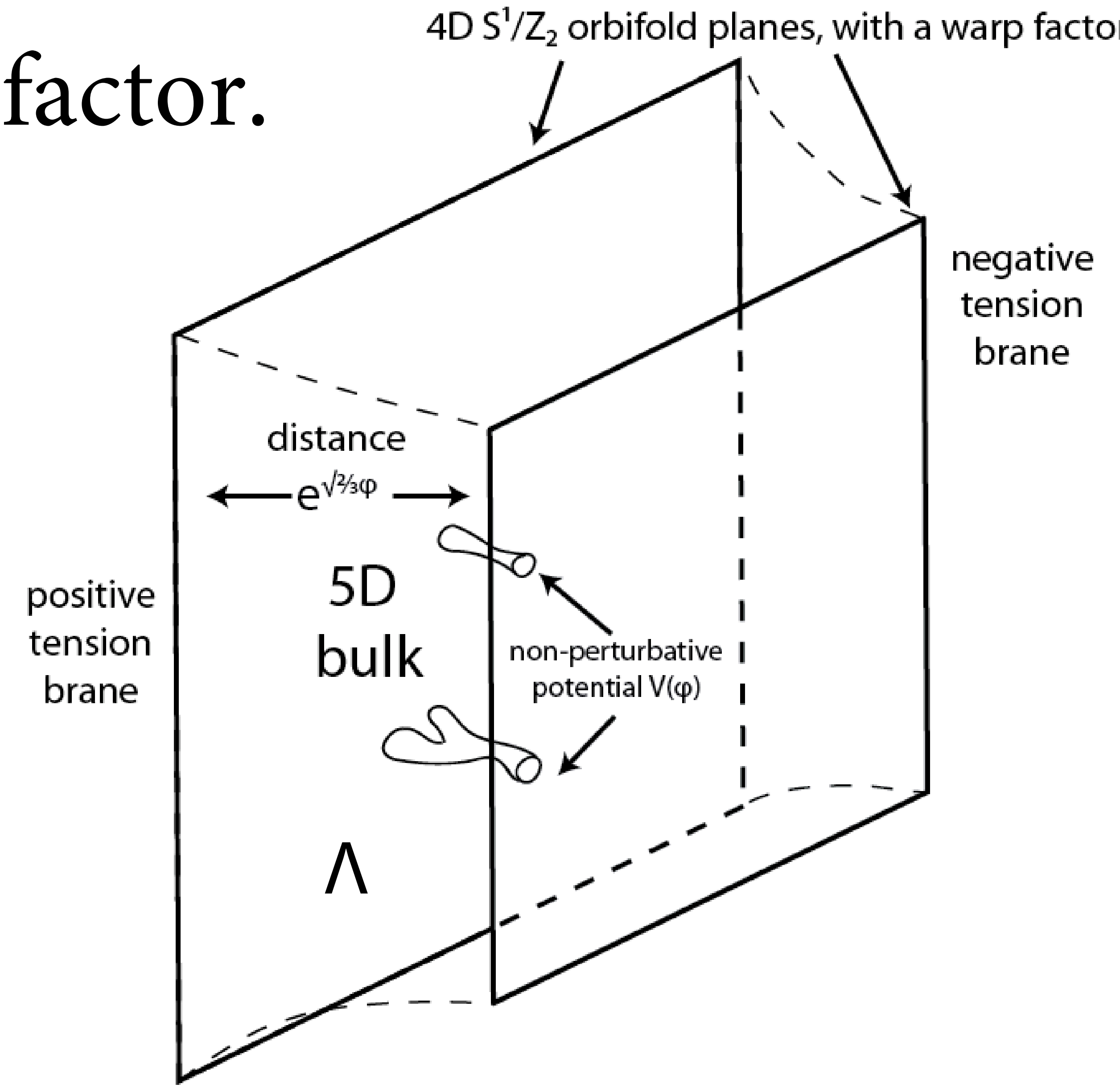
Warped models

- A *warped* extra dimension

$$ds^2 = \Omega(y)^2 a(t)^2 \eta_{\mu\nu} dx^\mu dx^\nu + (dy)^2$$

where $\Omega(y)$ is the warp factor can improve (or exacerbate) the situation as in the Randall-Sundrum models, heterotic M-theory, etc…

The warp factor.



4D low-energy effective action of heterotic M-theory (in Brans-Dicke frame)

$$S = \frac{\pi\rho V}{\kappa^2}\int_{\mathcal{M}^4}\sqrt{-h}d^4x\Big[e^c R - 0\times e^c(\partial c)^2 - 3\left(1+\tfrac{1}{3}\xi\alpha_0 e^c\right)\partial_\mu C\partial^\mu \bar{C}$$

$$-\tfrac{3}{8}e^{-c}CC\bar{C}\bar{C} - \frac{3k^2}{4}\left(1-\tfrac{1}{3}\xi\alpha_0 e^c\right)CC\bar{C}\bar{C}$$

$$-\frac{V}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\int_{\mathcal{M}^4}\sqrt{-g}\,d^4x\left((1+\xi\alpha_0 e^c)\,\mathrm{tr}(F^{(1)})^2 + (1-\xi\alpha_0 e^c)\,\mathrm{tr}(F^{(2)})^2\right)$$

(Lukas, Ovrut and Waldram, 1997)

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$$- \tfrac{3}{8} e^{-c} C C \bar{C} \bar{C} - \frac{3 k^2}{4} \left(1 - \tfrac{1}{3} \xi \alpha_0 e^c\right) C C \bar{C} \bar{C}$$

$$- \frac{V}{8 \pi \kappa^2} \left(\frac{\kappa}{4\pi}\right)^{2/3} \int_{\mathcal{M}^4} \sqrt{-g}\, d^4 x \left((1 + \xi \alpha_0 e^c) \operatorname{tr}(F^{(1)})^2 + (1 - \xi \alpha_0 e^c) \operatorname{tr}(F^{(2)})^2 \right)$$

- Violations of the weak equivalence principle and variation of α are naturally suppressed at higher order (at observational thresholds, depending on the Calabi-Yau manifold)

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$$- \tfrac{3}{8} e^{-c} C C \bar{C} \bar{C} - \frac{3k^2}{4}\left(1 - \tfrac{1}{3} \xi \alpha_0 e^c\right) C C \bar{C} \bar{C}$$

$$- \frac{V}{8\pi\kappa^2} \left(\frac{\kappa}{4\pi}\right)^{2/3} \int_{\mathcal{M}^4} \sqrt{-g}\, d^4 x \left((1 + \xi \alpha_0 e^c) \operatorname{tr}(F^{(1)})^2 + (1 - \xi \alpha_0 e^c) \operatorname{tr}(F^{(2)})^2 \right)$$

- Brans-Dicke parameter is zero, as in the S^1/Z_2 case

Results

	$w+1$	$\lvert \Delta G_{\rm BBN}/G \rvert$	$H_0^{-1}\lvert \dot{\alpha}_{\rm EM} \rvert$	$\lvert \eta \rvert$	$\lvert \omega \rvert$
Observations	$\lesssim 0.3$	$\lesssim 0.4$	$\lesssim 10^{-7}$	$\lesssim 10^{-12}$	
Scalar fields					
Minimally interacting	$\lesssim 0.3$	—	—	—	—
Brans-Dicke	$\lesssim 0.3$	$\lesssim 2 \times 10^{-3}$	—	—	$\gtrsim 40,000$
General	$\lesssim 0.3$	$\lesssim 10^{-4}$	$\lesssim 10^{-7}$	$\lesssim 10^{-12}$	$\gtrsim 10^{8}$
Compactifications					
S^1/Z_2	$\lesssim 0.3$		—	—	0
negative tension	$\lesssim 0.3$		—	—	$-3/2$
positive tension	$\lesssim 0.3$	—	—	—	$\gg 1$
String inspired					
Heterotic M-theory	$\lesssim 10^{-6}$		$\lesssim 10^{-7}$	$\approx 10^{-12}$	0
Runaway dilaton	$\lesssim 0.3$	$\lesssim 10^{-4}$	$\lesssim 10^{-7}$	$\lesssim 10^{-13}$	$\gtrsim 10^{8}$
Cosmic chameleon	—	—	—	$\lesssim 10^{-12}$	$\approx 10^{12}$

Conclusions

- It is important to look for dynamical dark energy (equivalently a cosmological scalar field) using other approaches than the equation of state
- Scalar field dark energy is likely to lead to violations of the equivalence principle which satisfy general relations
- It is a challenge to construct models in which the violations are naturally small, although some plausible mechanisms have emerged
- The optimal strategy for testing different dark energy models can be different
- Some of the tools of compactification – orbifolds and warped extra dimensions – can be used to suppress deviations from the equivalence principle

It’s over.