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Perturbations in a Regular Bouncing Universe

T.J. Battefeld, G. Geshnizjani, PERTURBATIONS IN A REGULAR BOUNCING UNIVERSE. [HEP-TH 0503160]

Thorsten J. Battefeld, Ghazal Geshnizjani, A NOTE ON PERTURBATIONS DURING A REGULAR BOUNCE. [HEP-TH 0506139]

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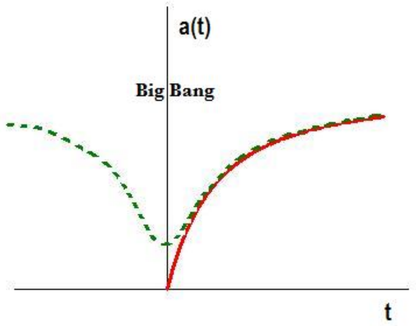
Singularity Problem

In Standard cosmology:

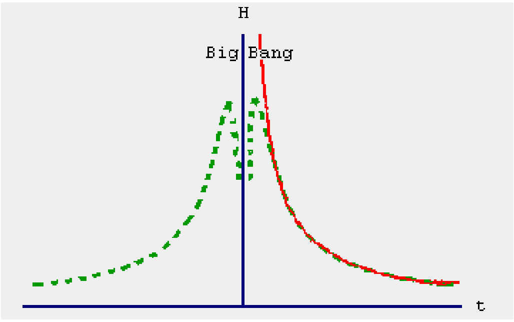
$$t \rightarrow 0 \quad \Rightarrow \quad a(t) \rightarrow 0 \quad \Rightarrow \begin{cases} H \rightarrow \infty \\ \rho \rightarrow \infty \end{cases}$$



⇒ Both Energy Density and Curvature diverge ⇒ Singularity !

both physical and Mathematical!

Bouncing Scenarios:



String/M-theory has inspired new cosmological scenarios to solve the singularity problem, in which a long period of accelerated (growing-curvature) evolution turns into a standard (decreasing-curvature) FRW-type cosmology, after going smoothly through a big bang-like event (Pre-big bang scenario: M. Gasperini and G. Veneziano, Cyclic scenario: , J. Khoury, et. al.).

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Challenges

Describing the **transition** between the two regimes.

Computing, in a **reliable** way, the final spectrum of amplified quantum fluctuations to be compared with present data on **CMB** radiation and **large-scale structure**.

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Modified version of the Randall-Sundrum (RS) scenario:

4D space-time + 1 extra time-like dimension

Y. Shtanov and V. Sahni,
Phys. Lett. B 557, 1 (2003)[arXiv:gr-qc/0208047].

$$G_{\mu\nu} = -\kappa^2 T_{\mu\nu} + \tilde{\kappa}^4 S_{\mu\nu} \quad \Rightarrow \quad H^2 = \frac{\kappa^2}{3}(\rho_+ - \rho_-)$$

$$\kappa^2 = 8\pi/M_p^2, \; \tilde{\kappa}^2 = 8\pi/\tilde{M}_p^3$$

5D Planck mass

$$\rho_+ := \rho \text{ and } p_+ := p.$$

$$\rho_- := \frac{\rho^2}{2\lambda},$$

$$p_- := \frac{\rho}{2\lambda}(2p+\rho)$$

$$6\kappa^2/\tilde{\kappa}^4$$

$\uparrow \rho \Rightarrow$ A Regular Bounce at t=0

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Background:

$$\rho_+(t) = r a(t)^{-n}$$
$$\rho_-(t) = \frac{r^2}{2\lambda} a(t)^{-2n}$$

Friedmann Eq.

$$a(t) = \left[\frac{r}{2\lambda}\left(1 + \frac{n^2}{6}\kappa^2\lambda t^2\right)\right]^{1/n}$$

For radiation dominated background : $(n = 4)$

$$a(x) = a_0\left(1 + x^2\right)^{1/4}$$

$\hookrightarrow t/t_0$

$$a_0 = (r/2\lambda)^{1/n} \qquad t_0 := \sqrt{\frac{6}{\lambda}}\frac{1}{n\kappa}$$

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Computing the final spectrum of quantum fluctuations

- **Scalar perturbations:**

Perturbed Einstein equations

$$\nabla^2\Phi - 3\mathcal{H}(\mathcal{H}\Phi + \Phi') = \frac{a^2}{2}\kappa^2\rho_0\xi\left(1 - \frac{\rho_0}{\lambda}\right)$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = \frac{a^2}{2}\kappa^2\rho_0\xi \times\left[w - (1+2w)\frac{\rho_0}{\lambda}\right],$$

$$[\Phi\mathcal{H} + \Phi']_{,i} = -\frac{a^2}{2}\kappa^2\rho_0 V_{,i}(1+w)\left(1 - \frac{\rho_0}{\lambda}\right)$$

Goes to 0 at x=1

(boundaries of the region where the null energy condition (NEC) is violated.)

↳ Σ! $\rho + P = 0$

$$\left(1 - \frac{2}{(1+x^2)}\right)\ddot{\Phi} + \frac{1}{2}\frac{x}{1+x^2}\left\{\left[5 - \frac{18}{(1+x^2)}\right]\right\}\dot{\Phi} - \left\{\left[\frac{1}{3} - \frac{10}{3}\frac{1}{(1+x^2)}\right]\left(\frac{-\tilde{k}^2}{(1+x^2)^{1/2}}\right) + \frac{1}{(1+x^2)^2}\right\}\Phi = 0$$

$$\tilde{k} := \frac{k t_0}{a_0}$$

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A note on regularity of the Bardeen potential in longitudinal gauge at the boundaries of the region where the null energy condition (NEC) is violated.

$$\begin{cases} \nabla^2\Phi - 3\mathcal{H}(\mathcal{H}\Phi + \Phi') = \frac{a^2}{2}\kappa^2\left(\rho_{(a)}\xi_{(a)} \pm \rho_{(b)}\xi_{(b)}\right) \\ \Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = \frac{a^2}{2}\kappa^2\left(w_a\xi_{(a)}\rho_{(a)} \pm w_b\xi_{(b)}\rho_{(b)}\right), \\ [\Phi\mathcal{H} + \Phi']_{,i} = -\frac{a^2}{2}\kappa^2\left(\rho_{(a)}V_{(a),i}(1+w_a) \pm \rho_{(b)}V_{(b),i}(1+w_b)\right), \end{cases}$$

+ **Adiabaticity**

$\Downarrow$

$$\begin{aligned} 0 = {} & \Phi''\left[\overbrace{\rho_a(1+w_a) \pm \rho_b(1+w_b)}^{\emptyset \text{ at } \eta_{nec}}\right] \\ & + 3\mathcal{H}\Phi'\left[\rho_a(1+w_a)^2 \pm \rho_b(1+w_b)^2\right] \\ & + \Big[- \left(w_a(w_a+1)\rho_a \pm w_b(w_b+1)\rho_b\right)\nabla^2 \\ & + 2\mathcal{H}'\left(\rho_a(1+w_a) \pm \rho_b(1+w_b)\right) \\ & + \mathcal{H}^2\left(\rho_a(1+w_a)(1+3w_a) \pm \rho_b(1+w_b)(1+3w_b)\right)\Big]\Phi \end{aligned}$$

P. Peter and N. Pinto-Neto, Phys. Rev. D 66, 063509 (2002)

$\varphi = \varphi_a + \varphi_b$

$\Downarrow$

$$\nabla^2\Phi_l - 3\mathcal{H}(\mathcal{H}\Phi_l + \Phi_l') = \frac{a^2}{2}\kappa^2\rho_{(l)}\xi_{(l)},$$

$$\Phi_l'' + 3\mathcal{H}\Phi_l' + (2\mathcal{H}' + \mathcal{H}^2)\Phi_l = \frac{a^2}{2}\kappa^2 w_l\xi_{(l)}\rho_{(l)}$$

$\Downarrow$

$$\begin{aligned} 0 &= \Phi_l'' + 3\mathcal{H}(1+w_l)\Phi_l' \\ &\quad + \left(-w_l\nabla^2 + 2\mathcal{H}' + (1+3w_l)\mathcal{H}^2\right)\Phi_l \end{aligned}$$

no reason to be consistent with energy conservation conditions:

$$\frac{\Phi_a'}{\rho_{(a)}(1+w_a)} = \frac{\Phi_b'}{\rho_{(b)}(1+w_b)}.$$

e.g. Adiabaticity

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$$A(\eta)\Phi'' + B(\eta)\Phi' + C(k,\eta)\Phi = 0,$$

$$B = -A'$$

energy conservation for each fluid requires:

$$\rho_l' = -3\mathcal{H}\rho_l(1+w_l)$$

$$A(\delta) = A_1\delta + A_2\delta^2 + A_3\delta^3 + \dots,$$
$$B(\delta) = -A_1 - 2A_2\delta - 3A_3\delta^2 + \dots,$$
$$C(\delta) = C_0 + C_1\delta + C_2\delta^2 + \dots,$$

- $\Phi_1 = \delta^2 + \frac{2A_2 - C_0}{3A_1}\delta^3 + \dots + \alpha_{n+1}\delta^{n+1} + \dots$ → **Wronskian technique**

$$\alpha_{n+1} = \frac{\sum_{i=2}^{n}[i(3+n-2i)A_{n+2-i} - C_{n-i}]\alpha_i}{(n-1)(n+1)A_1}$$

- $\Phi_2 = -\frac{A_1}{2} - \frac{8A_2 - C_0}{6}\delta + A_3\delta^2\ln(|\delta|) + O(\delta^2)$

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Back to:

$$(1-\frac{2}{(1+x^2)})\ddot{\Phi}+\frac{1}{2}\frac{x}{1+x^2}\left\{[5-\frac{18}{(1+x^2)}]\right\}\dot{\Phi}$$
$$-\left\{[\frac{1}{3}-\frac{10}{3}\frac{1}{(1+x^2)}](\frac{-\tilde{k}^2}{(1+x^2)^{1/2}})+\frac{1}{(1+x^2)^2}\right\}\Phi=0$$

Matching different approximate solutions at transition points:

$|x| \sim 0 \rightarrow$ Bounce

$|x| \sim 1 \rightarrow \eta_{nec}$

$|x| \sim \sqrt{\left(\frac{3}{\tilde{k}^2}\right)^{2/3}-1} \rightarrow \tilde{x}$

Bunch-Davis initial condition:

$$x \ll -\sqrt{\left(\frac{3}{\tilde{k}^2}\right)^{2/3}-1} \Rightarrow \Phi_k = \alpha\frac{3^{3/4}}{2}\frac{1}{\tilde{k}^{3/2}x}\left(i-\frac{\sqrt{3}}{2}\frac{1}{\tilde{k}\sqrt{-x}}\right)\exp\left(i\frac{2}{\sqrt{3}}\sqrt{-x}\tilde{k}\right)$$

$$\xrightarrow{\tilde{k}^2 \ll 1} \varphi_k \sim \frac{i}{\tilde{k}^{3/2}}\left(\underbrace{\frac{4}{3}\tilde{k}^2}_{\text{const.}} - \underbrace{\frac{\sqrt{3}}{2}\frac{i}{\tilde{k}(-x)^{3/2}}}_{\text{growing}}\right)$$

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Post bounce:

$$\varphi = A + B x^{-3/2}$$

Pre-bounce growing mode

Const. Mode

Before horizon reentry $\sim k^{-7/3}$

Decaying Mode

Pre-bounce growing mode

Before horizon reentry $\sim k^{-1/2}$

It dominates

$$\Longrightarrow \quad n_s = -\frac{2}{3}$$

Ruled out as a realistic competitor to inflation but shows that bounce has an impact on the spectrum,

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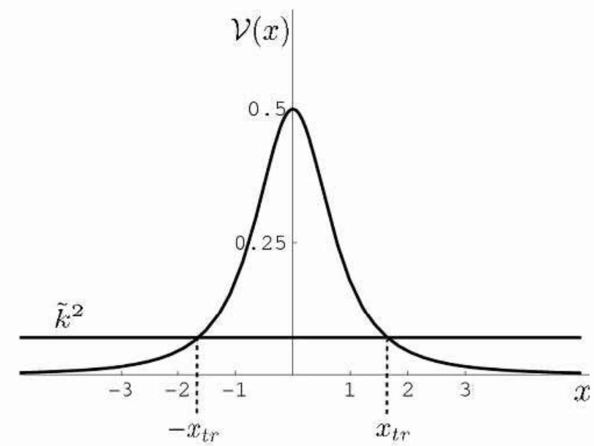
Tensor perturbation (gravity waves):

Perturbed Einstein equations (4D):

$$\mu'' - \nabla^2\mu - \frac{a''}{a}\mu = 0$$

ah

$$\mathcal{V}(x) := \left(\frac{t_0}{a_0}\right)^2 \frac{a''}{a} = \frac{1}{2}\frac{1}{(1+x^2)^{3/2}}$$



for $\tilde{k}^2 \ll \mathcal{V}(x)$:

$$\frac{\mu_k(\eta)}{a(\eta)} = A_2[1 - k^2\int^\eta \frac{d\tau}{a^2}\int^\tau d\varsigma\, a^2] + B_2\int^\eta \frac{d\tau}{a^2}[1 - k^2\int^\tau d\varsigma\, a^2 \int^\varsigma \frac{d\rho}{a^2}] + \cdots,$$

$$\int_0^\infty \frac{d\eta}{a^2} = 2.60\frac{t_0}{a_0^3} \longrightarrow \text{Finite!}$$

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$$\mu_k(x) = A_2 a_0 |x|^{1/2} + B_2 \frac{t_0}{a_0^2} \frac{|x|}{x} [2.6|x|^{1/2} - 2]$$

$$\tilde{k}^2 \gg V(x) \rightarrow \begin{cases} x < x_{tr} \Longrightarrow \mu_k(\eta) = \dfrac{1}{\sqrt{2k}}[A_1 e^{-ik(\eta-\eta_i)} + B_1 e^{ik(\eta-\eta_i)}] \\[2ex] x > x_{tr} \Longrightarrow \mu_k(\eta) = \dfrac{1}{\sqrt{2k}}[A_3 e^{-ik\eta} + B_3 e^{ik\eta}] \end{cases}$$

Matching $\Rightarrow$ Transfer function

The growing mode for h in the pre-collapse phase matches onto the constant mode in the post-collapse phase.

Bunch-Davis vacuum at the initial time: $n_t = 2$

Blue spectral index, no data to compare with yet!

Amplitude of power spectrum was dictated from the scales and details of the bounce and this result is in agreement with J. Martin, et. al. (2002)

Conclusions:

- Starting from a vacuum initial conditions for long wavelengths, this model is ruled out as a realistic model due to its predictions for scalar perturbations;
- However, we showed that the spectrum of final fluctuations is sensitive to details of the bounce, which leaves the door open for the possibility of a feasible bouncing scenario;
- We also developed a novel method that can be used for following perturbations through a general class of bouncing scenarios.