Spontaneous Isotropy Breaking: A Mechanism for CMB Multipole Alignments

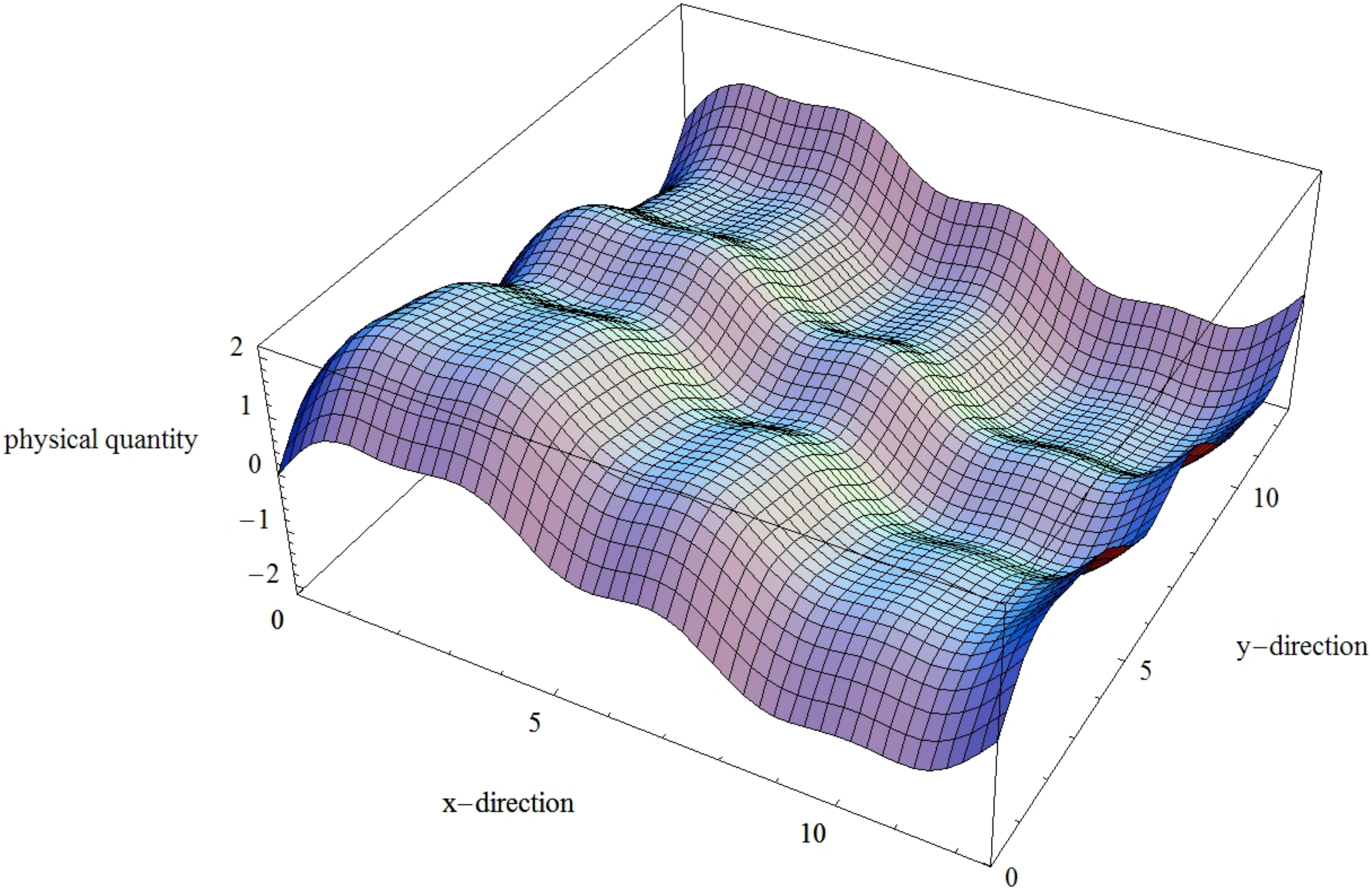
Christopher Gordon
University of Chicago (KICP)

Reference: Gordon, C., Hu, W., Huterer, D. and Crawford, T., [astro-ph/0509xx]

Possible Sources of directionality:

- One spatial dimension smaller than horizon (Tegmark, de Oliveira-Costa and Hamilton (2003)).

- One spatial dimension expanding at a different rate to the others (Jaffe, Banday, Eriksen, Gorski and Hansen (2005)).


physical quantity
2
1
0
−1
−2
0
5
10
x−direction
0
5
10
y−direction

Say quintessence had a gradient across the horizon in the z direction:

$$Q = Az + B$$

This gets mapped on to a sub-horizon modulation by the potential:

$$\begin{aligned} V &= V_0(1 + f\cos[Q/M_0]) \\ &= V_0(1 + f\cos[k_0 z + \delta]) \end{aligned}$$

Assume the field is light

$$\begin{aligned} \frac{\partial^2 V}{\partial Q^2}\frac{1}{H^2} &< \frac{\partial^2 V}{\partial Q^2}\frac{M_{\rm pl}^2}{V} \\ &\approx f\left(\frac{M_{\rm pl}}{M_0}\right)^2 \end{aligned}$$

Thus,

$$\frac{M_0}{M_{\rm pl}} \gg f^{1/2}\,.$$

Then the field will be frozen. This gives rise to a sub-horizon density perturbation that has Fourier components

$$\frac{\delta\rho_Q}{\rho_Q}(\mathbf{k}) = \frac{f}{2}e^{i\delta}(2\pi)^3\delta(\mathbf{k}-k_0\hat{\mathbf{x}}_3) + \frac{f}{2}e^{-i\delta}(2\pi)^3\delta(\mathbf{k}+k_0\hat{\mathbf{x}}_3)\,.$$

The curvature perturbation on co-moving hyper-surfaces is given by:

$$\begin{aligned}\zeta &= \zeta_i - \int_0^a \frac{da'}{a'}\frac{\delta p_T}{\rho_T + p_T} \\ &\approx \int_0^a \frac{da'}{a'}\frac{\delta\rho_Q}{\rho_m}\,.\end{aligned}$$

The matter density red-shifts as

$$\rho_m = \frac{\rho_Q}{a^3}\frac{\Omega_m}{\Omega_Q}\,.$$

So that

$$\zeta = \frac{a^3}{3}\frac{\delta\rho_Q}{\rho_Q}\frac{\Omega_Q}{\Omega_m}\,.$$

The Newtonian gravitational potential can be expressed in terms of ζ as

$$\Psi(\mathbf{k}, a) = \zeta - \frac{H}{a}\int_0^a \frac{da'}{H}(\zeta - \frac{\delta p_T}{\rho_T + p_T}) \;\approx\; \zeta - \frac{H}{a}\int_0^a \frac{da'}{H}(\zeta + \frac{\delta\rho_Q}{\rho_m})\,.$$

Then,

$$\Psi(\mathbf{k}, a) \approx \psi(2\pi)^3\delta(\mathbf{k} - k_0\hat{\mathbf{x}}_3) + \psi^*(2\pi)^3\delta(\mathbf{k} + k_0\hat{\mathbf{x}}_3)$$

where

$$\psi = -\frac{1}{3}\frac{\Omega_Q}{\Omega_m}\frac{f}{2}e^{i\delta}\left(a^3 - 4\frac{H(a)}{a}\int\frac{da'}{H(a')}a'^3\right) .$$

The integrated Sachs Wolfe effect due to the dark energy perturbations is given by

$$\frac{\Delta T(\hat{\mathbf{n}})}{T} = fw(\hat{\mathbf{n}}) = \int 2\Psi'(\mathbf{x} = D\hat{\mathbf{n}}, \eta)d\ln a\,,$$

The multipole moments are given by

$$fw_\ell \equiv \int d\hat{\mathbf{n}}Y_{\ell m}^*(\hat{\mathbf{n}})\frac{\Delta T}{T}(\hat{\mathbf{n}})\,.$$

With the Rayleigh expansion of a plane wave

$$\exp(i\mathbf{k}\cdot\mathbf{x}) = \sum_{\ell m} 4\pi i^{\ell} j_{\ell}(kD) Y^{*}_{\ell m}(\hat{\mathbf{k}}) Y_{\ell m}(\hat{\mathbf{n}})$$

they can be written as

$$f w_{\ell} = \int d\ln a \int \frac{d^3k}{(2\pi)^3} 4\pi i^{\ell} j_{\ell}(kD) Y^{*}_{\ell m}(\hat{\mathbf{k}}) 2\Psi'(\mathbf{k}, \ln a)$$

Using the solution for the Newtonian gravitation potential gives

$$w_\ell = -\frac{\Omega_Q}{\Omega_m} s_\ell \sqrt{4\pi(2\ell+1)} \int d\ln a\, j_\ell(k_0 D) I(a) a^3 \,,$$

$$s_\ell \equiv \cos\delta(-1)^{\ell/2}\delta_\ell^{\mathrm{e}} + \sin\delta(-1)^{(\ell+1)/2}\delta_\ell^{\mathrm{o}} \,,$$

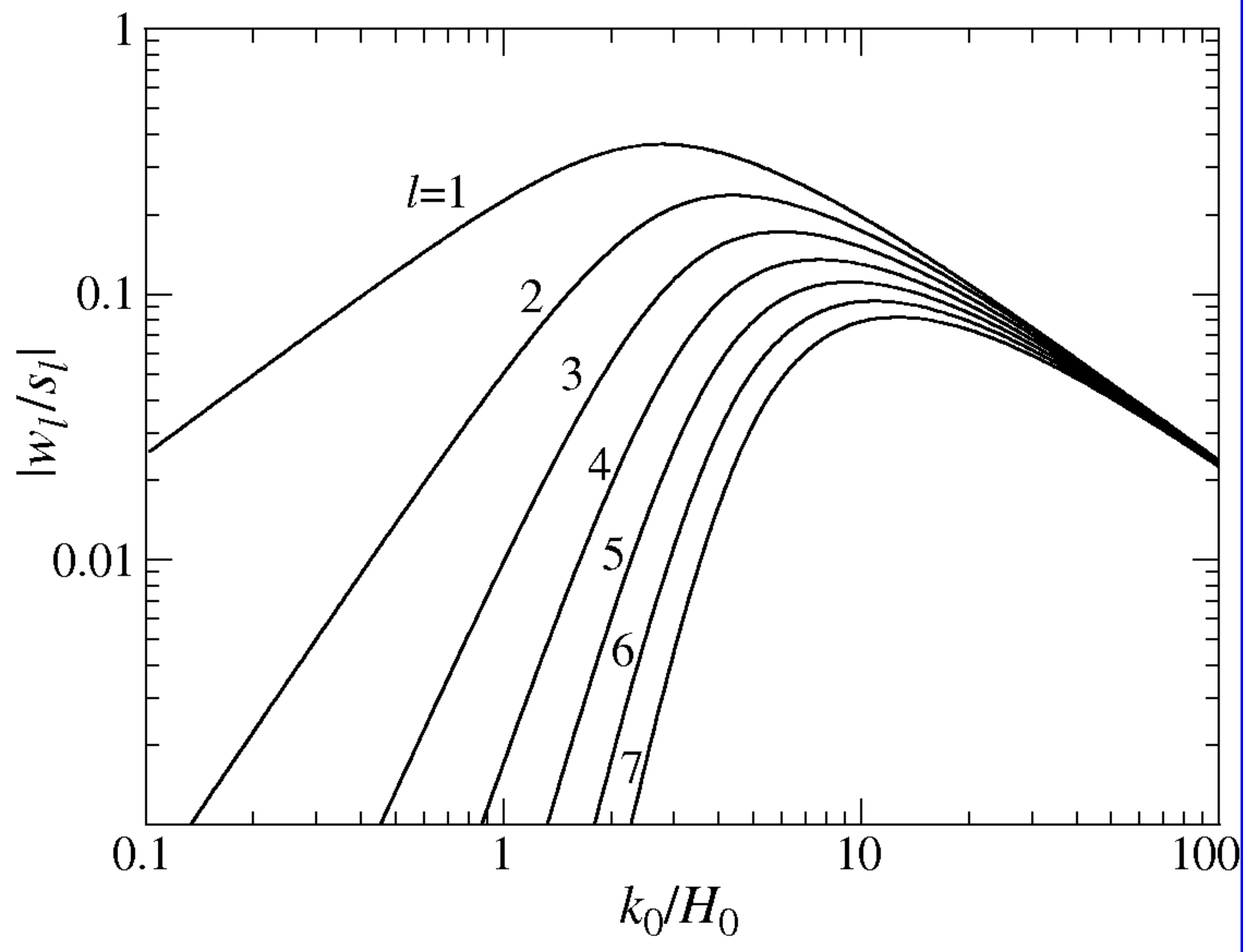
where $\delta_\ell^{\mathrm{e}} = 1$ if ℓ is even and 0 if ℓ is odd, and vice versa for δ_ℓ^{o}. Here we have defined

$$I(a) \equiv -\frac{8}{3a^3}\frac{d}{d\ln a}\left[\frac{H(a)}{a}\right]\int \frac{da'}{H(a')} a'^3 - \frac{2}{3} \,.$$

The spatial modulation projects onto an angular modulation with a weight given by the spherical Bessel function j_ℓ. For a superhorizon fluctuation

$$k_0 D = \frac{k_0}{H_0} H_0 D \sim \frac{k_0}{H_0} \ll 1$$

and so $j_\ell \propto (k_0/H_0)^\ell$.



1
0.1
0.01
$|w_l/s_l|$
0.1
1
10
100
k_0/H_0
l=1
2
3
4
5
6
7

What exactly needs to be fixed?

To test the quadrupole-octopole alignment, we take the normalized angular momentum (the t statistic of de Oliveira-Costa, Tegmark, Zaldarriaga, and A. Hamilton as generalized by Copi, Huterer, Schwarz, and Starkman.

$$(\Delta \hat{L})^2_\ell \equiv \frac{\sum_{m=-\ell}^{\ell} m^2 |a_{\ell m}|^2}{\ell^2 \sum_m |a_{\ell m}|^2} .$$

The statistic

$$(\Delta \hat{L})^2 \equiv (\Delta \hat{L})^2_2 + (\Delta \hat{L})^2_3 .$$

maximized over direction of the preferred axis captures both the alignment of the quadrupole and octopole and the planar nature of the octopole.

Then

$$\mathsf{Pr}\left[(\Delta\hat{L})^2 > (\Delta\hat{L})^2_{\mathsf{WMAP}}\middle|\,\Lambda\mathsf{CDM}\right] \approx 0.2\%$$

ℓ	m	$C_\ell^{\mathrm{fid}}(\mu\mathrm{K}^2)$	N_ℓ/C_ℓ	dipole	$\Delta L^2_{\mathrm{max}}$
2	0	1233	0.005	0.005	0.005
	1		0.005	0.013	0.003
	2		0.005	0.396	0.406
3	0	577	0.007	0.179	0.315
	1		0.007	0.079	0.003
	2		0.007	0.426	0.029
	3		0.007	2.155	2.560
4	0	322	0.009	3.929	1.641
	1		0.009	0.156	0.939
	2		0.009	0.052	0.508
	3		0.009	0.271	0.401
	4		0.009	0.406	0.180
5	0	202	0.011	0.035	0.014
	1		0.011	0.376	0.811
	2		0.011	0.035	1.254
	3		0.011	6.827	2.777
	4		0.011	0.077	2.769
	5		0.011	0.364	0.078

Multiplicative Modulation

$$T(\hat{\mathbf{n}}) \equiv A(\hat{\mathbf{n}}) + f[1 + w(\hat{\mathbf{n}})]B(\hat{\mathbf{n}})\,.$$

Multipole decomposition:

$$\begin{aligned}
T(\hat{\mathbf{n}}) &= \sum_{\ell m} t_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})\,, \\
A(\hat{\mathbf{n}}) &= \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})\,, \\
B(\hat{\mathbf{n}}) &= \sum_{\ell m} b_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})\,, \\
w(\hat{\mathbf{n}}) &\equiv \sum_{\ell} w_{\ell} Y_{\ell 0}(\hat{\mathbf{n}})\,.
\end{aligned}$$

The assumption of statistical isotropy for the underlying fields A and B requires that their covariance matrices satisfy

$$\begin{aligned} \langle a^*_{\ell m} a_{\ell' m'} \rangle &= \delta_{\ell\ell'}\delta_{mm'}C^{aa}_\ell \,, \\ \langle a^*_{\ell m} b_{\ell' m'} \rangle &= \delta_{\ell\ell'}\delta_{mm'}C^{ab}_\ell \,, \\ \langle b^*_{\ell m} b_{\ell' m'} \rangle &= \delta_{\ell\ell'}\delta_{mm'}C^{bb}_\ell \,. \end{aligned}$$

However statistical isotropy is not preserved in the observed temperature field $T(\hat{\mathbf{n}})$. We then get the convolution

$$t_{\ell m} = a_{\ell m} + f b_{\ell m} + f \sum_{\ell_1 \ell_2} R^{\ell_1 \ell_2}_{\ell m} b_{\ell_2 m}$$

with a coupling matrix written in terms of Wigner 3j symbols

$$
\begin{aligned}
R^{\ell_1\ell_2}_{\ell m} \equiv\ & (-1)^m \sqrt{\frac{(2\ell+1)(2\ell_1+1)(2\ell_2+1)}{4\pi}} \\
& \times \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & m & -m \end{pmatrix} w_{\ell_1} \, .
\end{aligned}
$$

The covariance matrix between the multipole moments then becomes

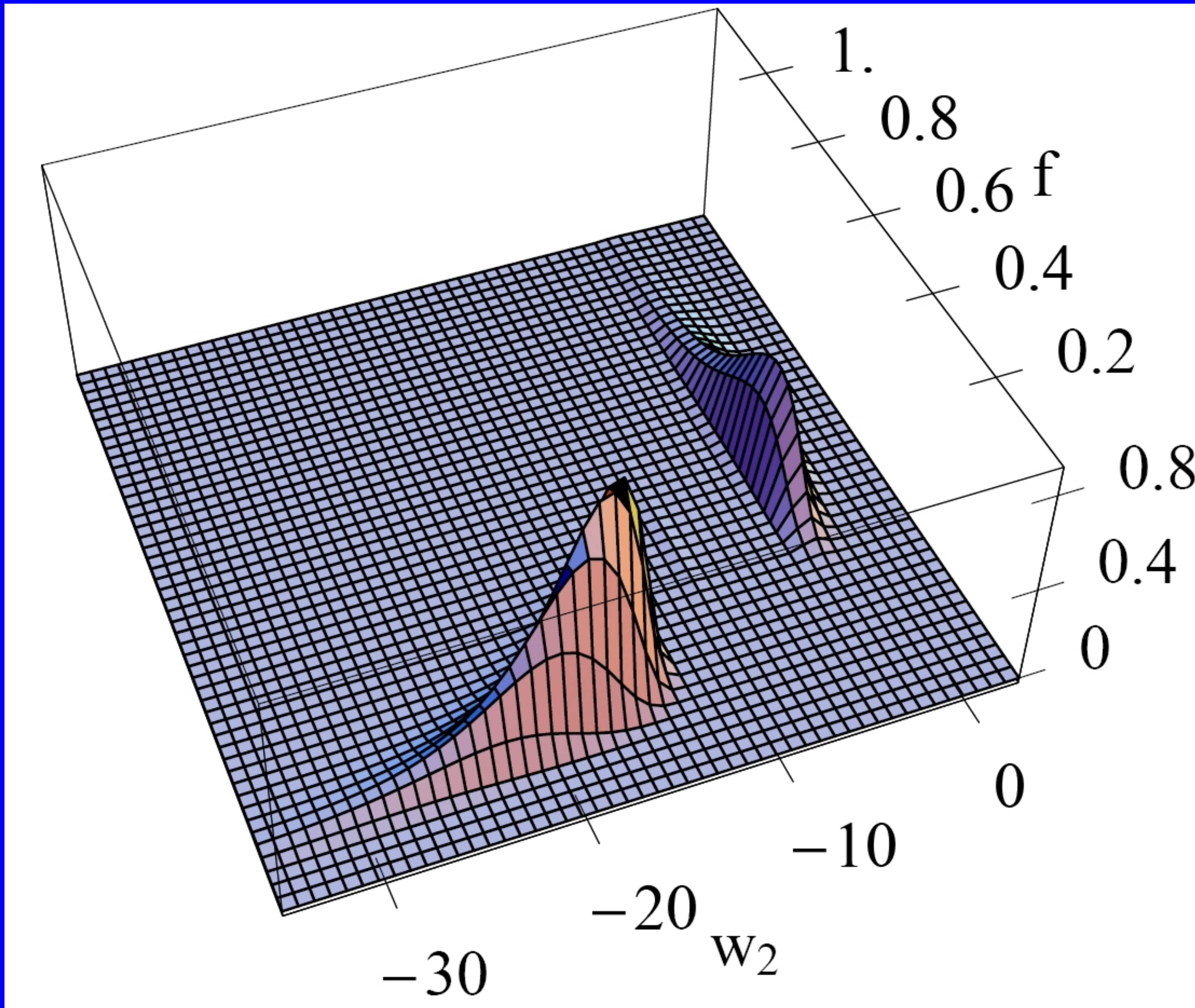
$$
\begin{aligned}
\langle t^*_{\ell m} t_{\ell' m} \rangle =\ & \delta_{\ell\ell'} [C^{aa}_\ell + 2f C^{ab}_\ell + f^2 C^{bb}_\ell] \\
& + f \sum_{\ell_1} \left[R^{\ell_1 \ell}_{\ell' m} (C^{ab}_\ell + f C^{bb}_\ell) + (\ell \leftrightarrow \ell') \right] \\
& + f^2 \sum_{\ell_1 \ell'_1 \ell_2} R^{\ell_1\ell_2}_{\ell m} R^{\ell'_1\ell_2}_{\ell' m} C^{bb}_{\ell_2} \, .
\end{aligned}
$$

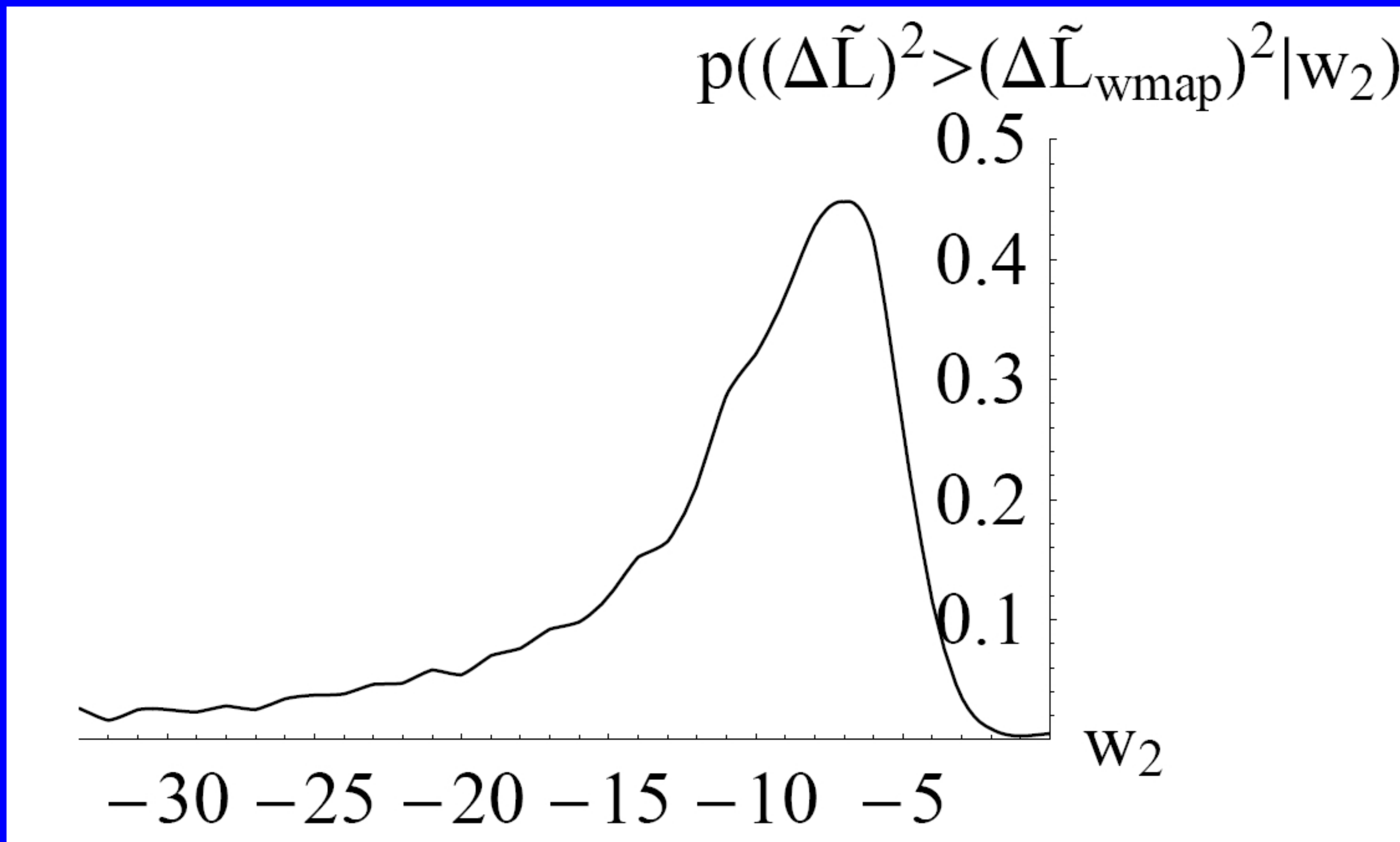
Assume,

$$w_\ell = w_2 \delta_{\ell 2} \, .$$

and

$$\begin{aligned} C_\ell^{aa} &= \begin{cases} C_\ell^{\mathrm{fid}}, & \text{if } \ell > 3 \\ 0, & \text{if } \ell = 2 \text{ or } \ell = 3 \end{cases} \\ C_\ell^{ab} &= 0 \\ C_\ell^{bb} &= \begin{cases} 0, & \text{if } \ell > 3 \\ C_\ell^{\mathrm{fid}}, & \text{if } \ell = 2 \text{ or } \ell = 3 \end{cases} \end{aligned}$$


1.
0.8
0.6
f
0.4
0.2
0.8
0.4
0
0
−10
−20
w_2
−30



$p((\Delta\tilde{L})^2 > (\Delta\tilde{L}_{wmap})^2 | w_2)$
0.5
0.4
0.3
0.2
0.1
w_2
−30 −25 −20 −15 −10 −5

$$
\begin{aligned}
& p\left((\Delta\tilde{L})^2 > (\Delta\tilde{L})^2 | a_{\ell m}\right) \\
= & \int_{w_2,f} p\left((\Delta\tilde{L})^2 > (\Delta\tilde{L})^2, w_2, f | a_{\ell m}\right) \, \mathrm{d}w_2 \, \mathrm{d}f \\
= & \int_{w_2,f} p\left((\Delta\tilde{L})^2 > (\Delta\tilde{L})^2 | w_2\right) p\,(w_2, f | a_{\ell m}) \, \mathrm{d}w_2 \, \mathrm{d}f
\end{aligned}
$$

Substituting in the previously evaluated quantities we get $p\left((\Delta\tilde{L})^2 > (\Delta\tilde{L})^2 | a_{\ell m}\right) = 0.07$, which is 28 times larger than the value for the fiducial model.

Conclusions

- WMAP data show anomalous alignments between $\ell = 2$ and $\ell = 3$.
- Superhorizon perturbations can lead to a subhorizon modulation.
- Possible in dark energy model but gives wrong modulation.
- Multiplicative modulation is needed.