Evolution of gravitational waves in the high-energy regime of brane-world cosmology

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1. Introduction

- ✓ Motivation
- ✓ Gravitational wave background
- ✓ Brane-world model
- ✓ High-energy effects
- ✓ Goal of our work

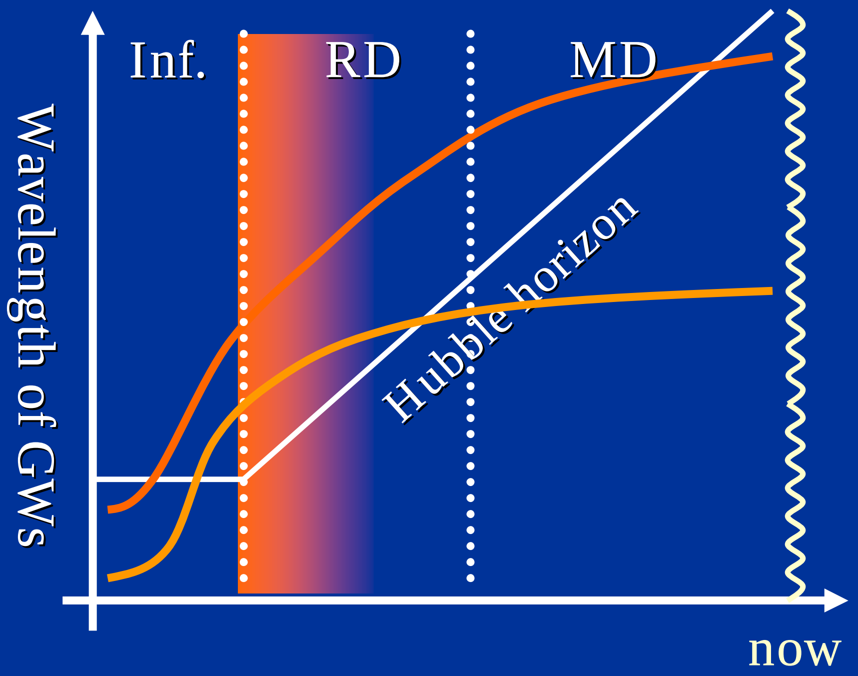
Motivation

- Gravitational wave background (GWB) generated in inflationary epoch can be a probe of extremely high-energy Universe.
- String theory or M-theory suggests that we live on a brane in 10/11 dimensional space-time.

Information about the extra-dimensions may be imprinted in the spectrum of GWB.

Gravitational wave background

- Quantum fluctuation of the space-time in the inflationary epoch.



- Inflationary epoch : Fluctuations exit from the horizon, and re-enter at late time.
- Inner the horizon :

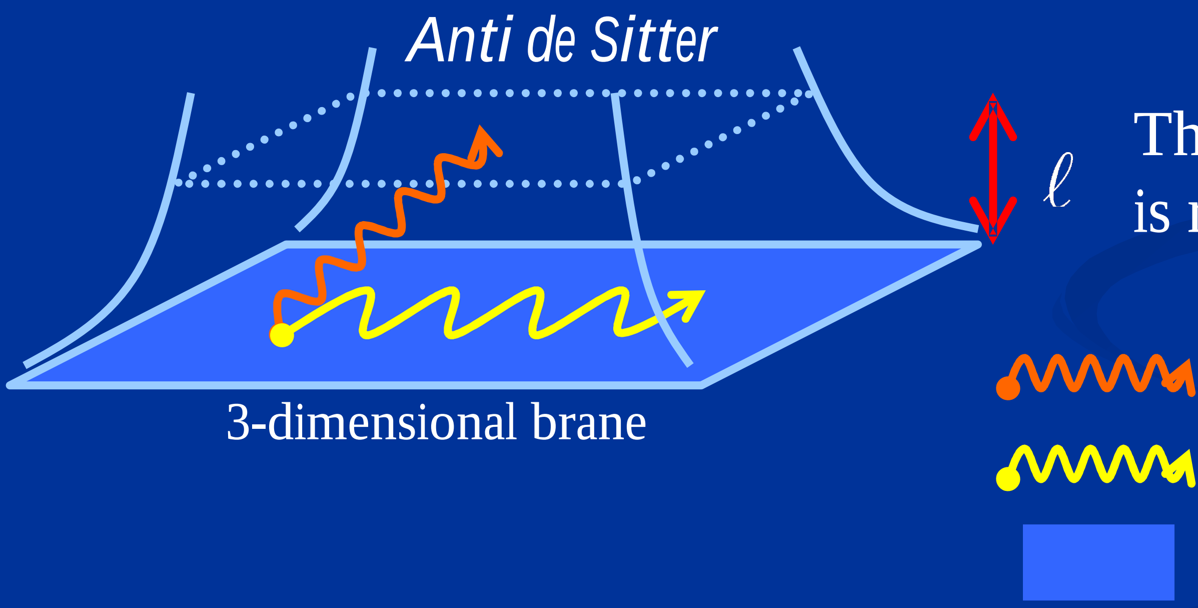
  The amplitude is damping as the scale factor evolves and the wavelength is red-shifted.

Brane-world model

- Randall-Sundrum II (single brane model)

Randall&Sundrum (1999)

  - The brane is embedded in 5D AdS space-time.



Chiaverini et al. (2003)

The scale of extra-dimension is restricted to < 0.1mm.

This feature of gravity modifies the expansion law of the Universe as well as the propagation of GWs.

High-energy effects

- Non-standard cosmological expansion Binetruy et al. (1999)

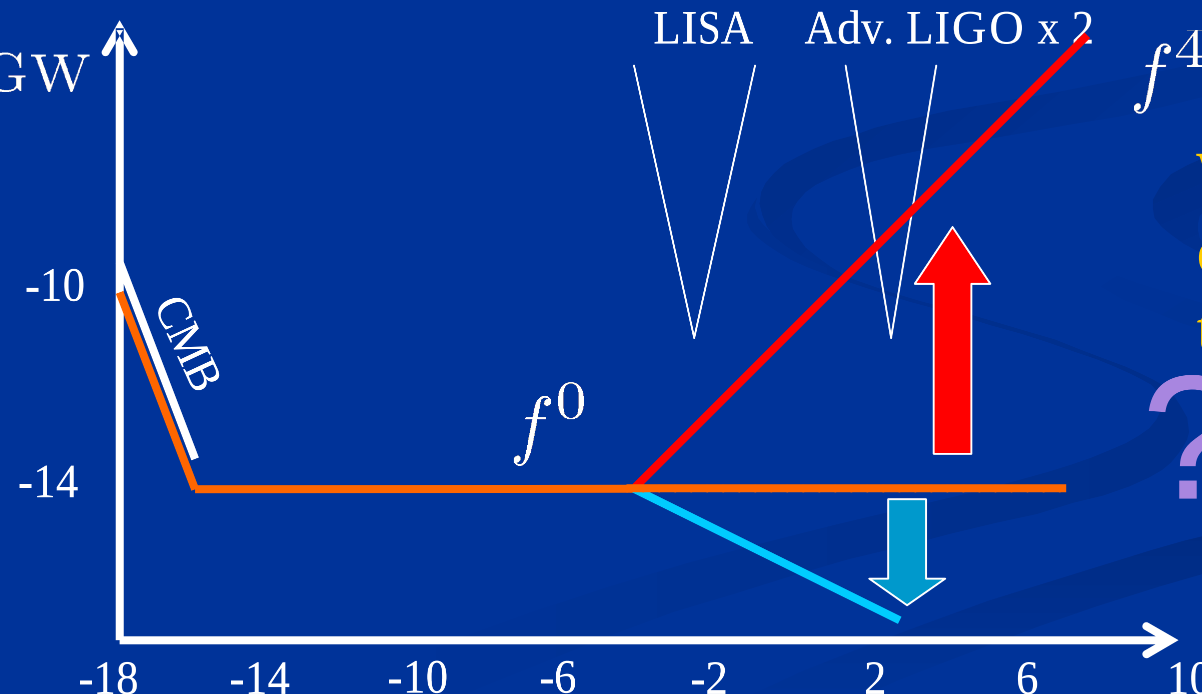
$$H^2 = \frac{8\pi G}{3}\rho\left(1+\frac{\rho}{2\lambda}\right) \longrightarrow a \propto (t^2 + 2\ell t)^{1/4}$$

$$a \propto t^{1/2} \quad \rightarrow \quad t^{1/4} \text{ (high energy RD)}$$

$$h \propto \frac{1}{a}$$

- KK-mode excitation

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log f} \propto f^2 h^2$$



Which effect overcomes the other ?

?

Goal of our work

construct the spectrum of GWB
in the brane-world scenario

- We can estimate the effect due to the high energy correction of Friedmann equations.
- However, the significance of KK-mode excitation is unknown.

→ We essentially have to perform numerical simulations.

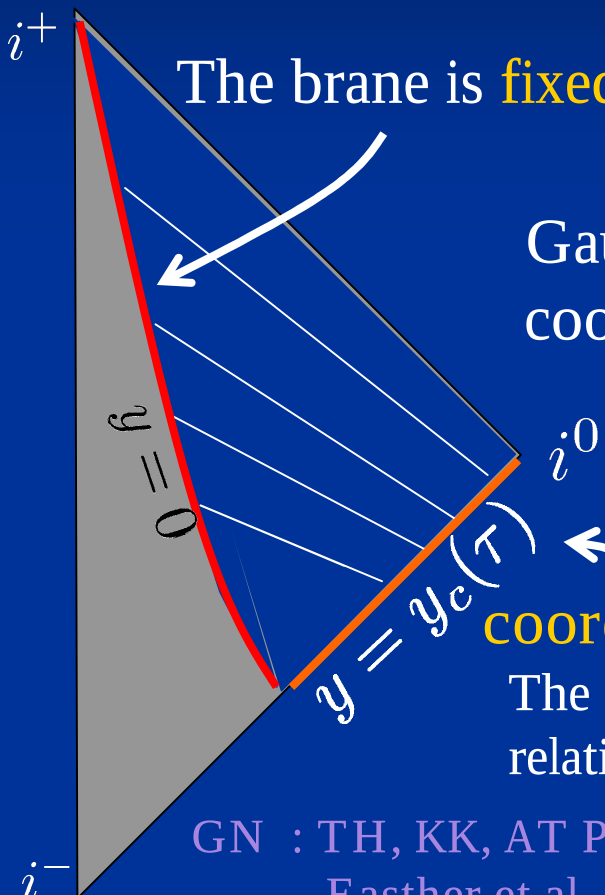
We choose a physically plausible initial condition, and must check the validity. Moreover, we investigate the dependence of results on the initial time.

- ✓ Poincare coordinates
- ✓ Cosmological expansion
- ✓ Wave equation
- ✓ Parameters
- ✓ Initial conditions
- ✓ Boundary conditions

Poincare coordinates

- Metric has no coordinate singularity in the bulk.

The brane is fixed in the bulk.

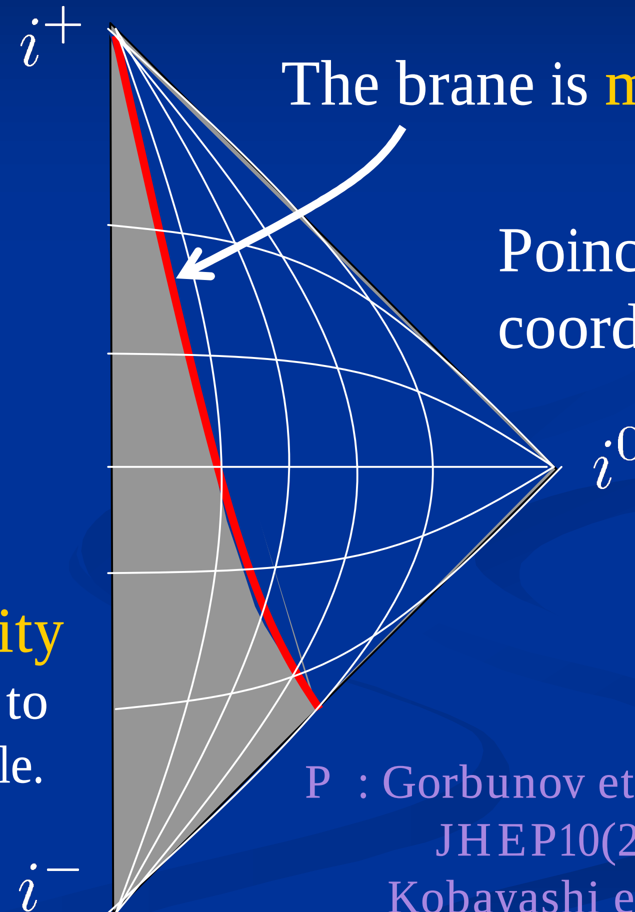


Gaussian Normal coordinate

coordinate singularity

The analyses are limited to relatively low energy scale.

The brane is moving.



Poincare coordinate

GN : TH, KK, AT PLB578(2004)269; Easther et al. JCAP10(2003)014; Battye et al. PRD69(2004)064040;

Null : Ichiki and Nakamura PRD70(2004)064017; astro-ph/0406606

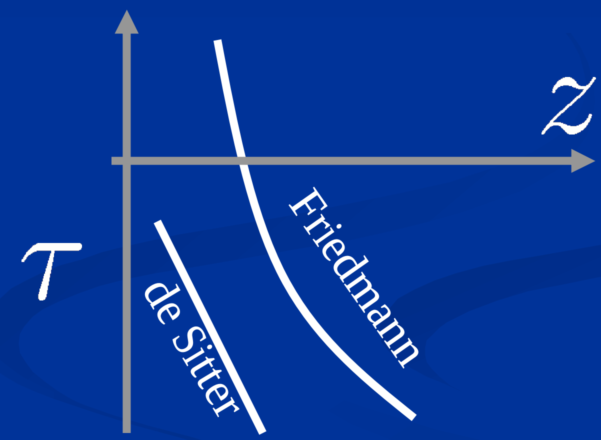
P : Gorbunov et al. JHEP10(2001)015; Kobayashi et al. PRD68(2003)044025; Kobayashi and Tanaka JCAP10(2004)015;

Cosmological Expansion

- The motion of brane determines the cosmological expansion of the brane universe.
  - Constant-speed

    = de Sitter universe
  - Deceleration

    = Friedmann universe



$$ds^2 = \frac{\ell^2}{z^2}(-d\tau^2 + d\mathbf{x}^2 + dz^2)$$

$$ds_{\mathrm{b}}^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$z_{\mathrm{b}} = \frac{\ell}{a(t)} \qquad \tau = T(t) \qquad \left(\dot{T}(t) = \frac{\sqrt{1 + H(t)^2}}{a(t)}\right)$$

Wave equation

- Tensor perturbations

$$ds^2 = \frac{\ell^2}{z^2}(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j + dz^2$$

- Wave equation

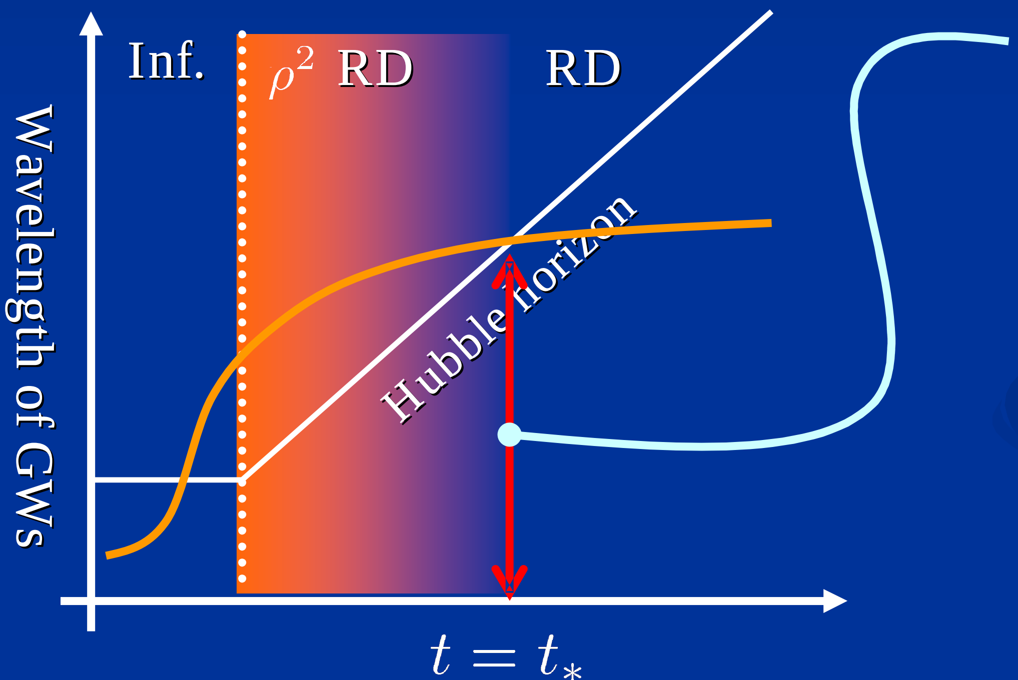
Splitting 3D spatial part of h_{ij} into Fourier modes $h(\tau, z; \mathbf{k})$

$$h(\tau, z; \mathbf{k}) = \sum_{A=+,\times} \int h_{ij} \epsilon^{ij}_{(A)} e^{i\mathbf{k}\cdot\mathbf{x}}\, d\mathbf{x}$$

$$\frac{\partial^2 h}{\partial \tau^2} - \frac{\partial^2 h}{\partial z^2} + \frac{3}{z}\frac{\partial h}{\partial z} + k^2 h = 0$$

Parameters

- The energy density at the horizon crossing



$$k = a_* H_*$$

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right)$$

$$\epsilon_* \iff k$$

$$\left(\epsilon(t) \equiv \frac{\rho(t)}{\lambda}\right)$$

- The scale of perturbation at the initial time

$$s_0 = \frac{a(t_0)H(t_0)}{k}$$

Initial conditions

Langlois et al. (2001)

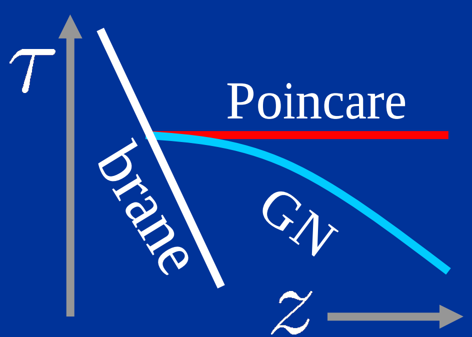
Inflation theories predict the significant suppression of KK-modes. ➔ There is only zero-mode initially.

Gaussian Normal : $h(\eta, y) = \mathrm{const.} \times (-k\eta)^{3/2} H^{(1)}_{3/2}(-k\eta)$

conformal time on brane

superhorizon scale

$$h(\eta, y) = \mathrm{const.}$$



However, in the Poincare coordinates, it is not trivial because of different spatial slicing.

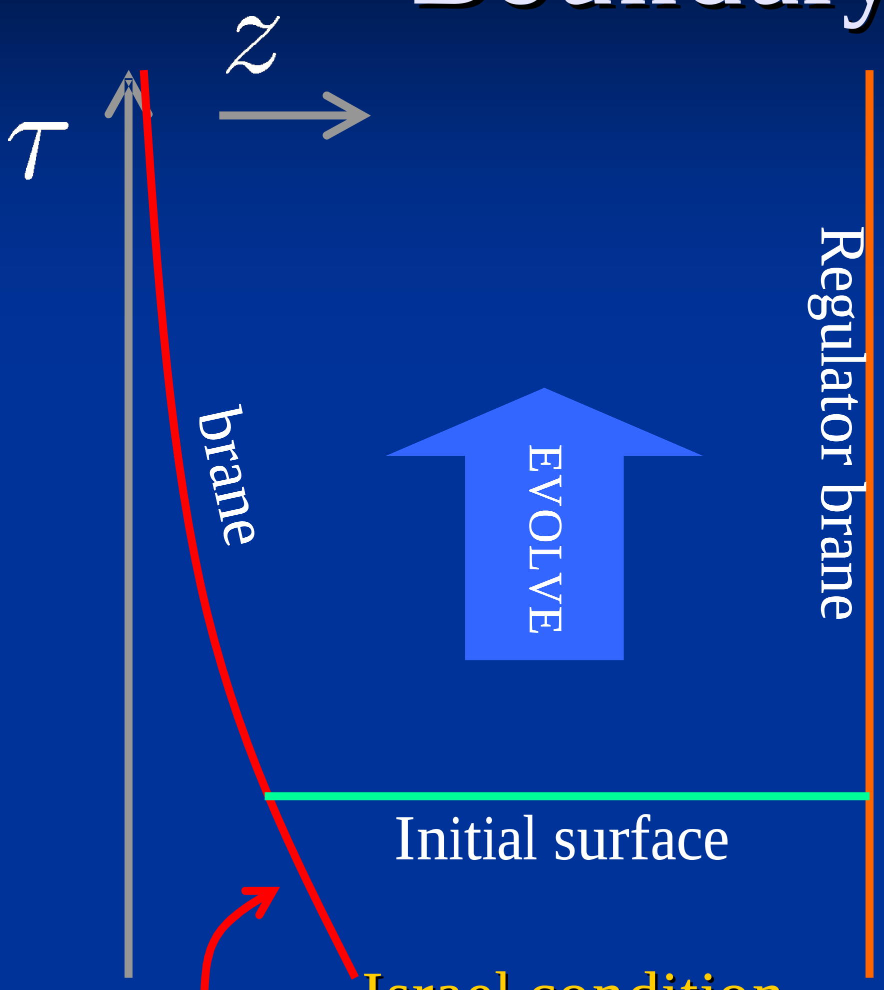
What is the zero-mode solution in the Poincare coordinates ?

Naively thinking :

$$h(\tau, z) = \mathrm{const.} \qquad \text{(superhorizon scale)}$$

Is it right ?

Boundary conditions



- Regulator brane must be far from physical brane. And we must check that the results do not depend on the position.
- Using 'Spectral method', we make simulations.

Israel condition

$$z_{\mathrm{brane}} = \frac{\ell}{a(t)} \qquad \left(\frac{\partial}{\partial \tau} - \frac{\sqrt{1+H^2\ell^2}}{H\ell}\frac{\partial}{\partial z}\right) h \Big|_{z=z(\tau)} = 0$$

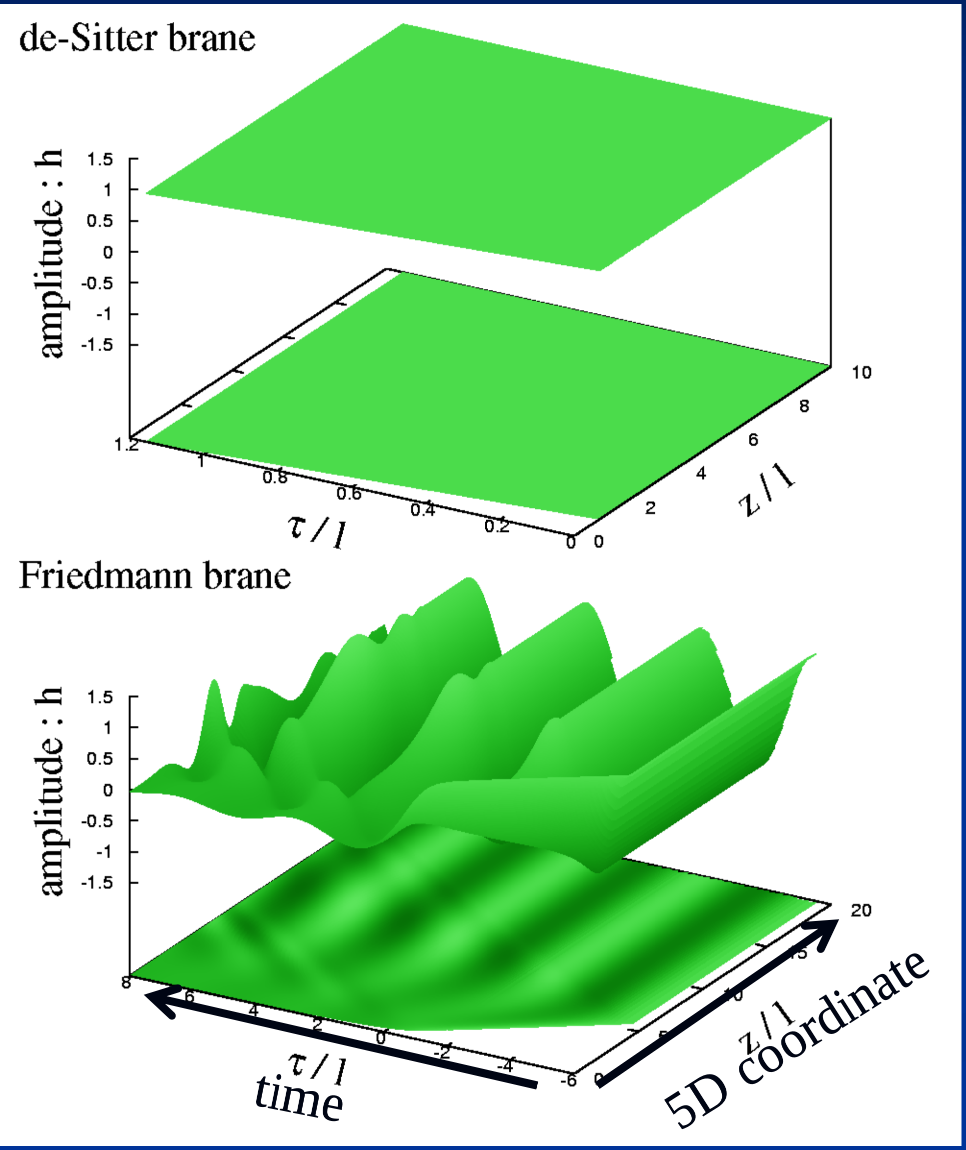
$$z_{\mathrm{reg}} = \mathrm{const.}$$

$$\frac{\partial h}{\partial z}\Big|_{z=z_{\mathrm{reg}}} = 0$$

3. Results

- ✓ Behavior in the bulk
- ✓ KK-mode excitation
- ✓ High-energy effects on spectrum
- ✓ Initial time dependence
- ✓ GWB spectrum
- ✓ EOS dependence

Behavior in the bulk



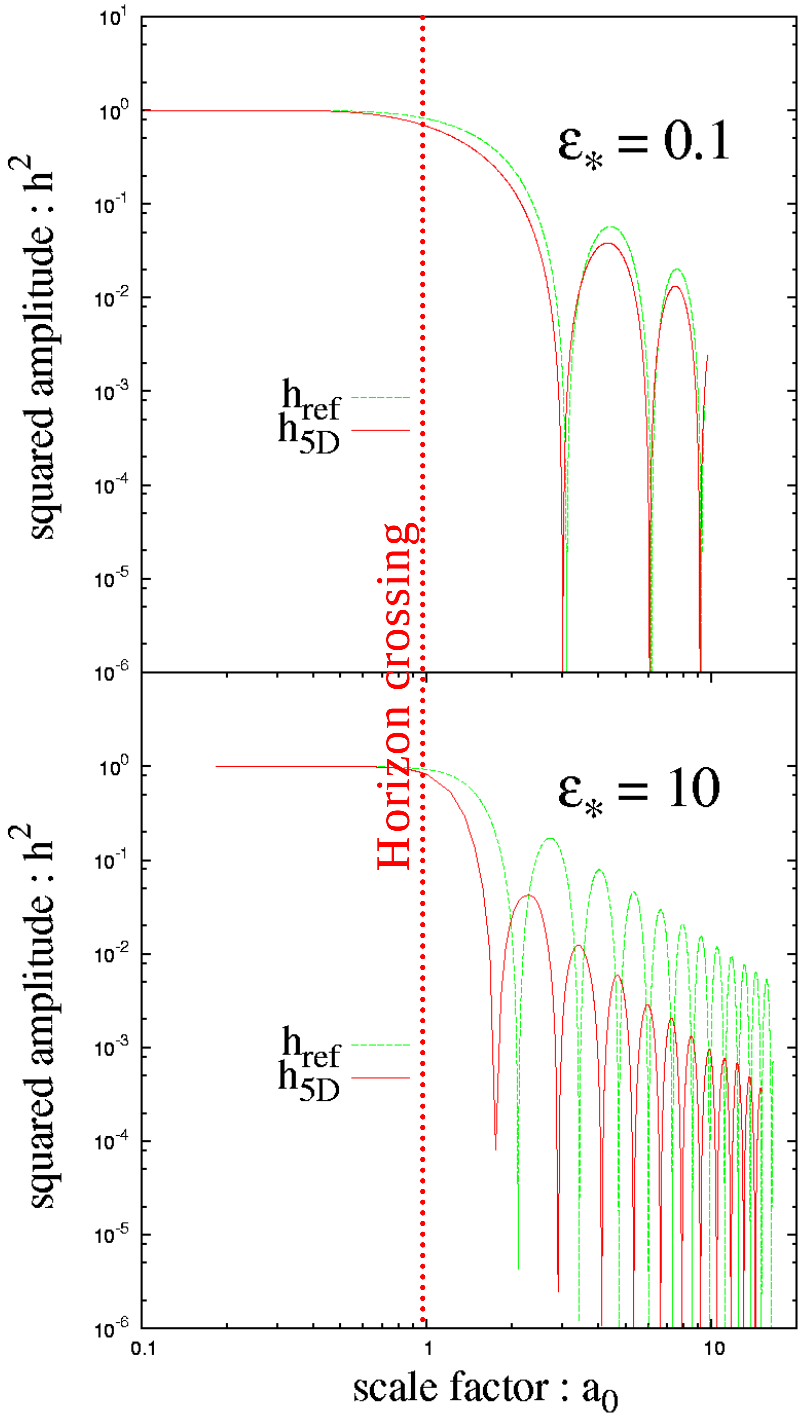
- In the case of de Sitter brane, the amplitude is frozen.

  → The initial condition

  $$h(\tau, z) = \mathrm{const.}$$

  is also valid in the Poincare coordinates.

- In the case of Friedmann brane, the brane motion causes significant excitations of KK-mode.



KK-mode excitation

- Reference wave

$$\frac{d^2 h_{\rm ref}}{dt^2} + 3H(t)\frac{dh_{\rm ref}}{dt} + \frac{k^2}{a(t)^2}h_{\rm ref} = 0$$

Including the 5D correction

Not including the KK-mode excitation

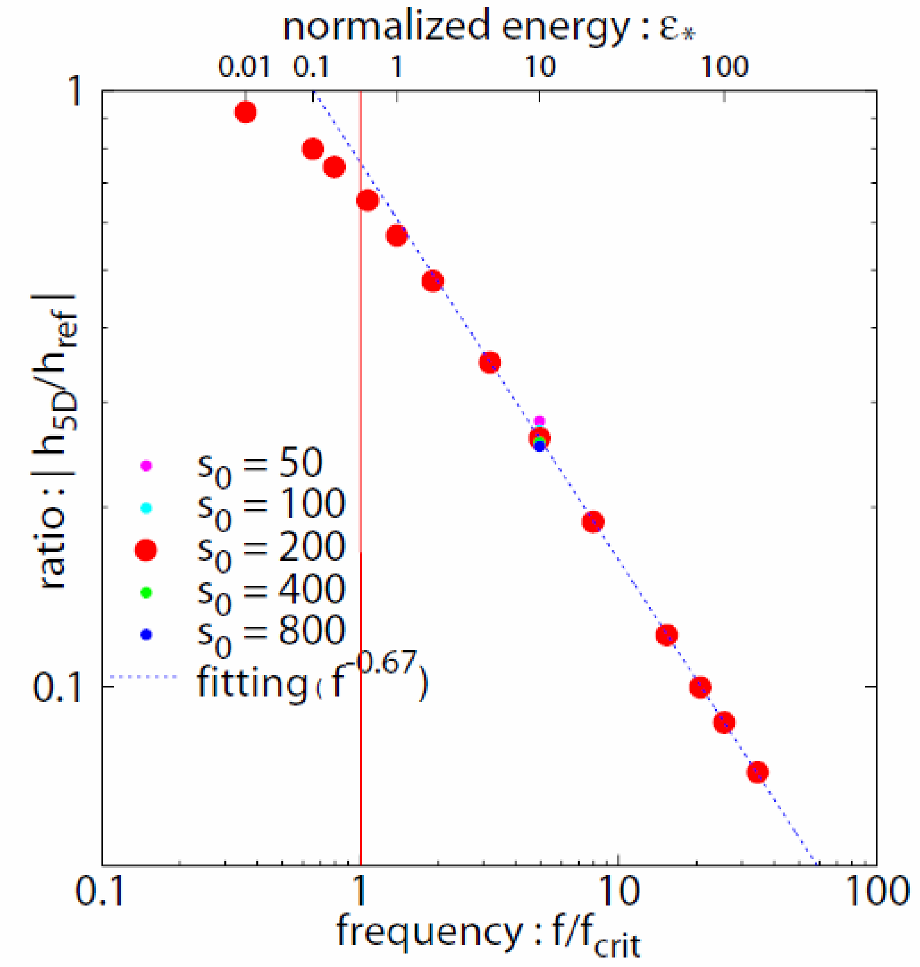
Reference wave

↕ ⋯⋯→ KK-mode excitation

simulation

- KK-mode excitation is significant in the high-energy (high frequency) case.

High-energy effects on spectrum



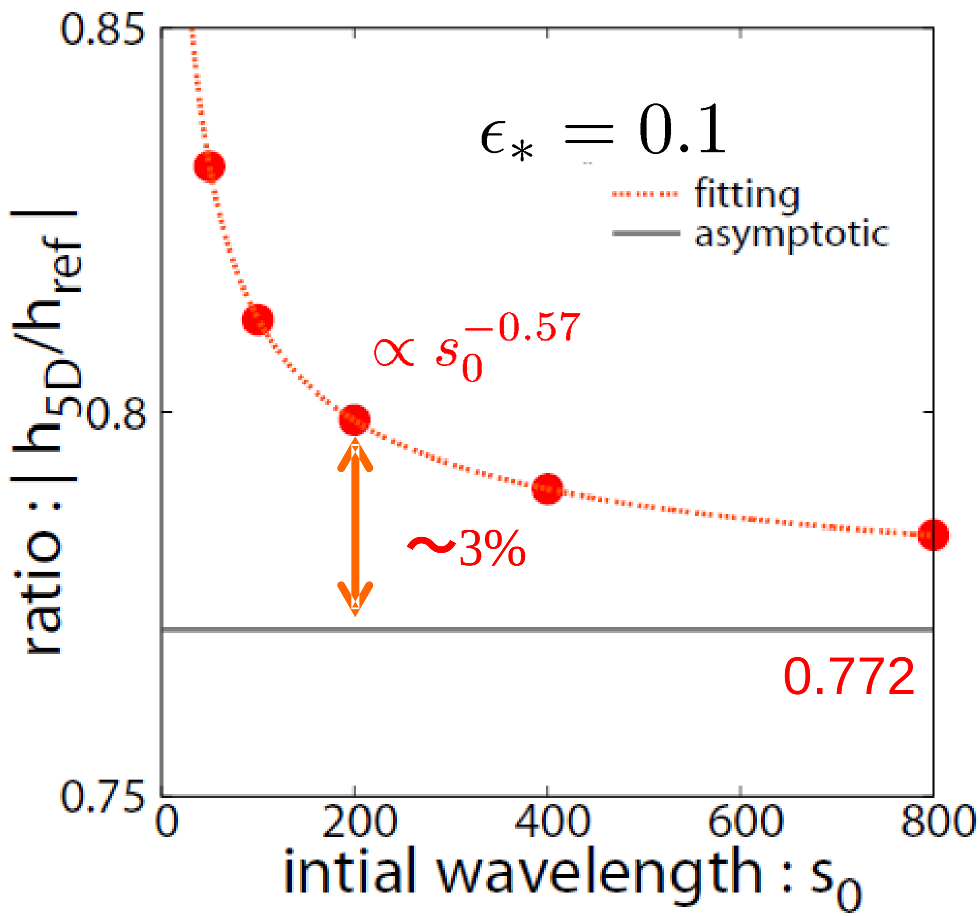
$\frac{h_{5D}}{h_{ref}}$ for various frequencies

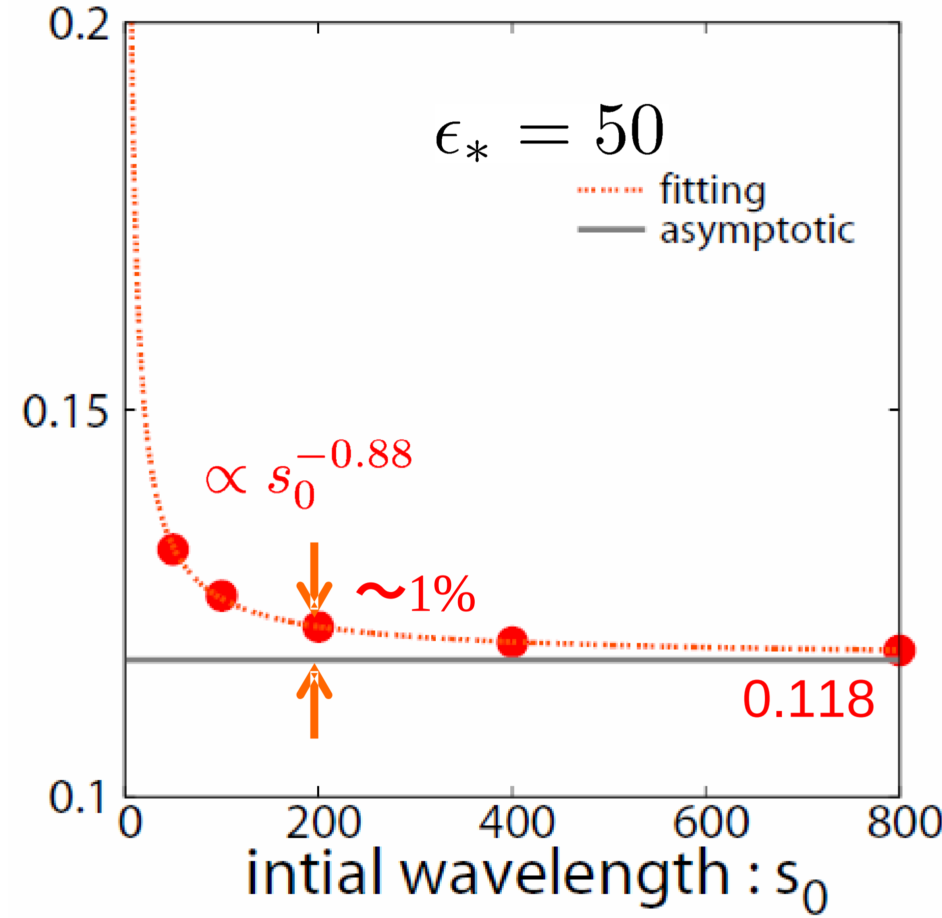
- Above the critical freq., the ratio is decreasing as $f^{-0.67}$ (fitting result)

Critical frequency : $H_* = \ell^{-1}$

- The initial time does not change the power-law index

Initial time dependence

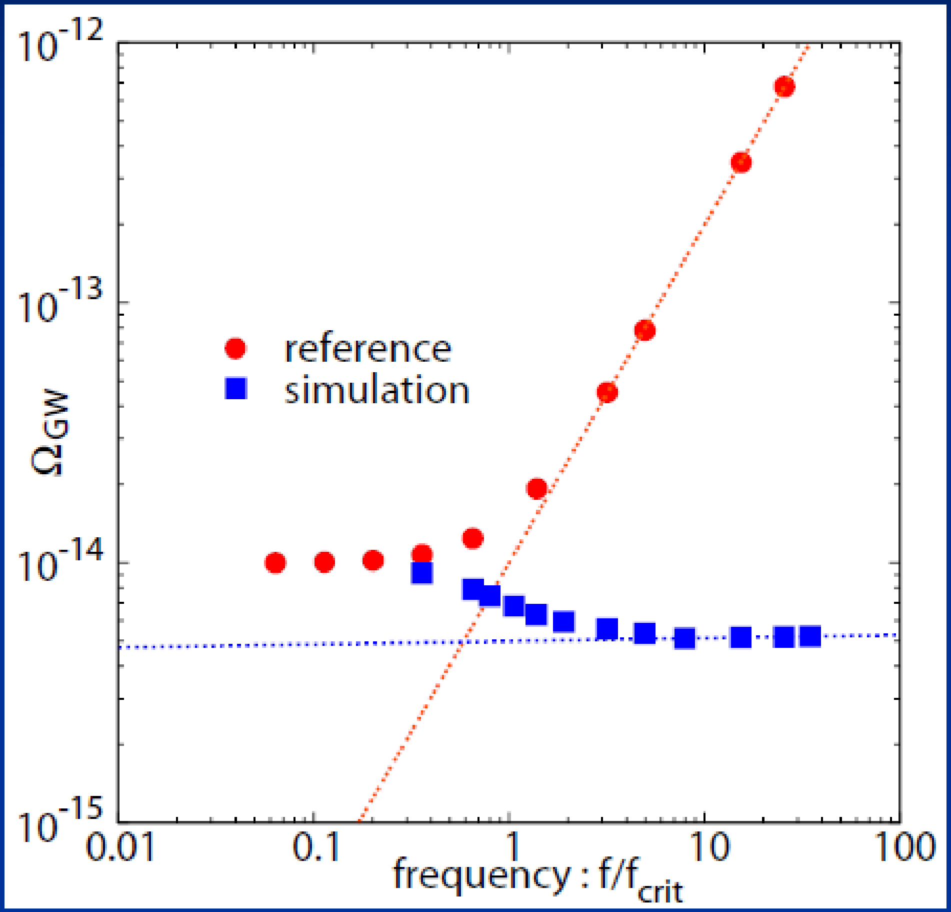




s_0 represents how the wavelength of GW is larger than the horizon scale :

$$s_0 = \frac{aH}{k}$$ at the initial time

GWB energy spectrum



Above the critical frequency

$$\Omega_{\rm GW}^{\rm ref}(f) \propto f^{4/3}$$

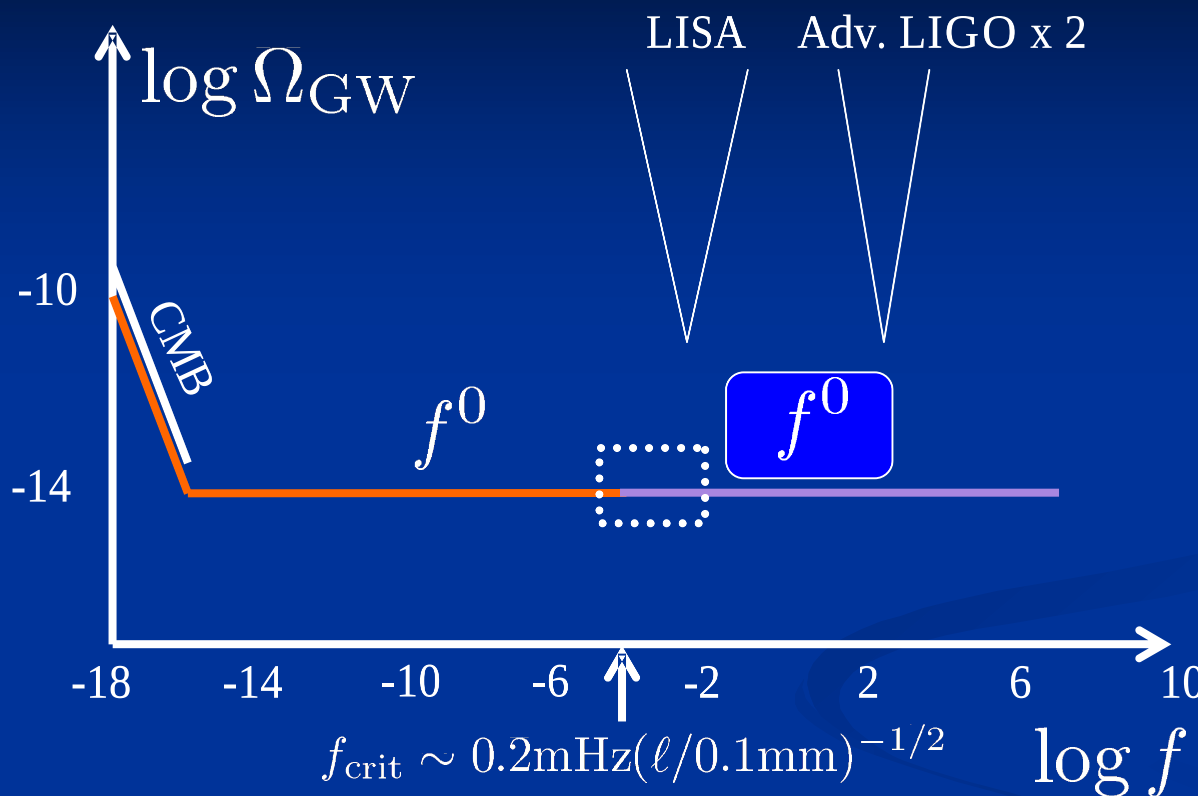
$$\frac{h_{\rm 5D}}{h_{\rm ref}} \propto f^{-0.67}$$

$$\Omega_{\rm GW}^{\rm 5D} \propto f^2 h_{\rm 5D}^2$$

$$\Omega_{\rm GW}^{\rm 5D} \propto f^0$$

Same spectrum as that of 4D theory

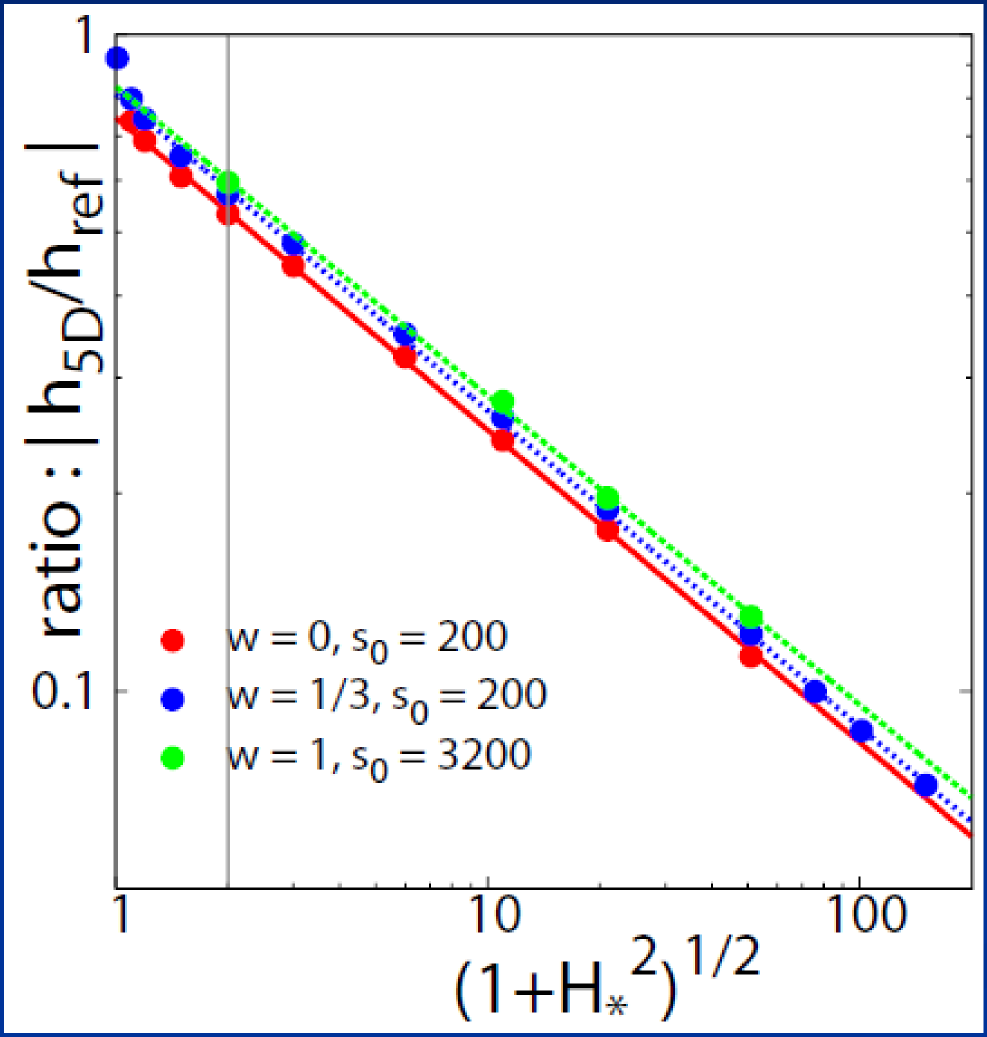
GWB energy spectrum



Extrapolating our result to higher frequency region, we obtain this spectrum. However, it is unknown whether such extrapolation can be justified.

→ further investigation

EOS dependence



Simulations for

w=0(dust),1/3(rad.),1.

In all cases, the spectrum becomes

$$\left|\frac{h_{\rm 5D}}{h_{\rm ref}}\right| \propto (1+H_*^2)^{-0.24\approx -1/4}$$

What is the physical meaning of this power-law ?

Hiramatsu, in preparation

Conclusion

- We investigated the evolution of GWs in a brane-world scenario using Poincare coordinates.
- We focused on two high-energy effects during the evolution.
  - Non-standard cosmological expansion
  - Kaluza-Klein mode excitation

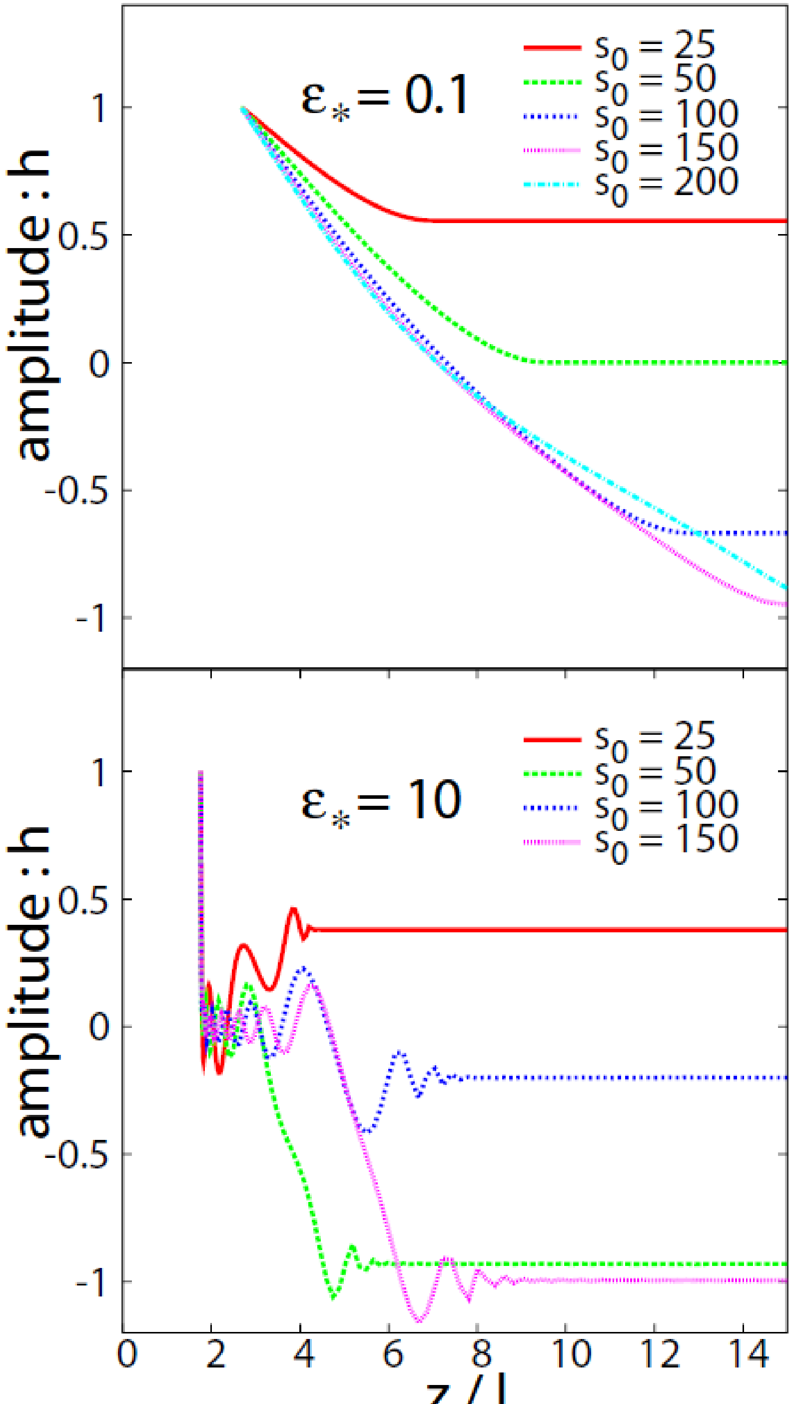
results

- We checked the validity of initial conditions which we imposed.
- The KK-mode excitation compensates the effect of non-standard cosmological expansion. → same as 4D theory
- The excitation may be insensitive to EOS.

future work

- Physical initial condition must be obtained by 5D QFT.
- Investigate the spectrum in much higher frequency region by analytical and numerical approach.
- Our work may provide a clue to understand the relationship between the motion of brane and the KK-mode excitation.

appendix



Dependence on initial time

← Snapshots of GWs in the superhorizon scale

S0 represents how the wavelength of GW is larger than the horizon scale :

$$s_0 = \frac{aH}{k} \quad \text{at the initial time}$$

- In the bulk, the behavior is quite sensitive to initial time.
- On the brane, amplitude tends to converge if we set the initial time early enough.

Formulae 1

- General solution in Poincare coordinates

$$\frac{\partial^2 h}{\partial \tau^2} - \frac{\partial^2 h}{\partial z^2} + \frac{3}{z}\frac{\partial h}{\partial z} + k^2 h = 0$$

$$h(\tau, z) = \int_0^\infty dm \left\{ \tilde{h}_1(m) z^2 H_2^{(1)}(mz) e^{i\omega\tau} + \text{c.c.} \right\}$$

- Friedmann brane cosmology

$$H^2 = \frac{\kappa_4^2}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) \qquad \dot{\rho} = -\gamma H \rho \qquad \gamma = 3(1+w)$$

$$a_0(t) = a_* \left(\frac{\gamma t^2 + 2t\ell}{\gamma t_*^2 + 2t_*\ell}\right)^{1/\gamma} \epsilon(t) = \frac{\rho(t)}{\lambda} = \frac{2\ell^2}{\gamma^2 t^2 + 2\gamma t \ell}$$

Formulae 2

- Gaussian normal coordinates

$$ds^2 = -n^2(t,y)\,dt^2 + a^2(t,y)\,d\mathbf{x}^2 + dy^2$$

$$a(t,y) = a_0(t)\left\{\cosh\left(\frac{y}{\ell}\right) - \left(1+\frac{\rho}{\lambda}\right)\sinh\left(\frac{y}{\ell}\right)\right\}$$

$$n(t,y) = e^{-y/\ell} + (2+3w)\frac{\rho}{\lambda}\sinh\left(\frac{y}{\ell}\right)$$

- De Sitter brane

$$ds^2 = \frac{\ell^2}{z^2}(-d\tau^2 + d\mathbf{x}^2 + dz^2)$$

$$\tau = \eta\cosh(y/\ell)$$
$$z = -\eta\sinh(y/\ell)$$

$$ds^2 = \frac{\ell^2}{\sinh^2(y/\ell)}\left\{\frac{1}{\eta^2}(-d\eta^2 + d\mathbf{x}^2) + \ell^{-2}dy^2\right\}$$