Parameterizing the Power Spectrum: Beyond the Truncated Taylor Expansion

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(Based on the work in collaboration with
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Outline

- Traditional Approach:

  Truncated Taylor Series Parameterization of the Power Spectrum

- Improved parameterization and its inflationary motivation

- Likelihood Analysis

- Conclusion/Discussion

Traditional Approach: Truncated Taylor Expansion of the Power Spectrum

$$\ln P(k) = \ln P_* + (n_* - 1)\ln\left(\frac{k}{k_*}\right) + n_*' \ln^2\left(\frac{k}{k_*}\right)$$

$$n - 1 \equiv \frac{d\ln P}{d\ln k}, n' \equiv \frac{dn}{d\ln k}$$

$$\left|(n_* - 1)\ln\left(\frac{k}{k_*}\right)\right| >> \left|n_*' \ln^2\left(\frac{k}{k_*}\right)\right|$$

$\Leftarrow$Not a trivial assumption, in particular for $|\ln k/k_*| \gtrsim 1$.

In fact, the current observations are consistent with

$$\left|(n_* - 1)\ln\left(\frac{k}{k_*}\right)\right| \sim \left|n_*' \ln^2\left(\frac{k}{k_*}\right)\right|$$

$$\boxed{\ln P(k) = \ln P_* + (n_* - 1)\ln\left(\frac{k}{k_*}\right) + n'_* \ln^2\left(\frac{k}{k_*}\right)}$$

- Standard slow-roll approximation (often valid for simple single inflation models)

A)Small: |n-1|<<1 $\boxed{\Leftarrow\text{Required by Observation}}$

B)Slowly varying: |n-1|>>|n'|>>|n''|>>…

$\boxed{\Leftarrow\text{ Required by neither of observation or theory}}$

Non-slowly varying small-roll parameters are generic, in particular, for multi-component inflation models

$\boxed{\text{Spectrum at high k is forced to be same as that at low k}}$

- General slow-roll approximation (E.Stewart '02) can accommodate |n-1|$\gtrsim$|n'|$\gtrsim$|n''|…

(Standard slow-roll is a special case of the general slow-roll)

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Improved parameterization and its inflationary motivation

$$\boxed{\ln P(k) = \ln P_* - Ak^{\nu}, n-1 = -\nu A k^{\nu}}$$

$$\boxed{\ln P(k) = \ln P_* + \frac{(n_* - 1)^2}{n'_*}\left[\left(\frac{k}{k_*}\right)^{\frac{n'_*}{n_*-1}} - 1\right]}$$

(The standard slow-roll corresponds to $\boxed{\nu = \left|\frac{n'_*}{n_* - 1}\right| << 1}$.

$\boxed{\ln P(k) = \ln P_* + (n_* - 1)\ln\left(\frac{k}{k_*}\right) + n'_* \ln^2\left(\frac{k}{k_*}\right)}$ for $\boxed{\left|(n_* - 1)\ln\left(\frac{k}{k_*}\right)\right| >> \left|n'_* \ln^2\left(\frac{k}{k_*}\right)\right|}$)

Our parameterization covers not only slow-roll inflation but also a much wider class of inflation models, using the same number of free parameters as the traditional truncated Taylor series parameterization

$$\boxed{n - 1 \equiv \frac{d\ln P}{d\ln k} = (n_* - 1)\left(\frac{k}{k_*}\right)^{\frac{n'_*}{n_*-1}}}$$

$$\boxed{n' \equiv \frac{dn}{d\ln k} = n'_*\left(\frac{k}{k_*}\right)^{\frac{n'_*}{n_*-1}}}$$

Improved parameterization and its inflationary motivation

$$\boxed{\ln P(k) = \ln P_* - Ak^\nu , n - 1 = -\nu A k^\nu}$$

General slow-roll formula: |n-1|≳|n'|≳|n''|…
(Stewart '02, Lee et al '05)

$$\boxed{\ln P = \mathrm{C} - B\xi^{-\nu}} \qquad \boxed{\left(\xi \equiv -\int \frac{dt}{a}\right)}$$

(concrete particle theory motivated example, e.g. Kadota&Stewart '03)

General slow-roll formula gives

$$\boxed{\ln P(k) = C - Ak^\nu \text{ for } \nu < 2} \qquad \boxed{\ln P(k) = C - Ak^2 \text{ for } \nu \geq 2}$$

$$\boxed{\frac{n'_*}{n_* - 1} = \begin{cases} \nu & \text{for } \nu < 2 \\ 2 & \text{for } \nu \geq 2 \end{cases}}$$

($n_*'/(n_*-1) \leq 2$ would be a consistency check for our analytical justification of the form of our parameterization)

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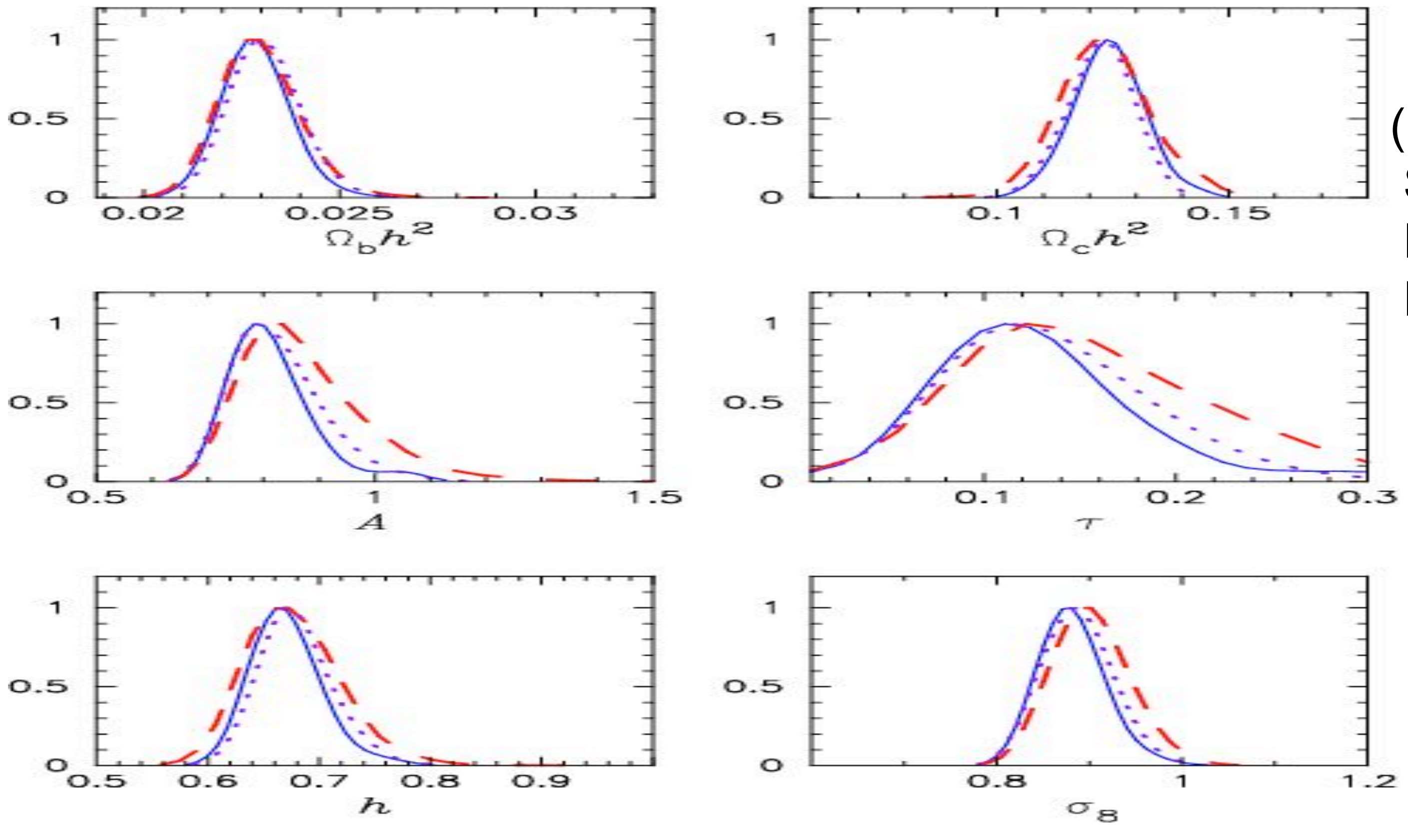
Likelihood analysis for the cosmological parameters

- Markov-Chain Monte-Carlo(MCMC) using the data from WMAP, ACBAR, CBI, VSA, SDSS and Lyman-α (covering up to k~5h/Mpc)

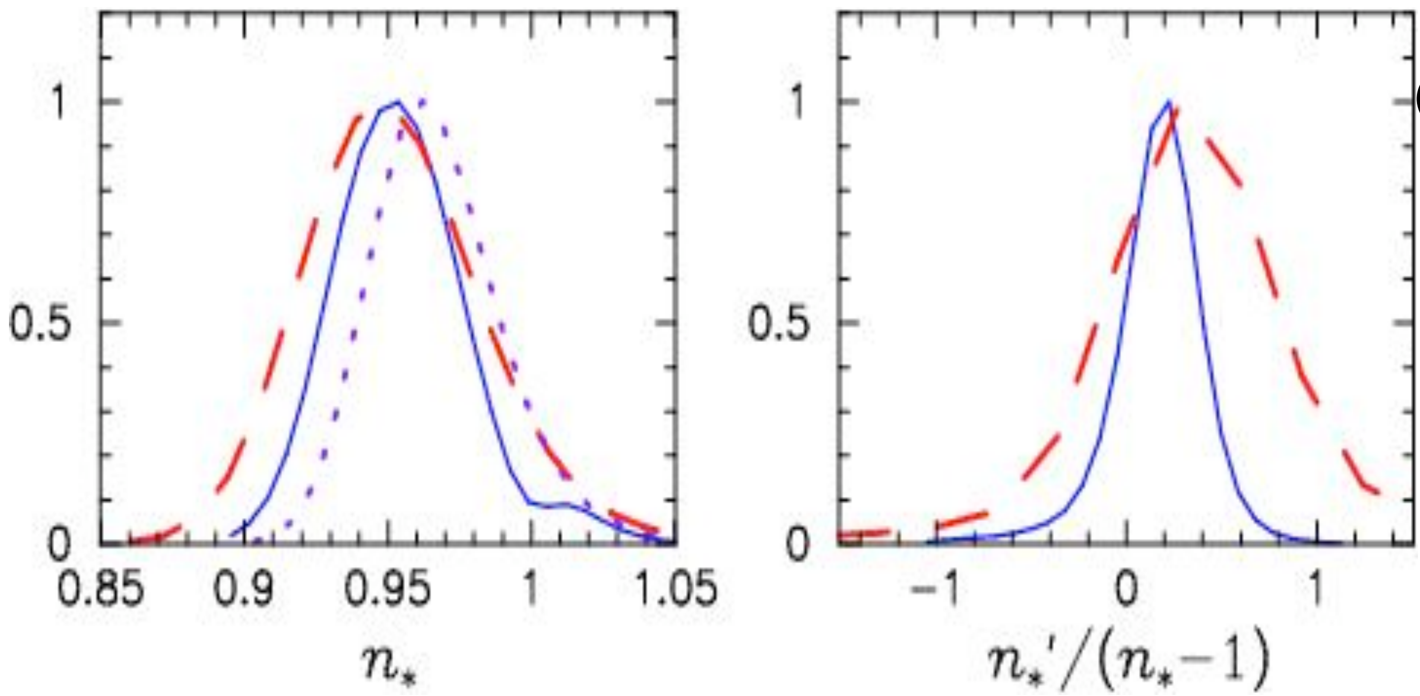


(Marginalized 1-D PDF
Solid:Ours,
Dashed:Taylor,
Dotted: Constant n)

Not a big difference within the currently available data, but would change once we get more data at higher k with better precision

Likelihood analysis for the inflationary parameters

Simple single component inflation models where the standard slow-roll approximation apply often leads to $|n'| \sim |n-1|^2 << |n-1|$



(Marginalized 1-D PDF
Solid:Ours,
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(Central values: ours $n'_* = -0.0087 \pm 0.0084$, trunceted Taylor $n'_* = -0.019 \pm 0.014$)

Our improved parameterization and even the truncated Taylor series expansion show that $|n'| \sim |n-1|$ is still consistent with the data.

$\left|(n_* - 1)\ln\left(\frac{k}{k_*}\right)\right| >> \left|n'_* \ln^2\left(\frac{k}{k_*}\right)\right|$ is not valid (our data covers $\Delta \ln k \sim 10$).

Conclusion/Discussion

- Presented the improved parameterization of the power spectrum which reduces to the traditional truncated Taylor series parameterization for slow-roll case, but have a better extension for the non-slow-roll cases.

- The analysis of current data indicates it is not consistent to parameterize the power spectrum via the truncated Taylor expansion.
- e.g. WMAP data analysis can be inconsistent.
- e.g. $|n-1| \sim n'$ is consistent with the data.
- Cosmological parameter ($h, \sigma_8, \Omega_b, \Omega_{cdm} \ldots$) estimations don't get altered for the current data, but the inflationary parameters do.

- Can be of great interest for modeling a small scale structure where no reason to assume a flat spectrum.