Constraining the Cosmological Parameters by the Cosmic Inversion Method

Noriyuki Kogo (Osaka/YITP),

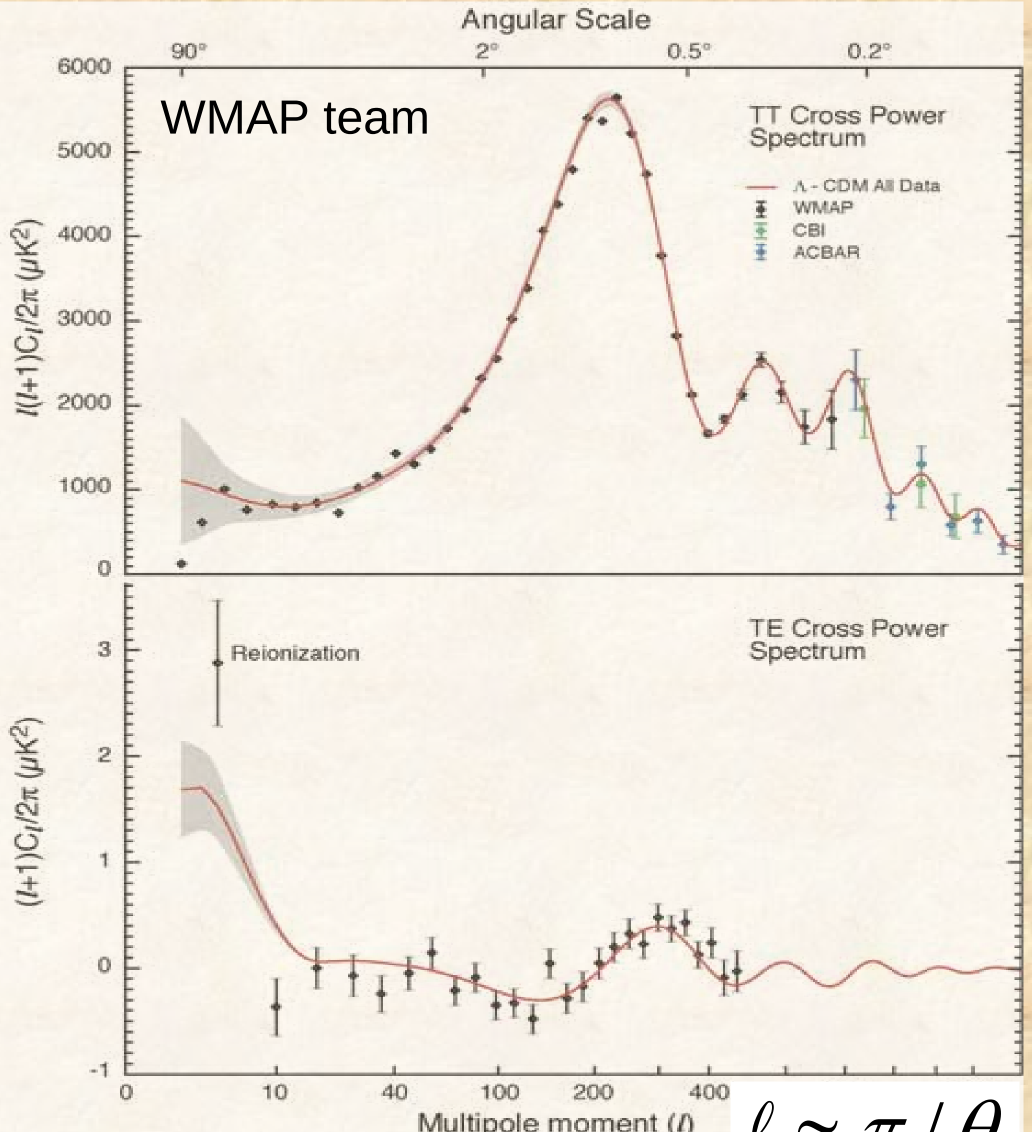
Misao Sasaki (YITP),

Jun’ichi Yokoyama (RESCEU)

astro-ph/0504471

1. Introduction

Parameter Estimation



$$\ell \approx \pi / \theta$$

Primordial spectrum

Assumption: $k^3 P(k) = A k^{n_s - 1}$

Best-fit parameters

$$h = 0.72 \pm 0.05$$
$$\Omega_b h^2 = 0.024 \pm 0.001$$
$$\Omega_m h^2 = 0.14 \pm 0.02$$
$$\tau = 0.166^{+0.076}_{-0.071}$$
$$A = 0.9 \pm 0.1$$
$$n_s = 0.99 \pm 0.04$$

“Precisely determined.”

But, obtained values depend on the assumption regarding a functional form of $P(k)$.

Cosmological Parameters & $P(k)$

The reason why the cosmological parameters are precisely determined is that the functional space of $P(k)$ is a priori restricted by the assumption of a simple functional form of $P(k)$.

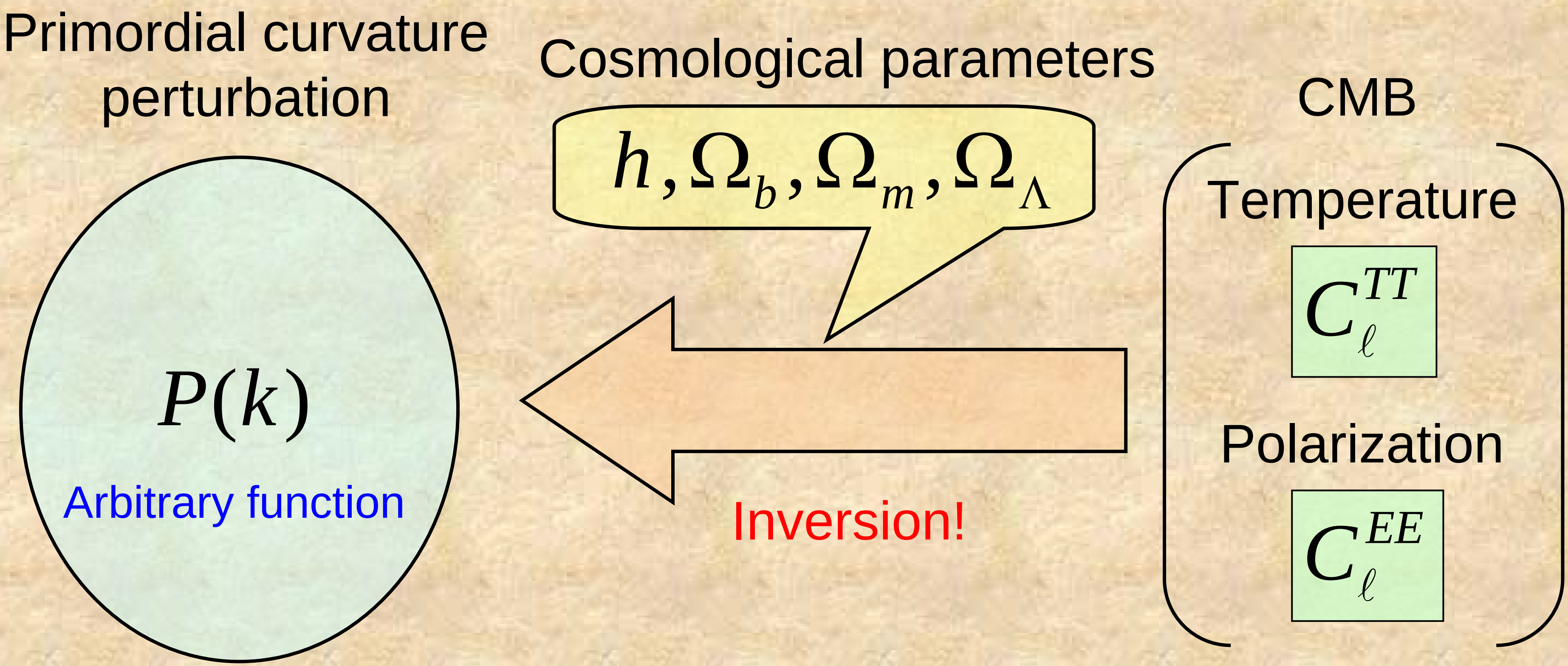
Precision of the cosmological parameters ⇔ Degree of freedom of $P(k)$

Trade-off

- W. H. Kinney, PRD **63**, 043001 (2001)
- T. Souradeep et al., astro-ph/9802262

Is it possible to constrain the cosmological parameters without any assumption on the functional form of $P(k)$?

"Cosmic Inversion"



Is it possible to constrain the cosmological parameters by requiring that resultant $P(k)$ is independent of the contribution of the polarization in our method?

2. Inversion Method

- M. Matsumiya, M. Sasaki, & J. Yokoyama, PRD **65**, 083007 (2002)
- M. Matsumiya, M. Sasaki, & J. Yokoyama, JCAP **0302**, 003 (2003)
- N. K., M. Sasaki, & J. Yokoyama, PRD **70**, 103001 (2004)

CMB Anisotropies

Assumptions: adiabatic fluctuations, Gaussianity, scalar modes only

- Temperature fluctuations

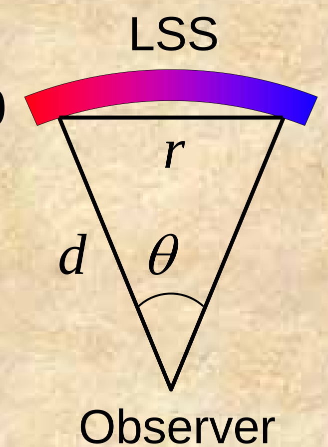
$$\Theta(\hat{\boldsymbol{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{T} Y_{\ell m}(\hat{\boldsymbol{n}}),$$

- Polarization

$$\left(Q \pm iU\right)(\hat{\boldsymbol{n}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left(a_{\ell m}^{E} \pm i a_{\ell m}^{B}\right) {}_{\pm 2}Y_{\ell m}(\hat{\boldsymbol{n}}).$$

B-modes vanish.

$z_* \approx 1089$



Angular power spectrum

$$C_\ell^{X\bar{X}} \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left\langle a_{\ell m}^{X\,*} a_{\ell m}^{\bar{X}} \right\rangle = \frac{2}{\pi} \int_0^\infty k^2 dk\, K_\ell^{X\bar{X}}(\eta_0, k) P(k), \quad X, \bar{X} = T \text{ or } E.$$

Primordial spectrum: $P(k)$

Kernel (Transfer functions): $K_\ell^{X\bar{X}}(\eta_0, k)$

- Thin LSS approximation

  Perform time integration of the transfer functions within the thickness of the last scattering surface (LSS).

- Small angle approximation

$$r = 2d \sin\frac{\theta}{2} \ll d, \quad \Leftrightarrow \quad \ell \geq O(10).$$

Inversion Formula

➢ Temperature + Polarization (TT + **α** EE)

$$-k^2 f^2(k)P'(k) + \left[-2k^2 f(k) f'(k) + kg^2(k) + \alpha k h^2(k)\right]P(k)$$
$$= S^{TT}(k) + \alpha S^{EE}(k).$$

$$S^{TT}(k) \equiv 4\pi \int_0^\infty dr \frac{1}{r}\frac{\partial}{\partial r}\left\{ r^3 \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \frac{2\ell+1}{4\pi} \frac{C_\ell^{TT,\,\mathrm{obs}}}{b_\ell^{TT,\,(0)}} P_\ell\left(1-\frac{r^2}{2d^2}\right)\right\} \sin kr,$$

$$S^{EE}(k) \equiv 4\pi \int_0^\infty dr\, r \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \frac{2\ell+1}{4\pi} \frac{(\ell-2)!}{(\ell+2)!} \frac{C_\ell^{EE,\,\mathrm{obs}}}{b_\ell^{EE,\,(0)}} P_\ell\left(1-\frac{r^2}{2d^2}\right) \sin kr.$$

α : free parameter

Correction factor

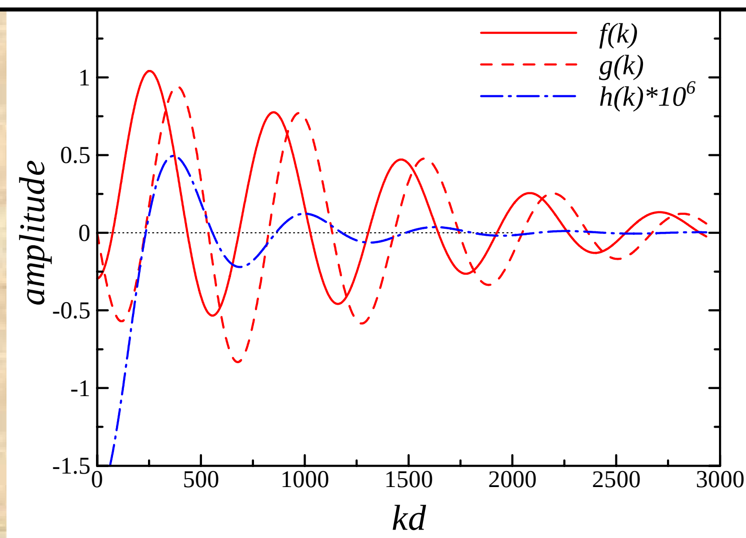
$$b_\ell^{XX,\,(0)} \equiv \frac{C_\ell^{XX,\,\mathrm{exact}\,(0)}}{C_\ell^{XX,\,\mathrm{approx}\,(0)}}, \quad \leftarrow \quad P^{(0)}(k) \propto k^{-3}.$$

∵ Almost independent of $P(k)$.

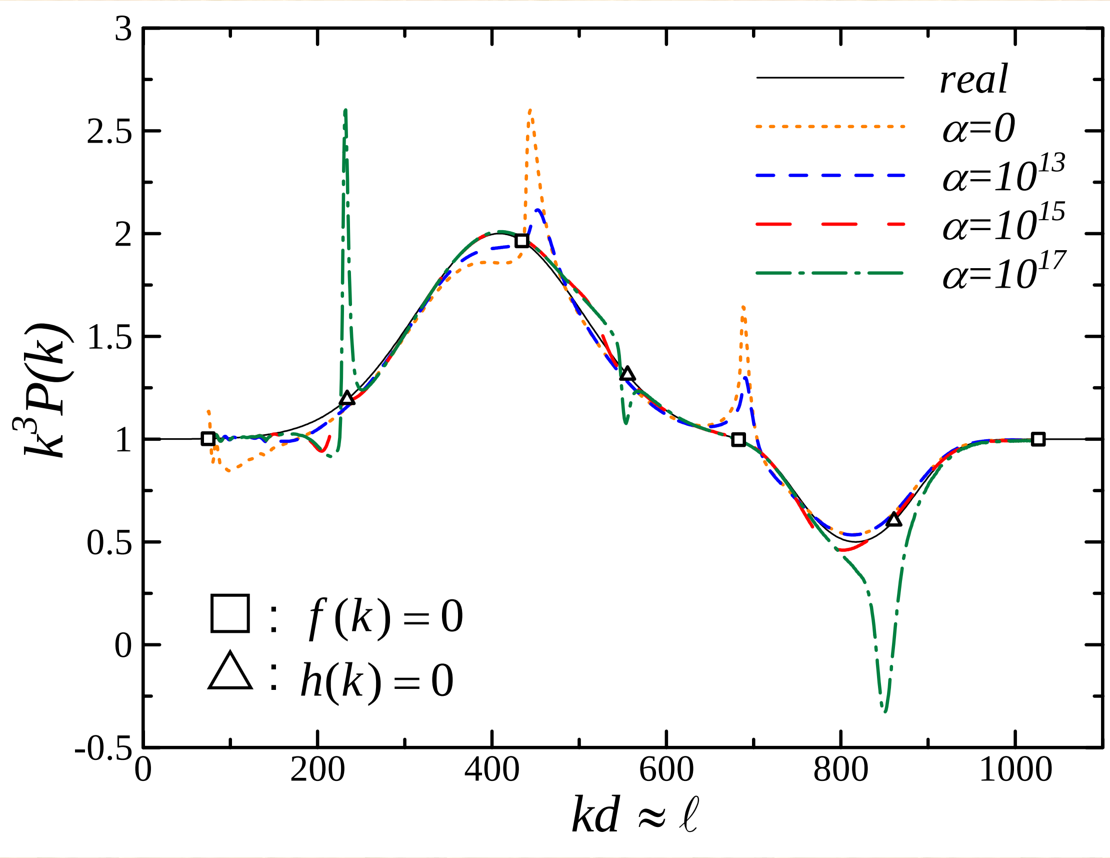
Boundary Conditions

$$P(k_s) = \frac{S^{TT}(k_s) + \alpha S^{EE}(k_s)}{k_s\left[g^2(k_s) + \alpha h^2(k_s)\right]}, \quad \text{for } f(k_s) = 0.$$

Time-integrated transfer functions



Test



Assumptions:

- Real $P(k)$ has a peak and dip.
- The cosmological parameters are exactly known.

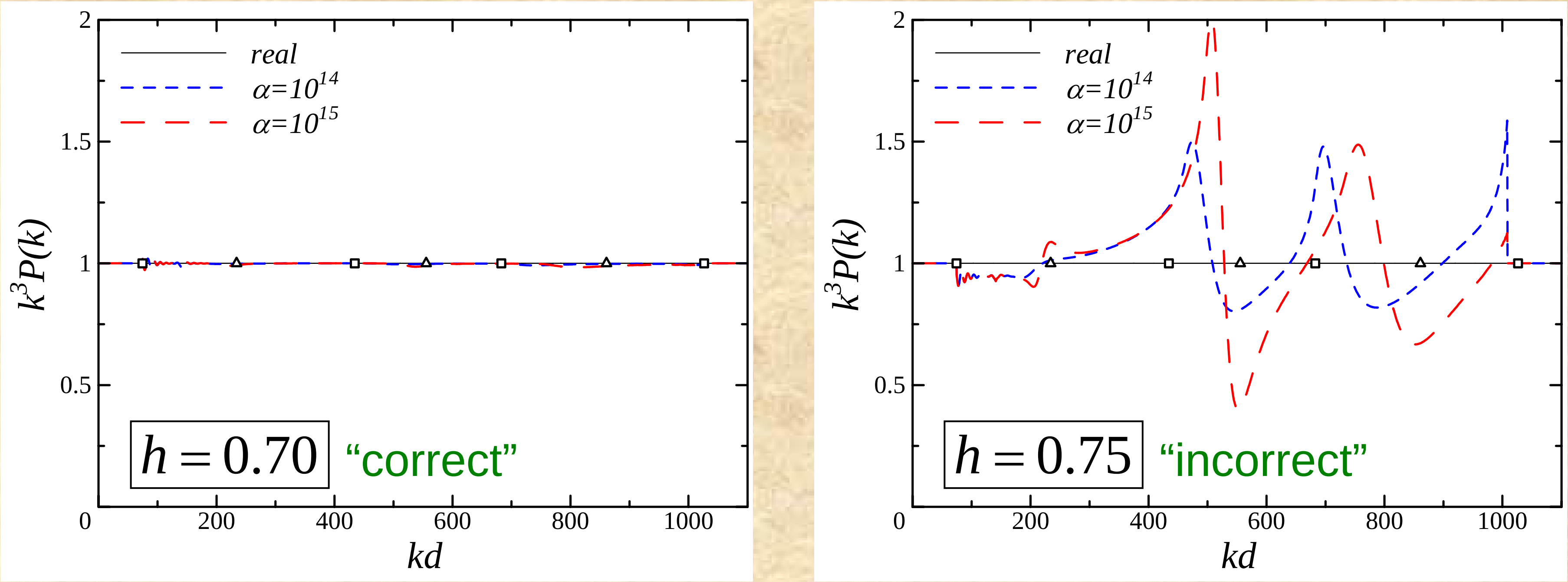
- $\alpha \to 0$: TT only
  Unstable near $f(k)=0$ (TT singularities).

- $\alpha \to \infty$: EE only
  Unstable near $h(k)=0$ (EE singularities).

- $\alpha = 10^{14-15}$: TT〜**α** EE
  No spurious sharp peak or dip near the singularities.
  ∵ Positions of the TT and EE singularities are different.

3. Constraining Cosmological Parameters

Strategy



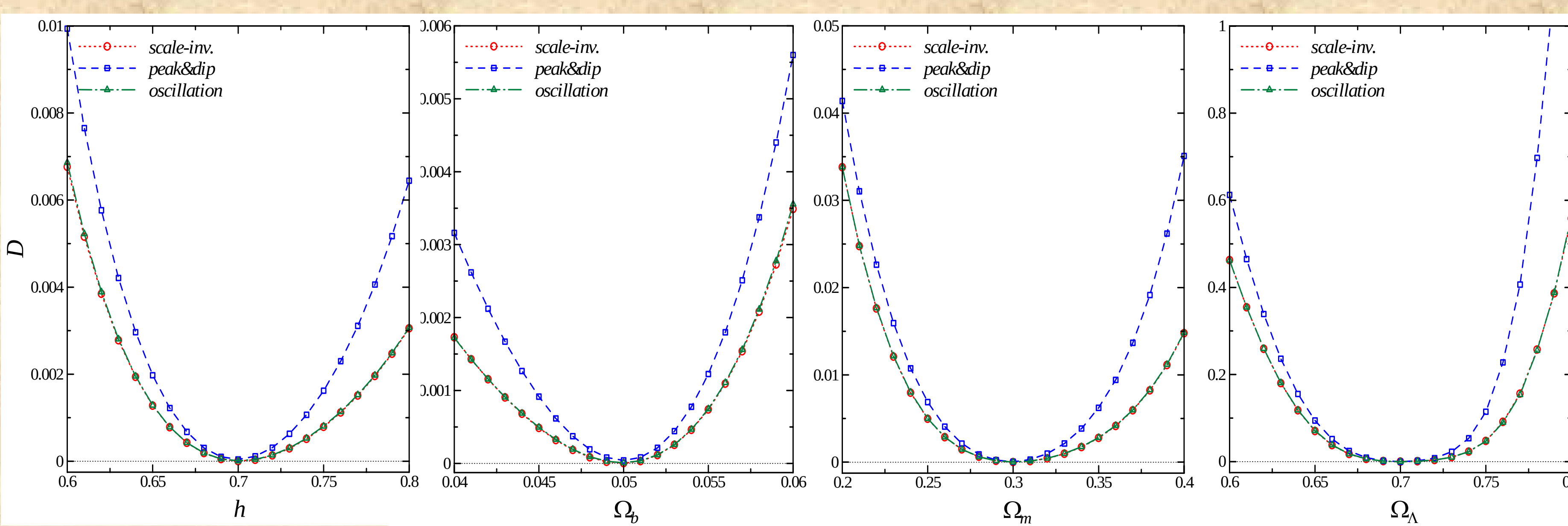
$$D \equiv \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \left[k^3 P_{\alpha_1}(k) - k^3 P_{\alpha_2}(k) \right]^2, \quad \alpha_1 \neq \alpha_2,$$

$P_\alpha(k)$: reconstructed $P(k)$ for a certain value of $\boldsymbol{\alpha}$.

We speculate that D takes its minimum value with respect to variation of the cosmological parameters at correct values of the these parameters.

Dependence of D

- With no observational error.
- For three different shape of $P(k)$.
- Assumed cosmological parameters are $h = 0.70,\ \Omega_b = 0.050,\ \Omega_m = 0.30,\ \Omega_\Lambda = 0.70.$
- Vary one of $h,\ \Omega_b h^2,\ \Omega_m h^2,\ \Omega_K = 1 - \Omega_m - \Omega_\Lambda,$ individually, with the others fixed.



We find that regardless of the shape of $P(k)$, D as a function of each cosmological parameter takes its minimum value at correct value of that parameter in any case.

Error Estimation

Make simulated data by drawing a random number from a Gaussian distribution with the PLANCK observational error around a theoretical spectrum.

↓

Calculate the value of D by varying the cosmological parameters and finding the minimum of D.

↓

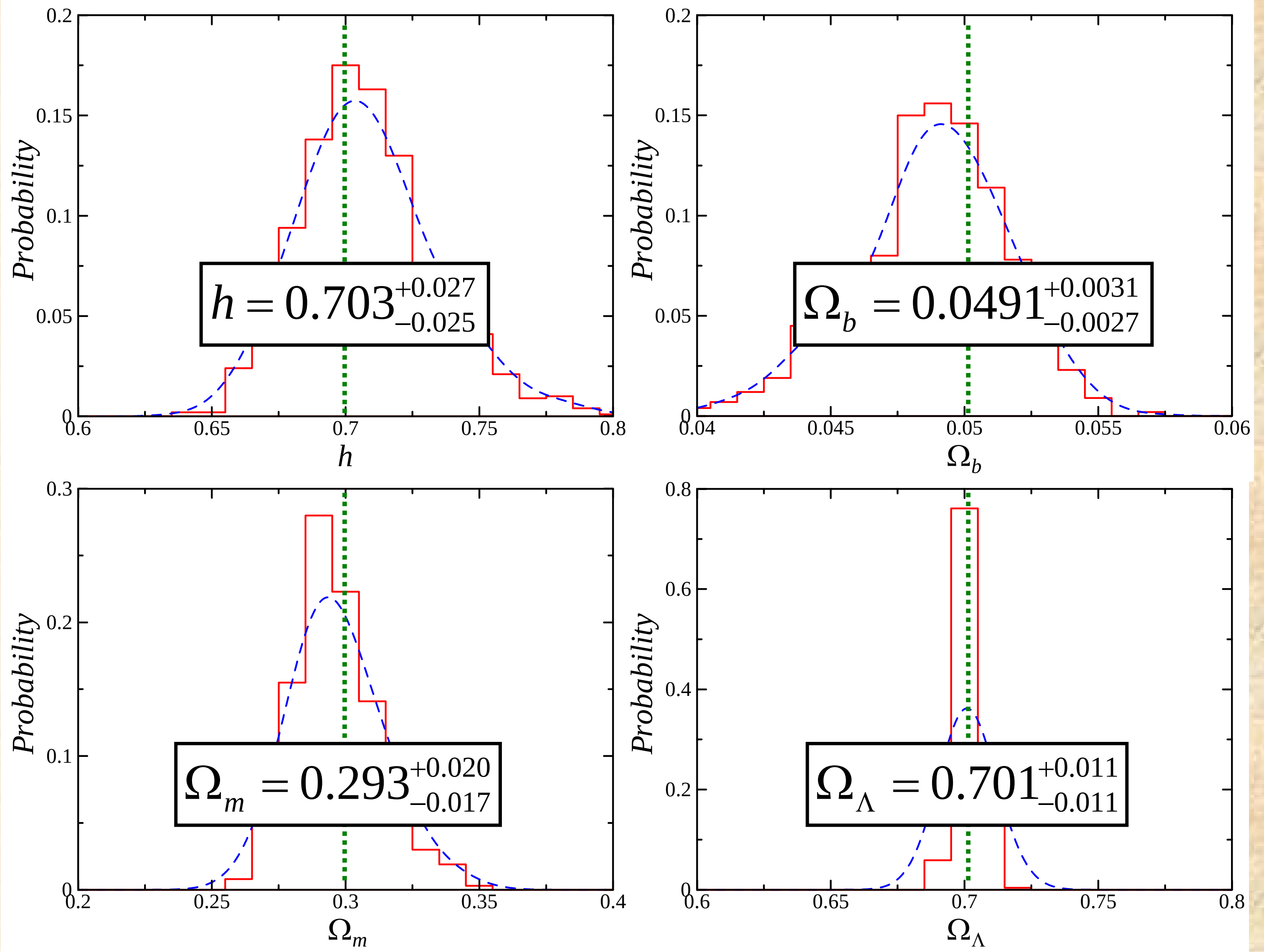
Construct histograms of the values of the cosmological parameters at the minimum of D from the 1000 realizations.

↓

Estimate probability distributions and the errors of the cosmological parameters.

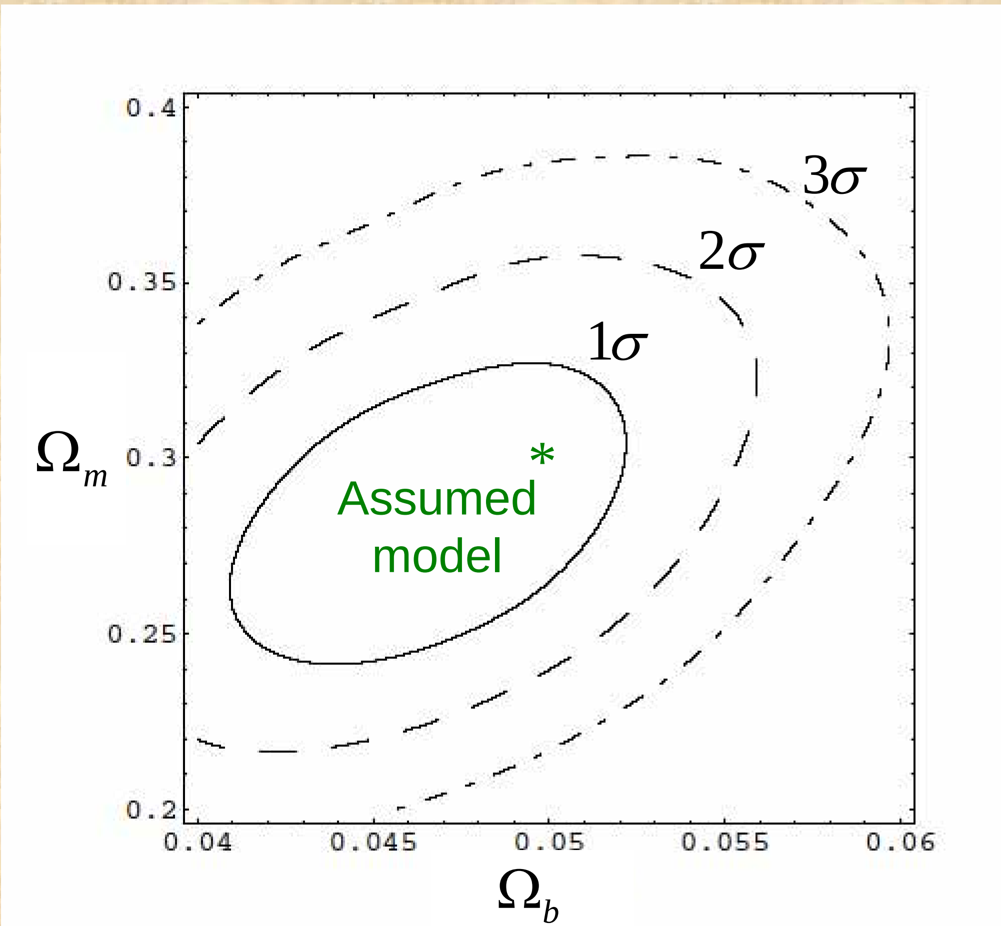
1D Parameter Search

Vary one of $h,\ \Omega_b h^2,\ \Omega_m h^2,\ \Omega_K$, individually, with others fixed.

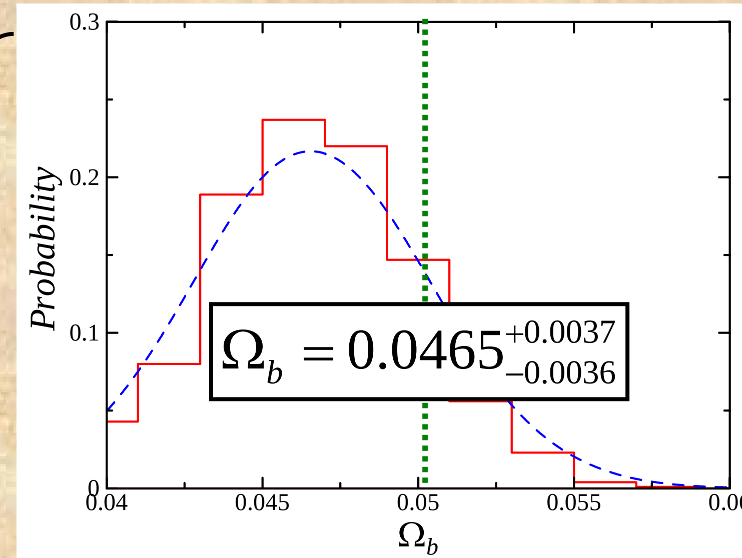


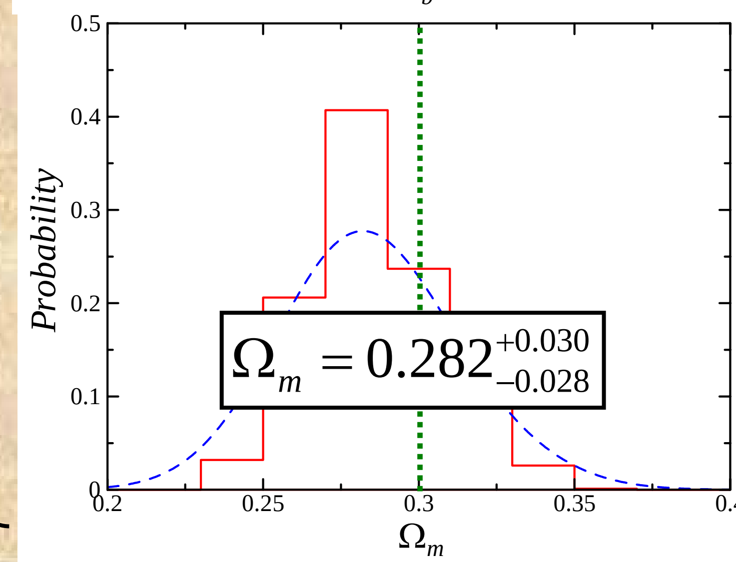
2D Parameter Search

Vary Ω_b, Ω_m with h, Ω_K fixed.



Projected probability distributions





4. Summary

• We have investigated the possibility to constrain the cosmological parameters in the context of the reconstruction of $P(k)$.

• We required that the reconstructed $P(k)$ dose not depend on the contribution of the polarization.

• We found that the cosmological parameters can be constrained without any assumption on the functional form of $P(k)$.

Future Issues

- Full multi-dimensional analysis.
- Tensor modes (B-mode polarization).