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Direct reconstruction of the quintessence potential

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Microsoft-free presentation

Aim: Obtain optimal constraints on the quintessence potential. We assume that quintessence has already been tested against alternative dark energy models.

Strategy: Parametrize the quintessence potential and predict observables as functions of those parameters. Constrain using MCMC parameter fitting.
For now, we focus on supernova data.

Standard approach:

Parametrize equation of state, eg

$$w(z) = w_0 + w_1 z + \ldots$$

Determine those and relate to the potential using some approximate relations.

Huterer and Turner 1999; Starobinsky 1998

Our approach:

Parametrize the quintessence potential

$$V(\Phi) = V_0 + V_1\Phi + V_2\Phi^2 + \ldots$$

We take $\Phi=0$ at the present epoch, and the field may roll in either direction.

Cf Direct reconstruction of the inflaton potential, Grivell and Liddle 2000

Features

- Numerical computation of observational quantities yields optimal constraints on the potential, and automatically gives error covariances.
- Present velocity of the scalar field is an arbitrary parameter, also to be fit from data.
- Treatment is very general and includes

  Scalar field rolling on a flat potential.

  Scalar field rolling uphill on a potential.

  Scalar field oscillations.

Connection to SNIA data

Data: 157 SNe from "gold" sample of Riess et al.

Fitting distance modulus

$$\mu_i - \mu(z_i;P) = \mu_i^* + \eta - 5\log D_{\mathrm{L}}(z_i;P)$$

where

$$\mu_i = m - M$$

$$\mu_i^* = m - M^*$$

$$\eta = 5\log(H_0\ \mathrm{Mpc}) + \Delta M - 25$$

$$\Delta M = M - M^*$$

$$D_{\mathrm{L}} = d_{\mathrm{L}} / H_0$$

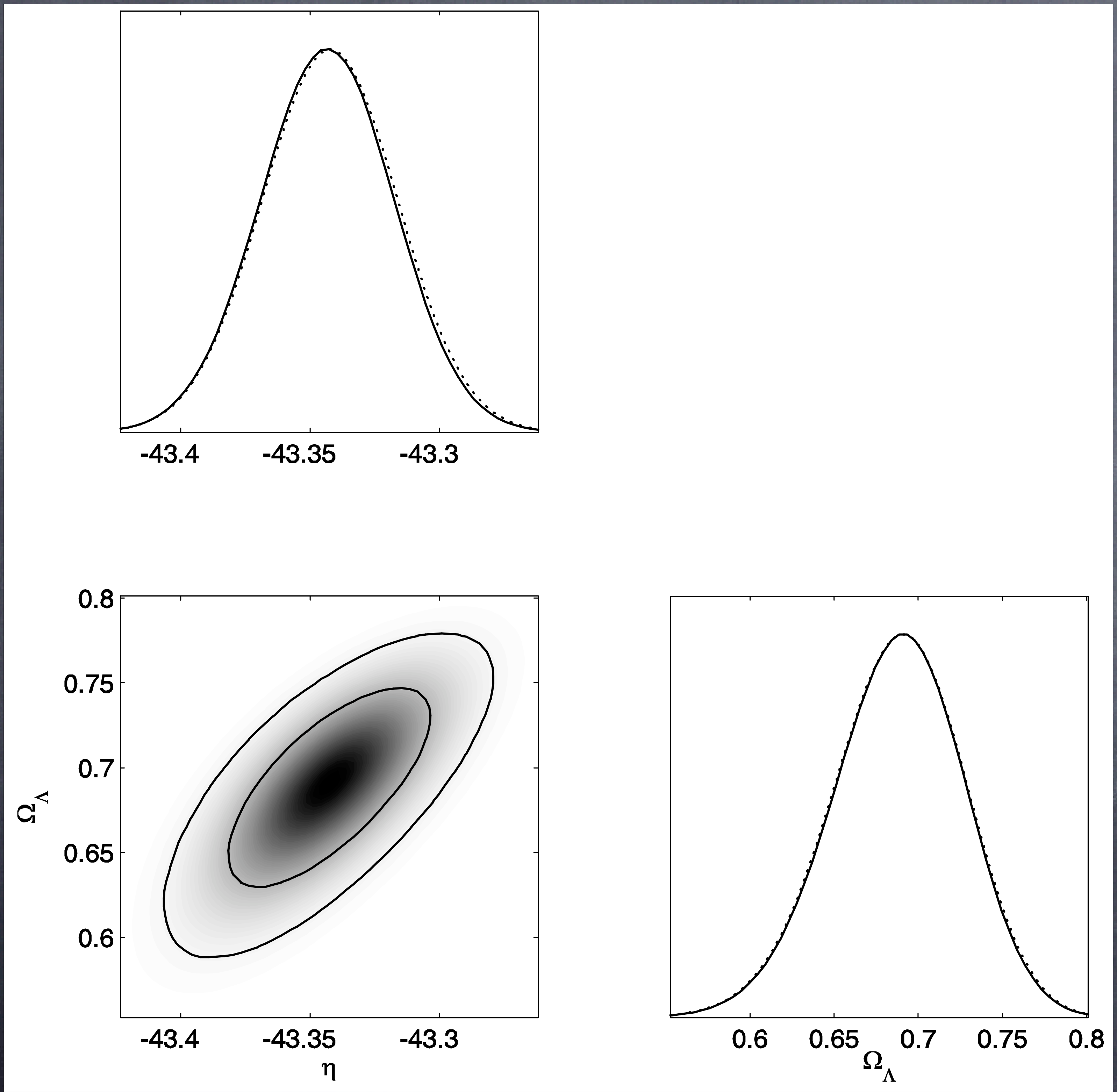
M=absolute magnitude of SN1A

M*=some fiducial estimate of M

$$\boxed{P = (\eta, \dot{\Phi}_0, V_0, V_1, \cdots, V_{D-3})}$$

Results: cosmological constant

D = 2

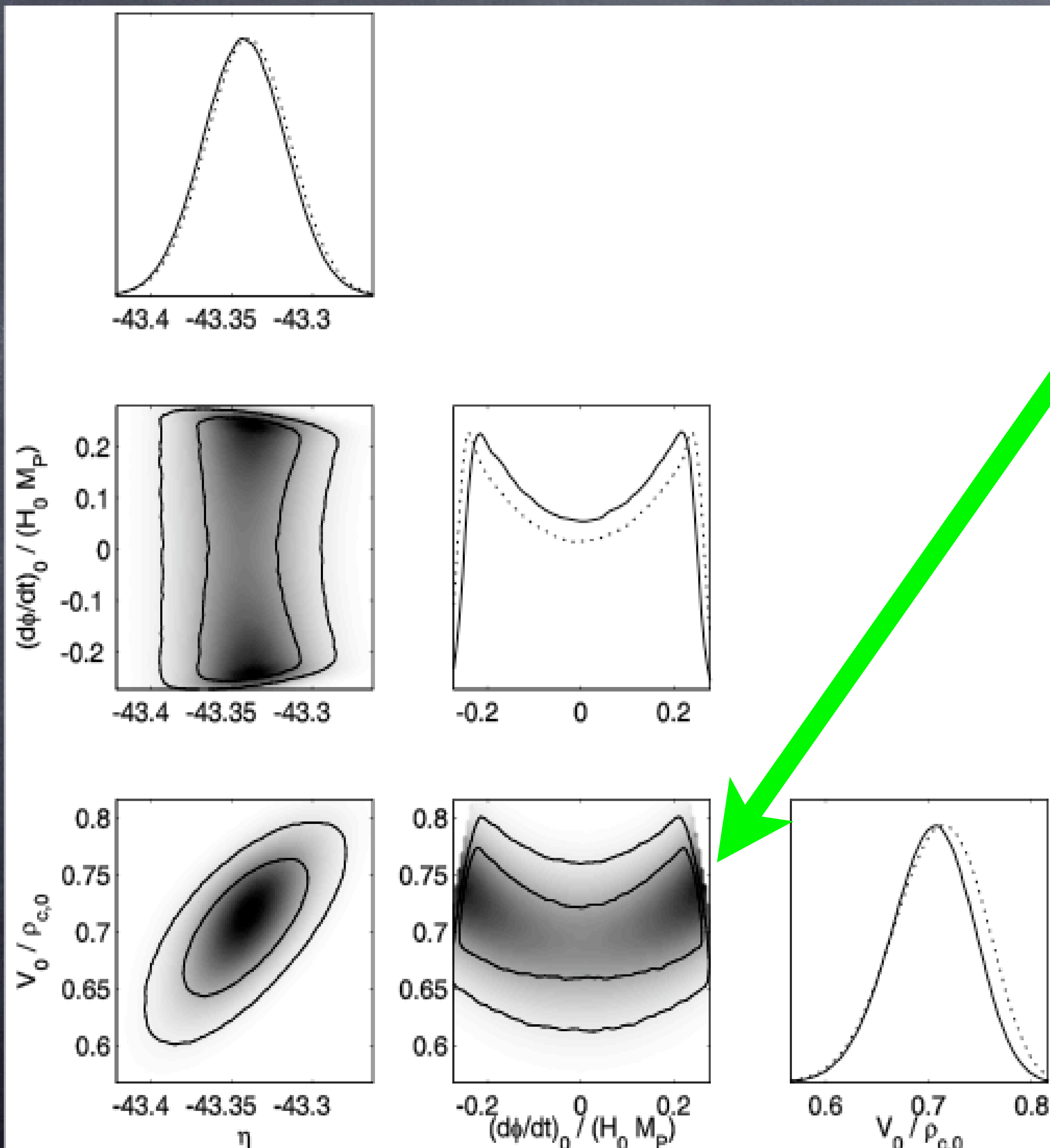


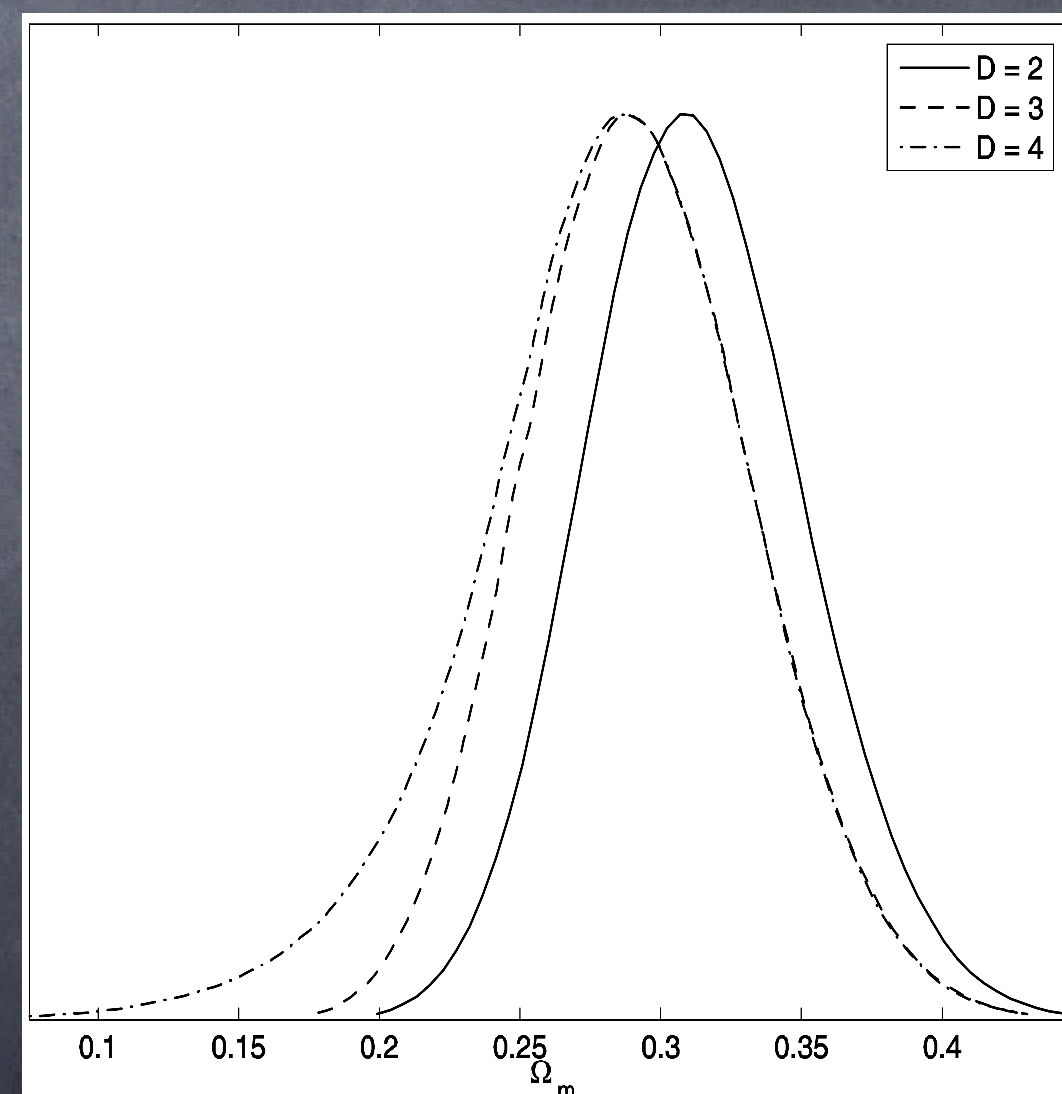
Nothing surprising

Results: flat potential, rolling field

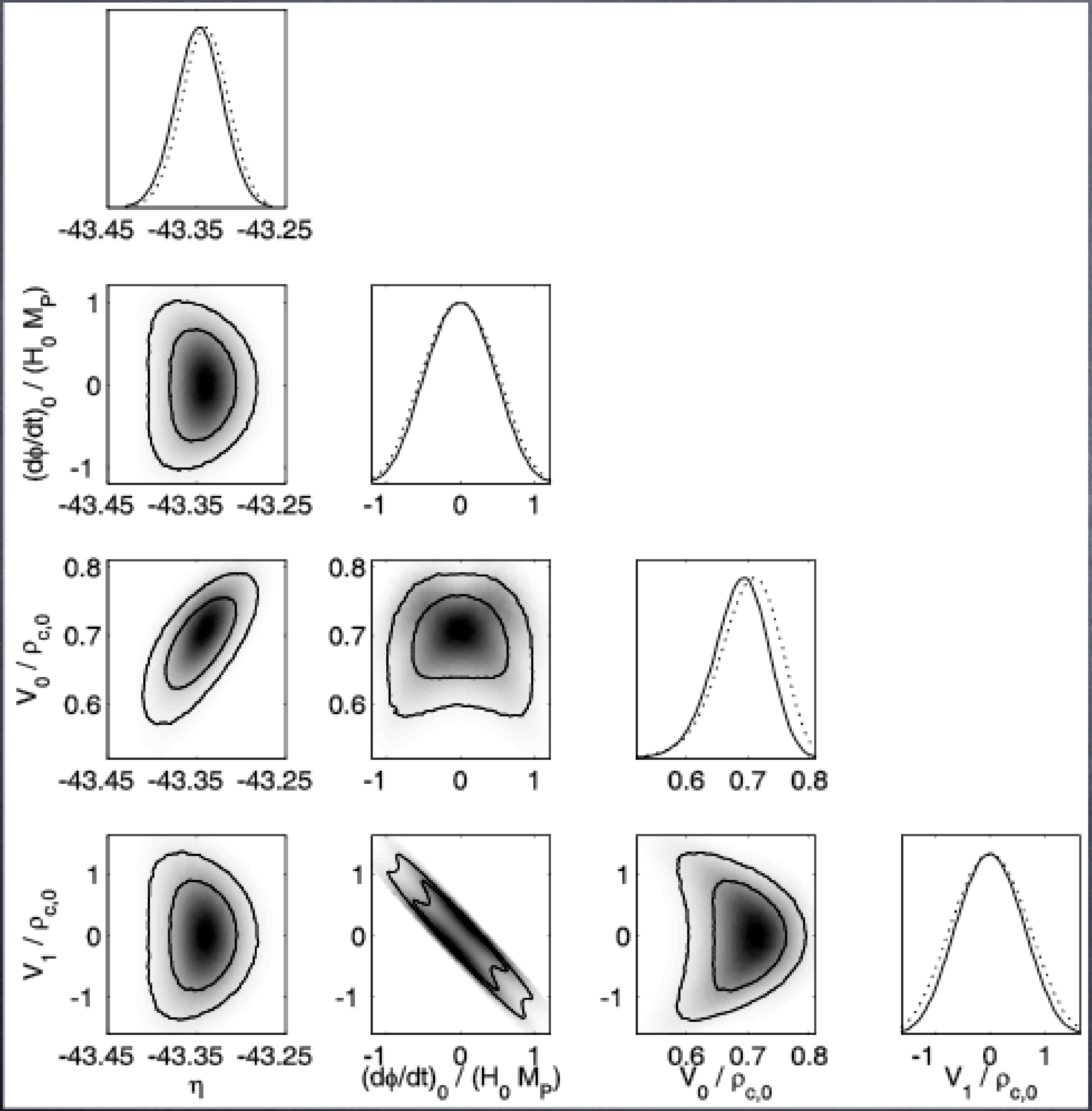
D = 3

Mild preference for field to roll, cut off by prior



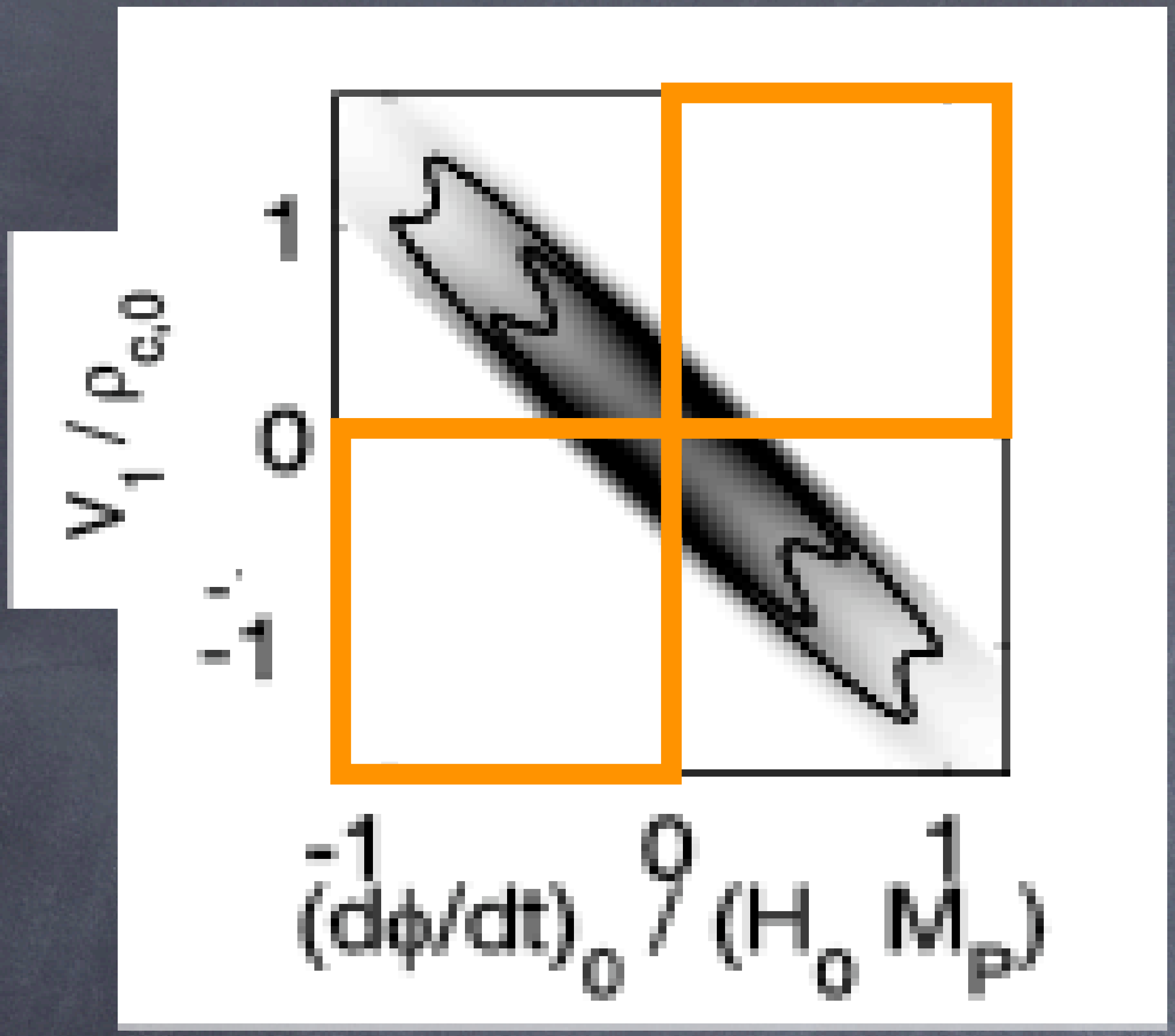


Results: linear potential, rolling field

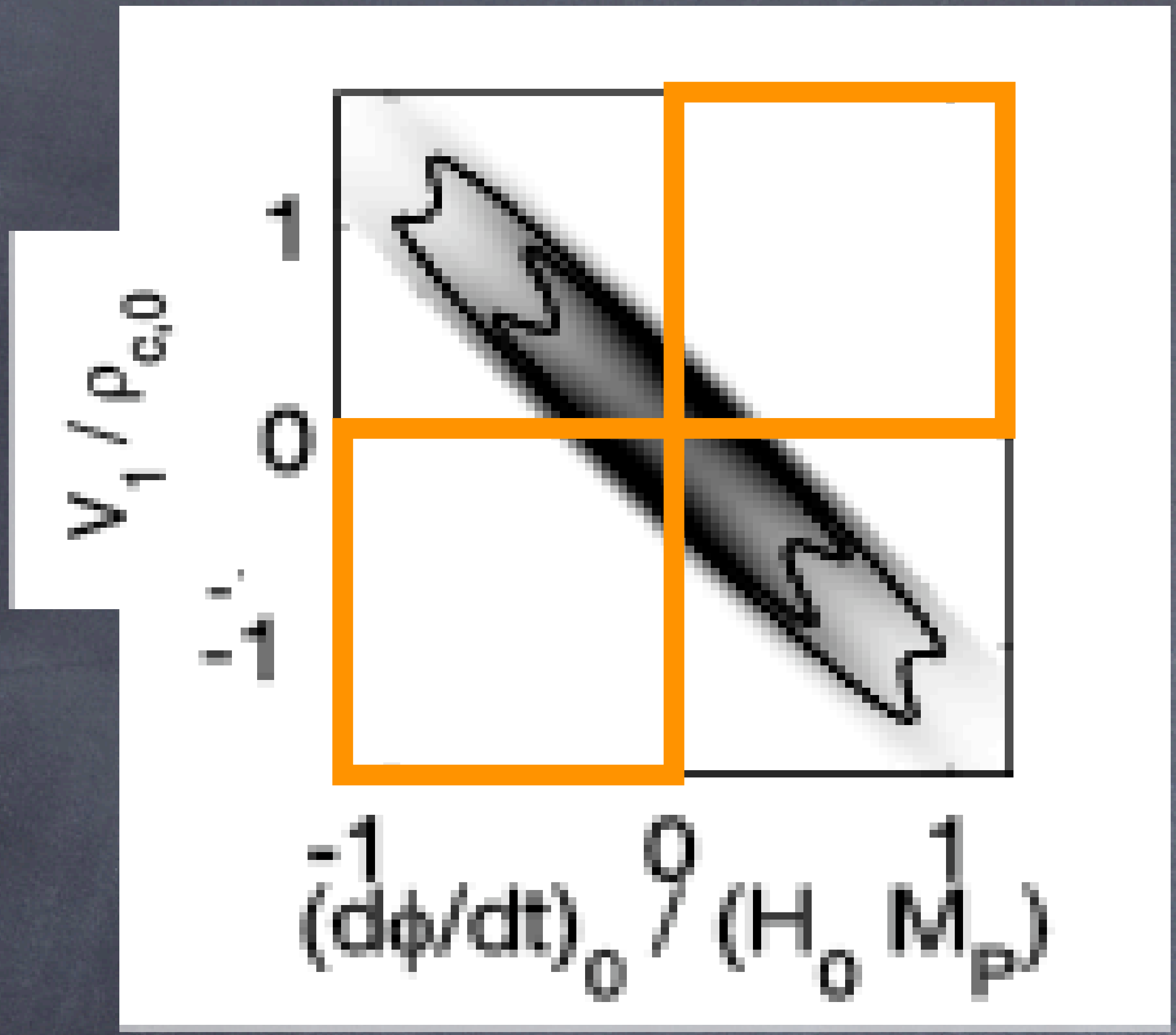


D = 4

We also did a quadratic potential but the system becomes essentially unconstrained by the data.

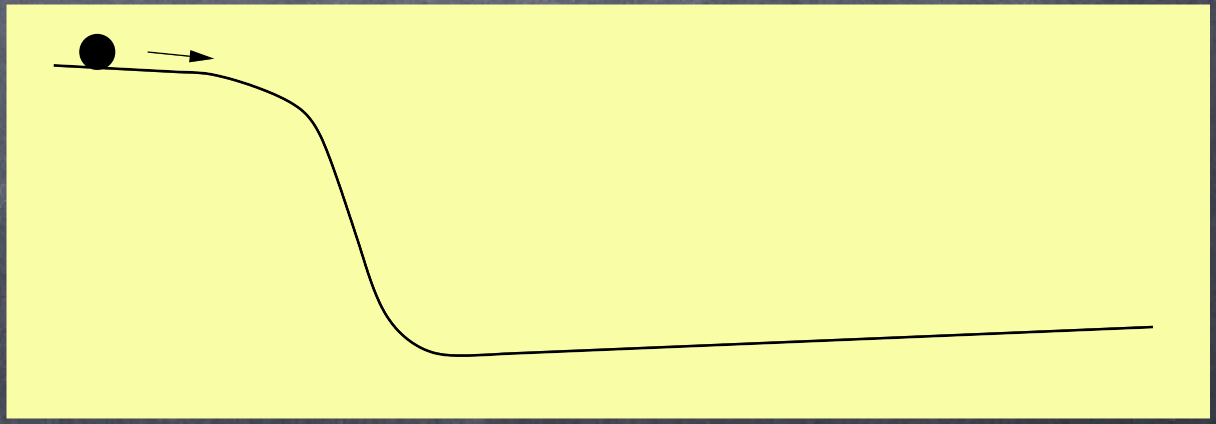


Boxes indicate regions where the field is presently rolling uphill. The maximum likelihood model is of this type.



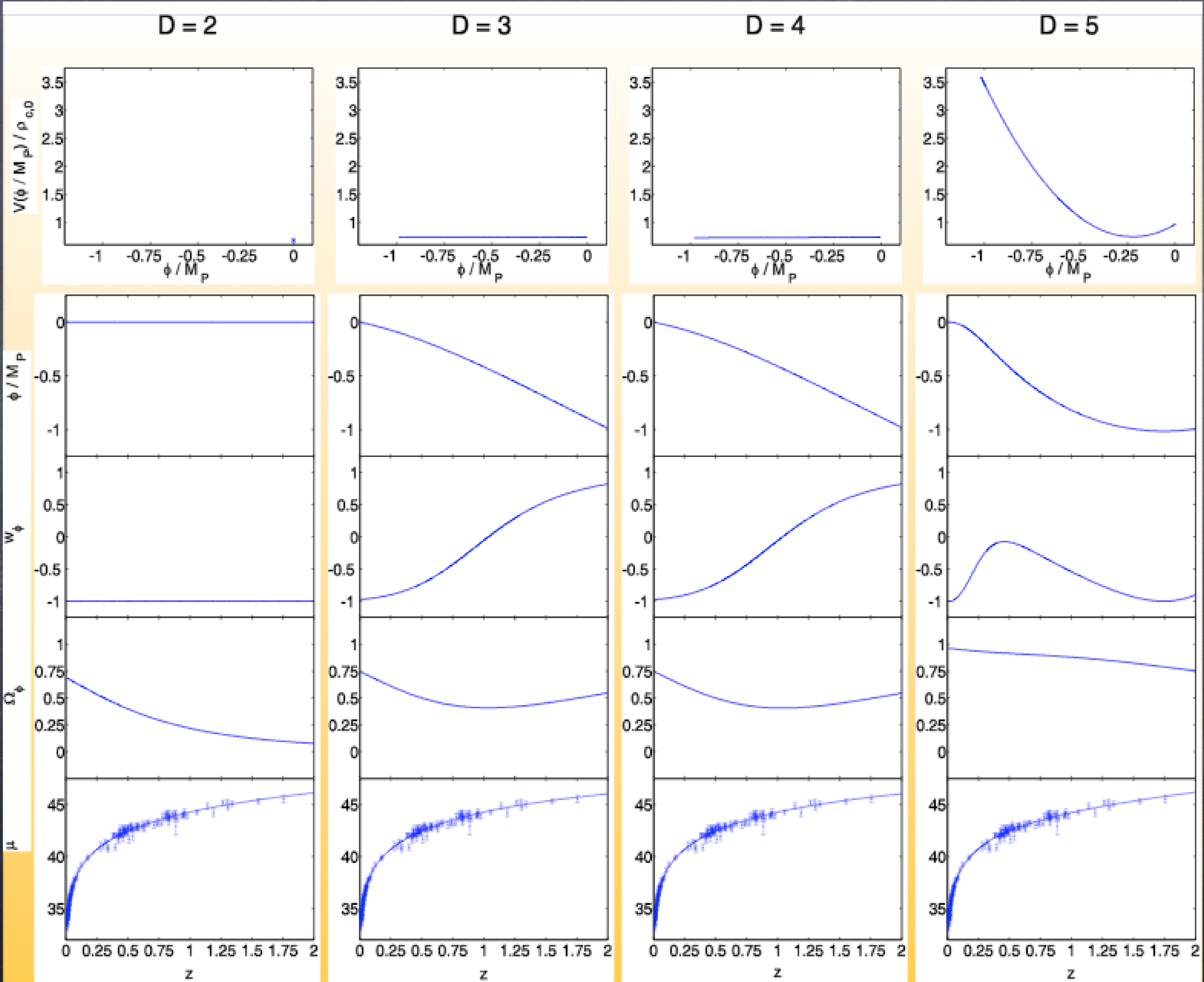
Boxes indicate regions where the field is presently rolling uphill. The maximum likelihood model is of this type.

Soon after our paper appeared, Csaki, Kaloper and Terning (astro-ph/0507148) produced a phenomenological model with the field rolling uphill.



Though it could just have been flat!

Properties of best-fit models



Model selection: which model is best?

Model selection statistics (eg Bayesian evidence, Parkinson's talk on Sunday) set up a tension between goodness of fit and model simplicity. We use an approximation to the evidence known as the Bayesian Information Criterion (BIC, see Liddle astro-ph/0401198).

$$\mathrm{BIC} = -2\ln \mathcal{L}_{max} + D \ln N$$

Best-fit models

	D = 2	**D = 3**	**D = 4**	**D = 5**
η	$-43.34^{+0.04}_{-0.04}$	$-43.34^{+0.04}_{-0.04}$	$-43.34^{+0.05}_{-0.05}$	-43.32
$\dot\phi_0/(H_0 M_{\mathrm{P}})$	–	0.25 $\lvert\dot\phi_0\rvert/(H_0 M_{\mathrm{P}}) \lesssim 0.29$	0.15 $\lvert\dot\phi_0\rvert/(H_0 M_{\mathrm{P}}) \lesssim 1.3$	-0.07
$V_0/\rho_{c,0}$	$0.69^{+0.06}_{-0.06}$	$0.74^{+0.06}_{-0.10}$	$0.73^{+0.07}_{-0.13}$	0.96
$V_1/\rho_{c,0}$	–	–	0.13 $\lvert V_1\rvert/\rho_{c,0} \lesssim 1.8$	1.99
$V_2/\rho_{c,0}$	–	–	–	4.50
$-2\ln\mathcal{L}_{\max}$	177.1	176.0	176.0	173.4
BIC_D	187.2	191.2	196.2	198.7
$\mathrm{BIC}_D - \mathrm{BIC}_2$	0	4.0	9.0	11.5

Model selection summary

	D=2	D=3	D=4	D=5
BIC	187.2	191.2	196.2	198.7

Cosmological constant model has the lowest BIC and so is the preferred fit, even though other models give higher maximum likelihood.

ΔBIC	Evidence
2	Positive
6+	Strong

Conclusions

- Optimal method for constraining quintessence potentials.
- Cosmological constant model is the preferred fit to the SN data.
- For SN data alone, allowing the field to roll is more strongly supported by the data than allowing the potential to be non-flat.
- Future directions: include spatial curvature (done), slow-roll approximation, incorporate more data types, extend model selection to compute Bayesian evidence ...