Moduli Stabilization

in

Heterotic Flux Compactifications

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Outline

- Introduction: Compactifications, moduli and flux
- Manifolds with SU(3) structure
- Heterotic string and SU(3) structure
- Half-flat mirror manifolds
- Generalized half-flat manifolds
- Conclusions and outlook

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Based on: hep-th/0507173 and hep-th/0408121

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In collaboration with: B. de Carlos (Sussex), S. Gurrieri (Kyoto), A. Micu (Sussex)

Compactification of $D = 10$ theory

Compactify on $d = 6$ space with topology X

Metric : $ds^2 = dx^\mu dx^\nu \eta_{\mu\nu} + g_{mn}(y, b^i) dy^m dy^n$,
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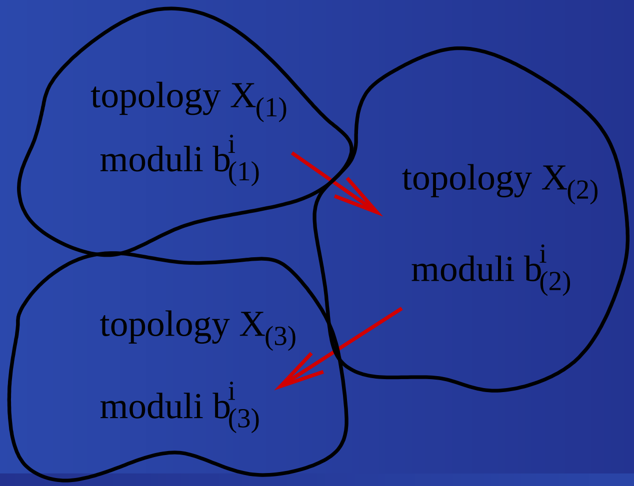
Schematic structure of moduli space:



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$d = 6$ dim. spaces with covariantly constant spinor η, $\nabla \eta = 0$ ($\rightarrow$ Ricci-flat)

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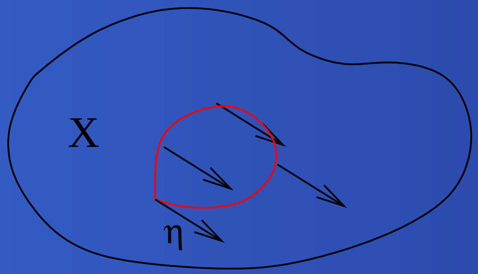

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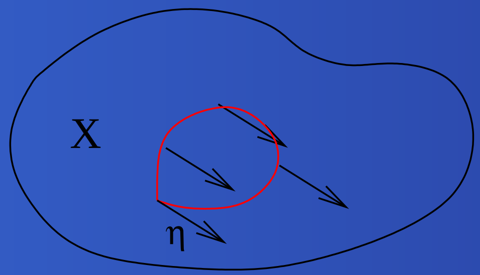

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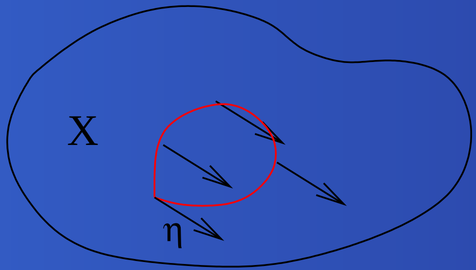

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Introduce forms:

$$J_{mn} = \eta^\dagger \gamma_{mn} \eta \qquad \text{Kahler form}$$
$$\Omega_{mnp} = \eta^\dagger \gamma_{mnp} \eta \qquad \text{complex structure}$$

Calabi-Yau spaces, cont'd

On a Calabi-Yau space

$$\begin{aligned} \nabla J &= 0 \Rightarrow dJ = 0 \\ \nabla \Omega &= 0 \Rightarrow d\Omega = 0 \end{aligned}$$

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Ricci-flat metric in one-to-one correspondence with $J = t^i \omega_i$ and $\Omega = \mathcal{Z}^A \alpha_A - \mathcal{G}_A \beta^A$, where ω_i, $i = 1, \ldots, h^{1,1}$ are two-forms and α_A, β^A, $A = 0, \ldots, h^{2,1}$ are three-forms.

Note: $d\omega_i = d\alpha_A = d\beta^A = 0$.

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Moduli
$$\left\{ \begin{array}{lll} t^i & \rightarrow & T^i \\ \mathcal{Z}^A & \rightarrow & Z^a \\ & & S \end{array} \right.$$

$t^i \rightarrow T^i$	Kahler moduli
$\mathcal{Z}^A \rightarrow Z^a$	complex structure moduli
S	dilaton

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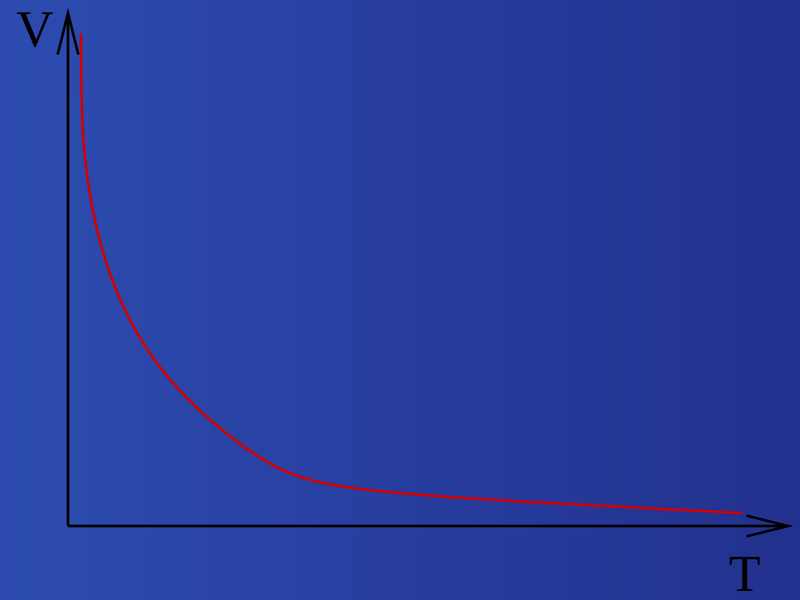
Typical shape of potential:



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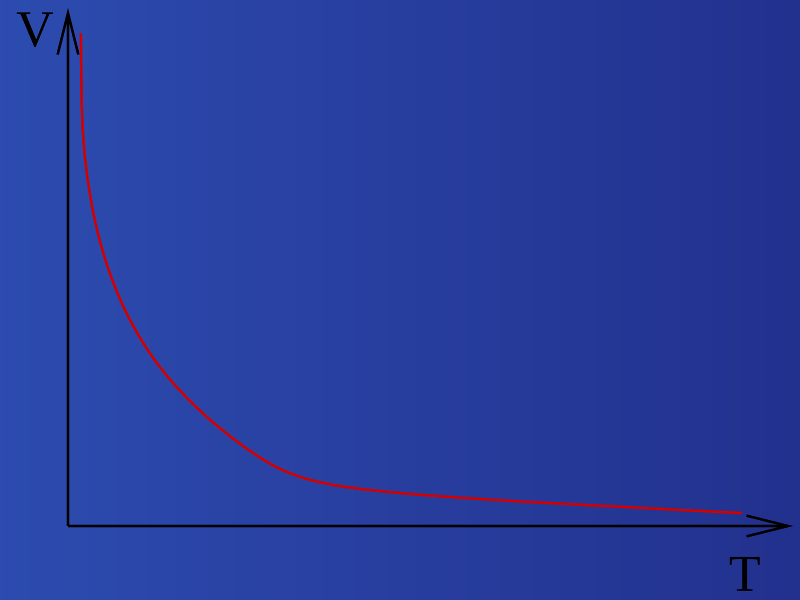

Hard to fix all moduli and potential very steep.

Flux

String theory contains p-forms $A_{I_1 \dots I_p}$ with field strengths $F = dA$.

Flux: Internal components of F are non-zero, $F_{m_1 \dots m_{p+1}} \neq 0$.

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$\rightarrow$ flux generates a $D = 4$ moduli potential

Example IIB

(Kachru et al., ’03)

Flux: NSNS 3-form H and RR 3-form F

Flux superpotential: $W_{\text{flux}} = \int_X (F - iSH) \wedge \Omega$

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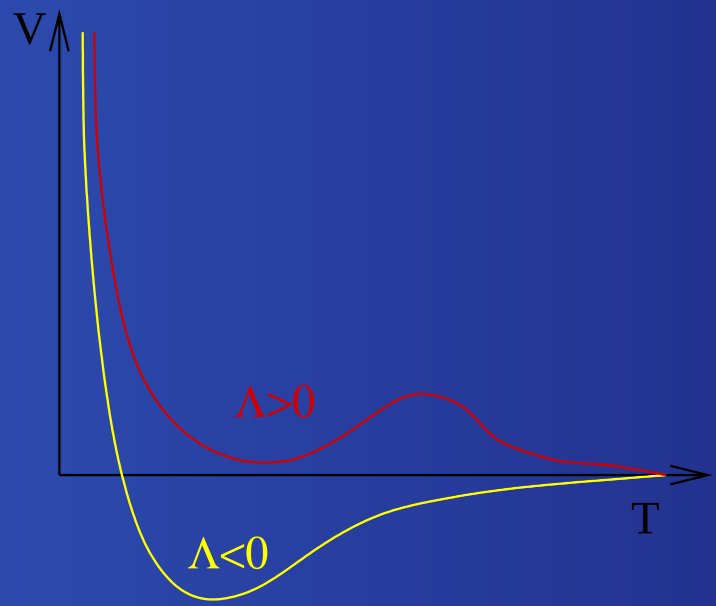

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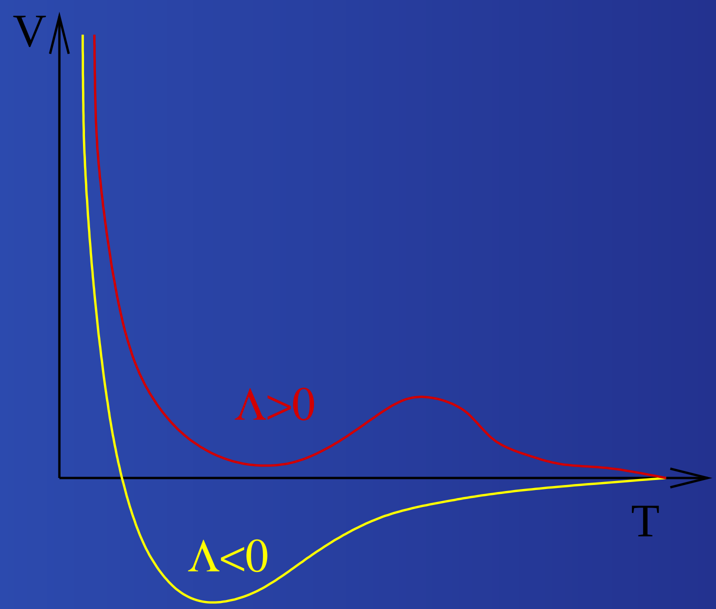

Can fix all moduli and less steep.

Heterotic string

Why heterotic?

- Gauge unification natural
- Large number of supersymmetric models
- Allows fermions from $\mathbf{16}$ of SO(10)

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Such manifolds can be classified by the $\mathrm{SU}(3)$ content of the torsion κ.

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Consistency of vacuum: $|\mathcal{W}| \ll 1$ at minimum.

Half-flat mirror manifolds

Arise in the context of type II mirror symmetry.
Characterized by: (Gurrieri, Louis, Micu, Waldram, '02)

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\begin{aligned}
d\omega_i &= e_i \beta^0 \\
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Magnetic torsion?

Simple STZ model

Three moduli: S, T, Z

Kahler- and superpotential:

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Validity: Need large radius, Re$(T) > 1$, large complex structure, Re$(Z) > 1$ and weak coupling Re$(S) > 1$.

Reminder, $D = 4$ supergravity

Supergravity potential:

$$V = e^K \left(K^{X\bar{Y}} F_X \bar{F}_{\bar{Y}} - 3|W|^2 \right)$$

where $F_X = \partial_X W + \partial_X K W$, $K_{X\bar{Y}} = \partial_X \partial_{\bar{Y}} K$.

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Supersymmetric vacua: $F_X = 0$

Cosmological constant: $V|_{F_X=0} = -3e^K |W|^2 \leq 0$

Simple STZ model, results

For supersymmetric vacua:

- All fields stabilized, except for one combination of Im(T) and Im(Z) which is flat.
- $x \equiv c$Re$(S) = 1/4 \longrightarrow$ barely weak coupling
- difficult to achieve Re$(T) > 1$ and Re$(Z) > 1$

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$\rightarrow$ hard to find consistent vacua

General case

Arbitrary number of moduli: S, T^i, Z^a

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$\rightarrow$ difficult to find consistent, supersymmetric vacua for half-flat mirror manifolds.

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Main problem: No heterotic derivation of such a model known.

Generalized half-flat manifolds

Suggested by gauging of $N = 2$ SUGRA

Characterized by:

(d'Auria et al., '04)

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d\omega_i &= p_{Ai}\beta^A - q_i^A \alpha_A \\
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$$\mathcal{W} = (q_i^A\mathcal{G}_A - p_{Ai}Z^A)T^i + \epsilon_A Z^A - \mu^A\mathcal{G}_A$$

If $dH = 0$ (standard embedding) $\mu^A p_{Ai} - \epsilon_A q_i^A = 0$.

Solution method

Superpotential: $W = \mathcal{W}(T^i, Z^a) + ke^{-cS}$

Find global solution T_0^i, Z_0^a, so that $\partial_i W(T_0, Z_0) = \partial_a W(T_0, Z_0) = 0$.

Define $\mathcal{W}_0 = \mathcal{W}(T_0, Z_0)$ and require $|\mathcal{W}_0| \ll 1$.

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Define $\mathcal{W}_0 = \mathcal{W}(T_0, Z_0)$ and require $|\mathcal{W}_0| \ll 1$.

Then a local solution exists nearby and

$$(2x+1)e^{-x} = \left|\frac{\mathcal{W}_0}{k}\right|$$

where $x = c\text{Re}(S)$.

Simple STZ model

Kahler- and superpotential:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) - 3\ln(Z+\bar{Z})$$

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Standard embedding: $\xi r - \epsilon q + \mu p - \rho e = 0$

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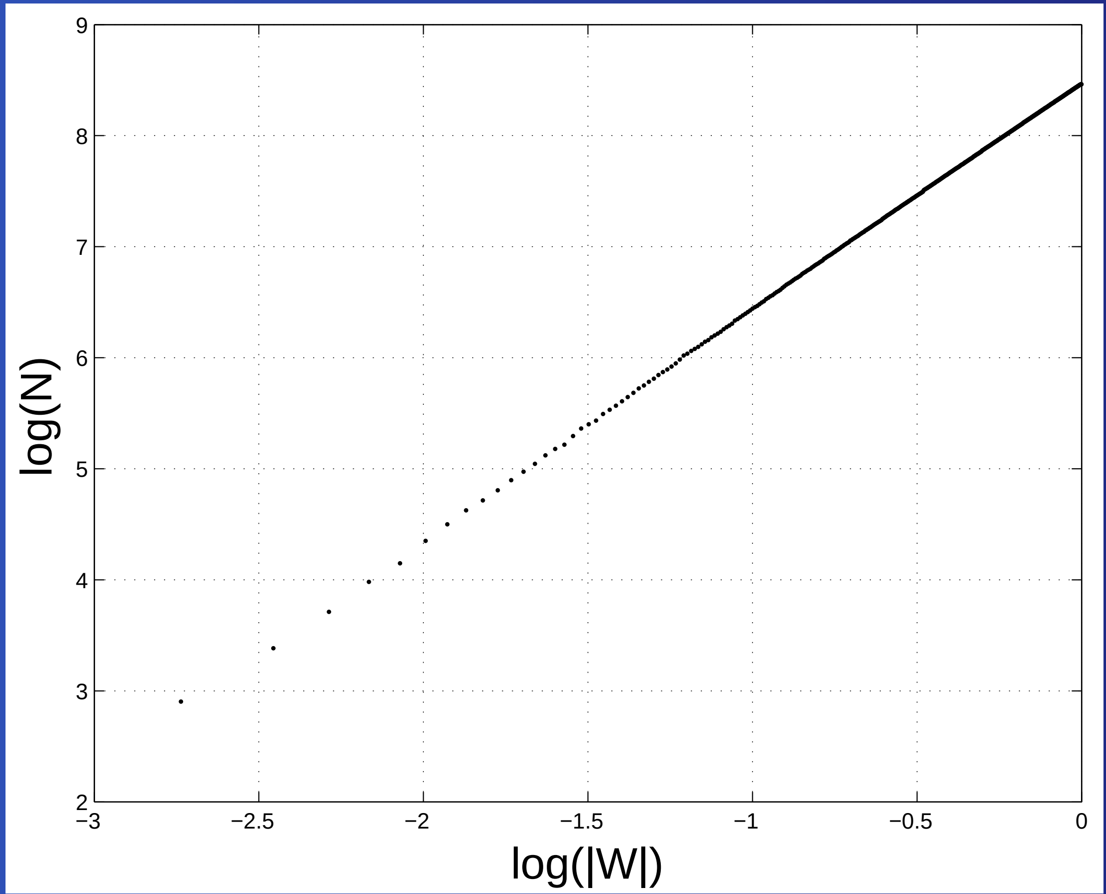
For standard embedding: $|\mathcal{W}_0| > 0.8$ always.

Non-standard embedding

Number of global solutions as a function of $|\mathcal{W}_0|$ for parameters in range $-70 \ldots 70$.



Non-standard embedding, cont'd

Let $N(M,w)$ be the number of solutions for parameters in the range $-M\ldots M$ and $|\mathcal{W}_0| < w$.

We find : $N(M,w) \simeq M^5 w^2$

$\rightarrow$ roughly 10^{-3} of vacua lead to a weak gauge coupling Re$(S) \simeq 24$.

Conclusion and outlook

- The “missing” RR flux in the heterotic string can be “replaced” by torsion of the internal space.
- It is difficult to obtain consistent vacua for half-flat mirror spaces and for more general spaces with standard embedding.
- Generalized half-flat spaces with non-standard embedding allow for a large number of consistent vacua.
- Gauge couplings weak enough for unification are obtained for a small fraction (e. g. $1/1000$) of these vacua.

Conclusion and outlook, cont'd

- The cosmological constant can probably be lifted to positive values by adding anti 5-branes.
- A number of theoretical issues need better understanding: construction of spaces, gauge field sector, quantization,...
- Analyze potentials away from vacua: cosmology, inflation?