Calculating the primordial non-gaussianity

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Overview

1. Non-gaussianity as a discriminator.

2. Defining the primordial curvature perturbation ζ.

3. Defining the non-gaussianity of ζ.

4. The magic formula for ζ.

4. The three main scenarios.

Non-gaussianity as a discriminator

Model $\Rightarrow$ Primordial Perturbation $\Rightarrow$ Observables

Primordial perturbation is:

- Mostly adiabatic
  - How much isocurvature?
  - How much tensor?
  - How much cosmic string, wall, texture?
- Almost scale-independent
  - How much tilt?
- Almost gaussian
  - How much non-gaussianity?
  - What kind (bispectrum, trispectum).
  - **How can we compute the non-gaussianity?**

My main messages

To particle theorists

- I will show you how to calculate the primordial non-gaussianity.
- Cf. spectral tilt: $n - 1 = 2\eta - 6\epsilon$ (Liddle/DHL 1992)

To astronomers

- Trispectrum, even higher correlators, could be as important as the bispectrum.
- I will give you the dependence of each correlator on the wave-vectors.

Defining the curvature perturbation ζ

1. Smooth the Universe on the scale 10^{-2} Mpc.
2. Consider $t < 100$ s (before horizon entry $\equiv$ 'primordial')
3. Work on slicing of uniform energy density ρ
4. Define local scale factor: $g_{ij} = \tilde{a}^2(\mathbf{x}, t)\delta_{ij}$
5. Define $\zeta \equiv \ln \tilde{a}(\mathbf{x}, t) - \ln a(\mathbf{x}, t) \equiv \delta N$

- In words, ζ is the perturbation in the number of e-folds of expansion, starting from a *flat* slice.
- ζ is conserved iff pressure $P(\rho)$ is a unique function.
- Its constant value at $t \sim 10$ s provides the **initial condition** for adiabatic perturbations.

The correlators

Spectrum $\mathcal{P}$, bispectrum† $f_{\rm NL}$, trispectrum†† $\tau_{\rm NL}$:

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') K_1 \mathcal{P} \quad (1)$$

$$-\frac{3}{5}\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \rangle = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'') K_2 \mathcal{P}^2 f_{\rm NL} \quad (2)$$

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \zeta_{\mathbf{k}'''} \rangle_{\rm c} = (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''') K_3 \mathcal{P}^3 \tau_{\rm NL} \quad (3)$$

where the kinematic factors depend on the wave-vectors:

$$K_1 \equiv 2\pi^2/k^3 \quad (4)$$

$$K_2 \equiv K_1(k) K_1(k') + \text{5perms} \quad (5)$$

$$K_3 \equiv K_2 K_1(|\mathbf{k}+\mathbf{k}''|) + \text{23perms} \quad (6)$$

† Komatsu/Spergel 2000; Maldacena 2003

†† Boubekeur/DHL 2005

Theory and Observation

SLOW-ROLL INFLATION PREDICTION:†

$\mathcal{P}$, $f_{\rm NL}$ and $\tau_{\rm NL}$ almost scale-independent.

OBSERVATION:

1. $\mathcal{P} = (5 \times 10^{-5})^2$ (WMAP+SDSS)
2. $|f_{\rm NL}| \lesssim 100 \ll \mathcal{P}^{-1/2}$ (WMAP)
3. $|\tau_{\rm NL}| \lesssim 10^8 \ll \mathcal{P}^{-1}$ (COBE !!!!)

- From 2 and 3, ζ is almost gaussian.
- Observation eventually will give (absent detection)
    - $|f_{\rm NL}| \lesssim 1$ and $|\tau_{\rm NL}| \lesssim ??????$

† DHL/Rodriguez 2005

Magic formula 1 for $\zeta^\dagger$

$$\zeta(\mathbf{x}, t) = \delta N \quad (7)$$

Here $N(\phi_i(\mathbf{x}), \rho(t))$ is number of e-folds of expansion *starting* from a flat slice just after horizon exit (field values $\phi_i(\mathbf{x})$) and *ending* on a uniform-density slice at time t. We're interested in $t \sim 10\,$s.

- The magic formula gives ζ knowing evolution of a family of unperturbed local universes (separate universe picture††)
- It's not cosmological perturbation theory!

$\dagger$ Sasaki/Stewart 1995, DHL/Malik/Sasaki 2005

$\dagger\dagger$ Liddle/Malik/DHL/Wands 1999

Magic formula 2 for $\zeta^\dagger$

$$\zeta(\mathbf{x},t) = \sum_i \frac{\partial N}{\partial \phi_i}\delta\phi_i(\mathbf{x}) + \frac{1}{2}\sum_{ij}\frac{\partial^2 N}{\partial\phi_i\partial\phi_j}\delta\phi_i(\mathbf{x})\delta\phi_j(\mathbf{x}) \tag{8}$$

- The $\delta\phi_i$ have spectrum $(H/2\pi)^2$ and are practically gaussian††
- Magic formula gives $\mathcal{P} = (H/2\pi)^2 \sum N_i^2$, and determines non-gaussianity if it's significant.

NOTE: We discount exotic inflation models (D-celeration, ghost inflation) giving non-gaussian inflaton perturbation $\delta\phi$.

$\dagger$ Sasaki/Stewart 1995; DHL/Rodriguez 2005

$\dagger\dagger$ Seery/Lidsey 2005, DHL/Zaballa 2005

Scenario 1: The inflaton ϕ generates ζ

The standard scenario

$$\zeta \simeq N_\phi \delta\phi. \quad (9)$$

- Spectrum† $\mathcal{P} \simeq (H^2/2\pi\dot{\phi})^2$
- Tilt†† $n - 1 = 2\eta - 6\epsilon$
- Negligible non-gaussianity†††.

†Starobinsky 1982, etc.

††Liddle/DHL 1992

†††Seery/Lidsey 2005, see also Maldacena 2003

Scenario 2: Curvaton[†] σ generates ζ

Not exotic[††]; many 'curvaton' candidates.

$$\zeta = N_\sigma \delta\sigma + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma)^2 \equiv \zeta_{\rm g} - \frac{5}{3} f_{\rm NL} \zeta_{\rm g}^2 \tag{10}$$

Prediction[†††]:

$$\mathcal{P} = \frac{H_* r}{3\pi} \frac{g'}{g} \tag{11}$$

$$-(3/5) f_{\rm NL} = -1 - \frac{1}{2} r + \frac{3}{4} \frac{1}{r} \left(1 + \frac{g g''}{g'} \right) \tag{12}$$

$$\tau_{\rm NL} = 4 (3 f_{\rm NL}/5)^2 \tag{13}$$

† DHL/Wands 2002; Moroi/Takahashi 2002; Enqvist/Sloth 2002

†† Discovered unknowingly; Hamaguchi/Murayama/Yanagida, 2001

††† DHL/Wands 2002; Okamoto/Hu 2002; Bartolo/Matarrese/Riotto 2004; DHL/Rodriguez 2005

Scenario 3: mixture of 1 and 2

Mainly inflaton with some curvaton. *Not exotic* eg. GUT inflation with σ a string axion.

$$\zeta = N_\phi \delta\phi + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma)^2 \equiv \zeta_\phi + \zeta_\sigma \qquad (14)$$

predictions†: $f_{\rm NL} \sim 10^4 (\zeta_\sigma/\zeta_\phi)^3$ and $\tau_{\rm NL} \sim 10^{10} (\zeta_\sigma/\zeta_\phi)^4$.

APPLICATION: two-component inflation

$$V = V_0 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\sigma^2 \sigma^2, \qquad (\text{unperturbed } \sigma = 0) \qquad (15)$$

Magic formula††for $\langle B^2/A \rangle$ *agrees* with one 2nd-order cos. pert. calculation†††, *disagrees* with another††††, but anyhow negligible.

† DHL/Boubekeur 2005 †† DHL/Rodriguez 2005 ††† Malik 2005

†††† Enqvist/Vaihkonen 2004 from Acqaviva et. al. 2003

Final thoughts

1. We at last have a *complete* and *simple* understanding of primordial non-gaussianity.

2. Non-gaussianity may discriminate between models more strongly than spectral tilt, tensor, correlated or un-correlated isocurvature, cosmic string signals etc.

3. Most scenarios predict that at least one of the above will be observed. So we will learn a lot even if none of them are observed!