Simulations of the End of SUSY Hybrid Inflation and Non-Topological Soliton Formation

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SUSY Hybrid Inflation <=> Flat inflaton potential without extremely small couplings + No radiative corrections

Two classes of SUSY hybrid inflation model:

$$V = V_F + V_D \equiv \left| \frac{\partial W}{\partial \Phi_i} \right|^2 + \frac{g^2}{2} \left| \Phi_i^\dagger T^a \Phi_i \ (+\xi) \right|^2$$

- D-term Hybrid Inflation: $W = \lambda S \Phi_+ \Phi_-$

  Energy density driving inflation due to $U(1)_{FI}$ D-term, $V_D = g^2 \xi^2 / 2$
  => No η-problem in SUGRA (no $O(H)$ inflaton mass)

  Inflation ends with $U(1)_{FI}$-breaking transition
  => Cosmic string CMB problem if inflaton produces cosmological density perturbations
  => $g \lesssim 10^{-2}$ and $\lambda \lesssim 10^{-5}$ *[Endo et al hep-ph/0304126, Rocher and Sakellariadou, hep-ph/0412143]*

  $g, \lambda \approx 1$ possible if density perturbations are due to a 'curvaton'

- F-term Hybrid Inflation: $W = \lambda S (\overline{\Phi} \Phi - \mu^2)$

  Energy density driving inflation due to F-term, $V_F = \lambda^2 \mu^4$
  => η-problem unless minimal inflaton kinetic terms ($K \sim S^\dagger S$)

  Cosmic string CMB problem unless $\lambda \lesssim 10^{-2}$ *[Jeannerot and Postma, hep-ph/0503146]*.

  F-term scalar field dynamics similar to D-term with $\lambda / g \approx 1$.

D-term Inflation: $W = \lambda S \Phi_+ \Phi_-$

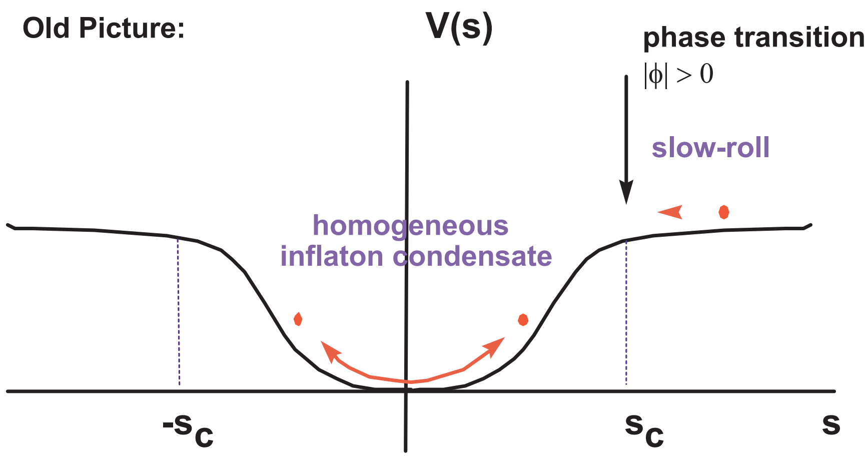
$$V(S, \Phi_+, \Phi_-) = \lambda^2 |S|^2 \left(|\Phi_+|^2 + |\Phi_-|^2\right) + \lambda^2 |\Phi_+|^2 |\Phi_-|^2 + \frac{g^2}{2}\left(|\Phi_+|^2 - |\Phi_-|^2 + \xi\right)^2 + V_{1-\mathrm{loop}}$$

Old picture of the end of SUSY hybrid inflation:

$|S| > S_c = g\xi^{1/2}/\lambda$ => Slow-roll inflation, $\Phi_+ = \Phi_- = 0$.

$|S| < S_c \Rightarrow m^2_{\Phi_-} < 0 \Rightarrow < \Phi_- > \neq 0$ => Phase transition ending inflation ($m_S^2 = \lambda^2 |\Phi_-|^2 \gg H^2$).

=> Spatially homogeneous coherently oscillating inflaton field [$\equiv$ Bose condensate of inflatons] which dominates until inflatons decay, 'reheating' the Universe



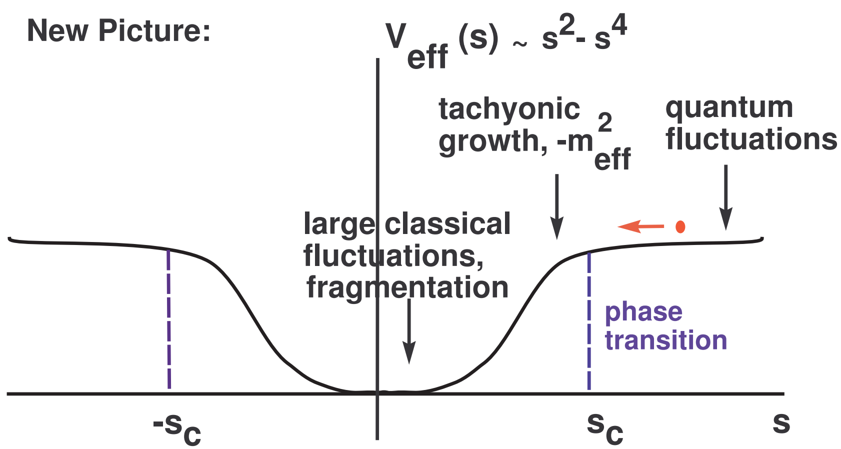
New Picture: Growth of quantum fluctuations => Highly inhomogeneous post-inflation era.

- Effective Theory: Minimize for $\Phi_-(S)$ as a function of $S \Rightarrow$

$$V_{eff}(S) = \lambda^2 \xi |S|^2 - \frac{\lambda^2}{2g^2}|S|^4 \;, \quad |S| \leq S_c = g\sqrt{\xi}/\lambda$$

  Attractive $-|S|^4$ interaction implies an inflaton condensate would be unstable with respect to spatial perturbations if it could form. *[JMcD PRD 66 (2002) 043525]*

- In fact: Sub-horizon quantum fluctuations of the inflaton undergo tachyonic amplification $(\propto e^{m_{\rm eff} t}) \Rightarrow$ quantum modes rapidly grow to become semi-classical.
  Subsequent classical evolution $\Rightarrow$ energy density dominated by spatial fluctuations $\delta\rho(\mathbf{x})/\overline{\rho} \sim 1$ before a homogeneous condensate can form = Tachyonic Preheating *[Felder et al, PRL 87 (2001) 011601; PRD 64 (2001) 123517]*



In D-term inflation, outcome depends on λ/g:

- $\lambda/g \lesssim 0.1 \Rightarrow$ inflaton perturbation growth remains linear ($\delta\rho/\rho \ll 1$) $\Rightarrow$ spatial perturbations of an coherently oscillating inflaton condensate
- $\lambda/g \gtrsim 0.1 \Rightarrow$ inflaton perturbations become non-linear ($\delta\rho/\rho \sim 1$) $\Rightarrow$ inhomogeneous energy density, no condensate formation [*From dominant mode growth in a semi-analytic model [Matt Broadhead and JMcD, Phys.Rev. D68 (2003) 083502] and simulations [Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]*].
- => Likely to occur generally for F-term inflation ($\lambda/g \approx 1$)
- Numerical solution of the scalar field equations is necessary to understand the evolution of the energy density lumps

Simplified model: Real inflation S, complex Φ_- $\quad [\Phi_+ = A^\mu = 0$ *throughout* $]$

[Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]

$$\ddot{S} + 3H\dot{S} - \frac{\nabla^2}{a^2} S = -\lambda^2 |\Phi_-|^2 S$$

$$\ddot{\Phi}_- + 3H\dot{\Phi}_- - \frac{\nabla^2}{a^2}\Phi_- = -\lambda^2 |S|^2 \Phi_- + g^2 \left(\xi - |\Phi_-|^2\right)\Phi_-$$

May replace quantum fields with classical fields + stochastic initial conditions *[Polarski and Starobinsky, gr-qc/9504030]* :

Initial Conditions at $S = S_c$ ($\Rightarrow$ massless fields):

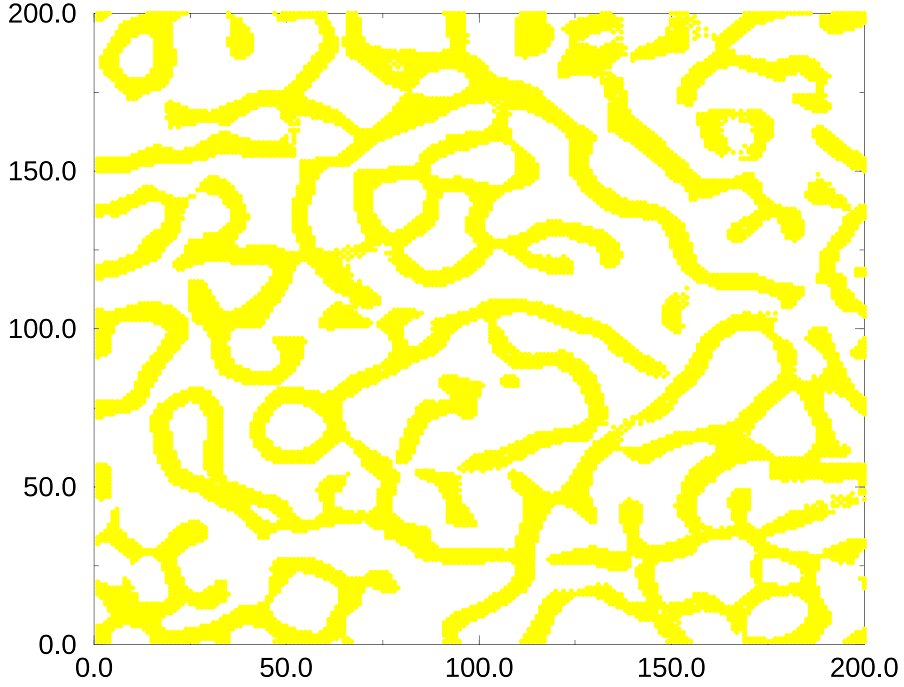
$$\delta\phi(\mathbf{x},0) = \frac{1}{\sqrt{V}} \sum q(\mathbf{k},0) e^{i\mathbf{k}.\mathbf{x}} \quad ; \quad \delta\dot{\phi}(\mathbf{x},0) = \frac{1}{\sqrt{V}} \sum p(\mathbf{k},0) e^{i\mathbf{k}.\mathbf{x}} \quad ; \quad \delta\phi \equiv \delta s,\ \delta\phi_1,\ \delta\phi_2$$

$q(\mathbf{k},0)$, $p(\mathbf{k},0)$ are Gaussian distributed:

$$P\left(q(\mathbf{k},0)\right) = \frac{1}{\pi |q(\mathbf{k},0)|^2_{rms}} \exp\left(-\frac{|q(\mathbf{k},0)|^2}{|q(\mathbf{k},0)|^2_{rms}}\right)$$

$$|q(\mathbf{k},0)|_{rms} = \frac{1}{\sqrt{2k}} \left(1 + \frac{H^2}{k^2}\right)^{1/2} \quad ; \quad |p(\mathbf{k},0)|_{rms} = \frac{H}{\sqrt{2k}} \left(1 + \frac{H^2}{k^2}\right)^{-1/2}$$

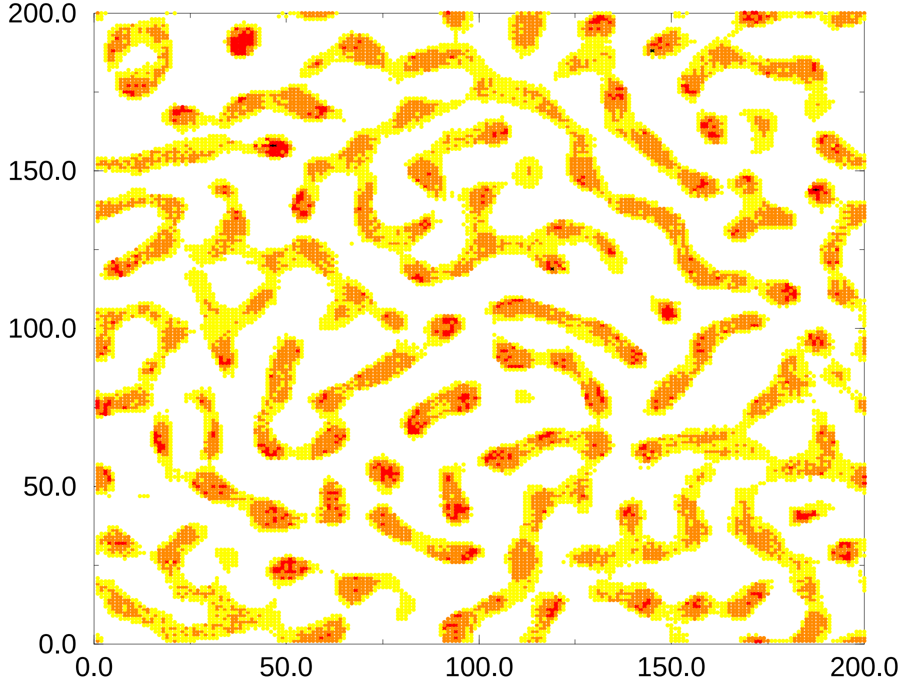
2-D Simulation: 200 $\times$ 200 lattice, $\lambda/g = 0.14$. $t = 1\tau_S$:



(Yellow = $\rho > (1-2)\overline{\rho}$; Orange = $\rho > (2-3)\overline{\rho}$; Red = $\rho > (3-4)\overline{\rho}$; Black = $\rho > 4\overline{\rho}$)

[Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]

$t = 3\tau_S$:



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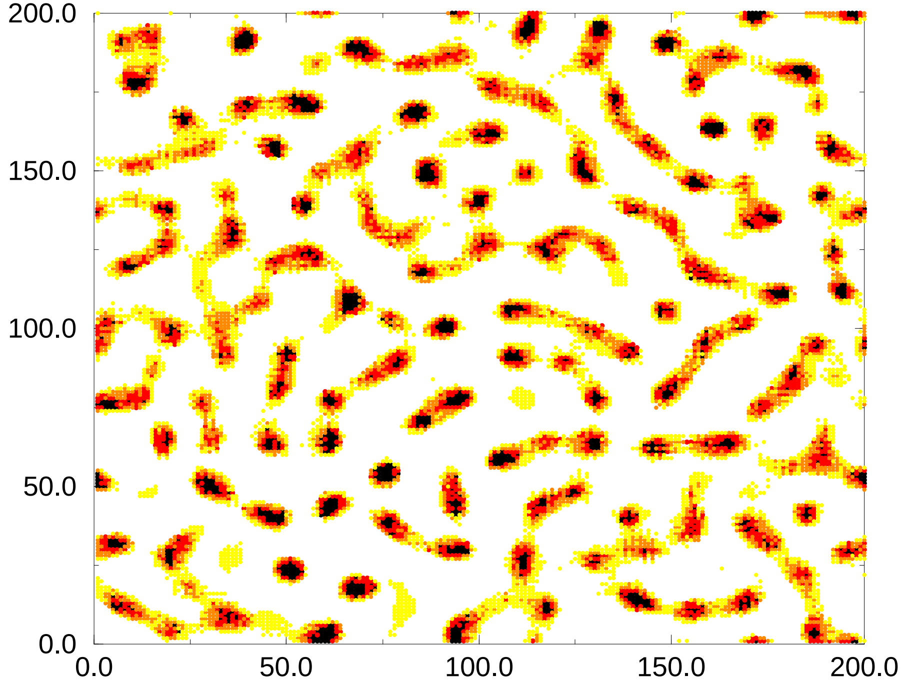
[Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]

$t = 4\tau_S$:



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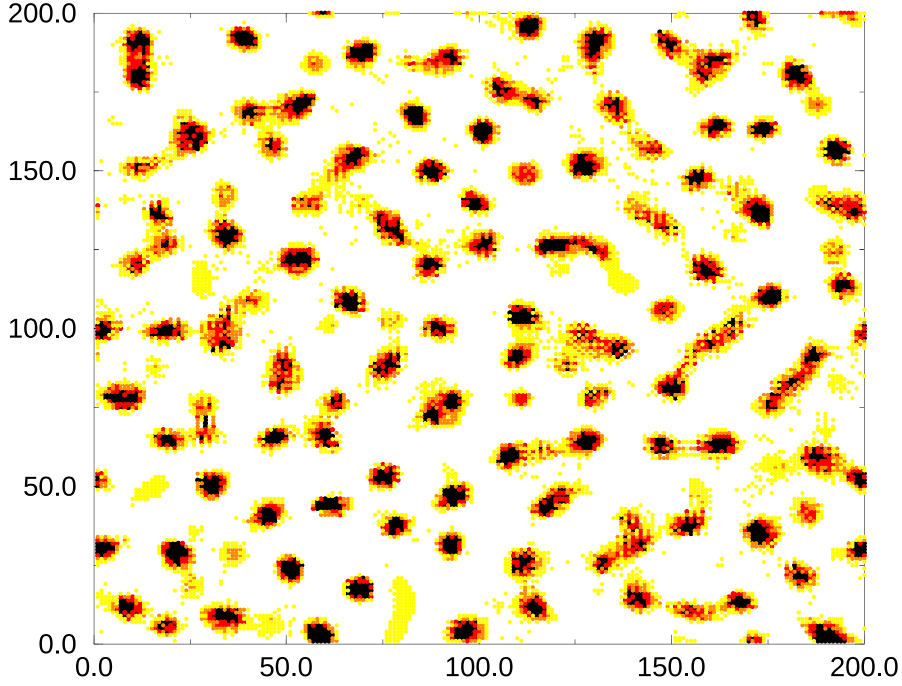
[Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]

$t = 5\tau_S$:



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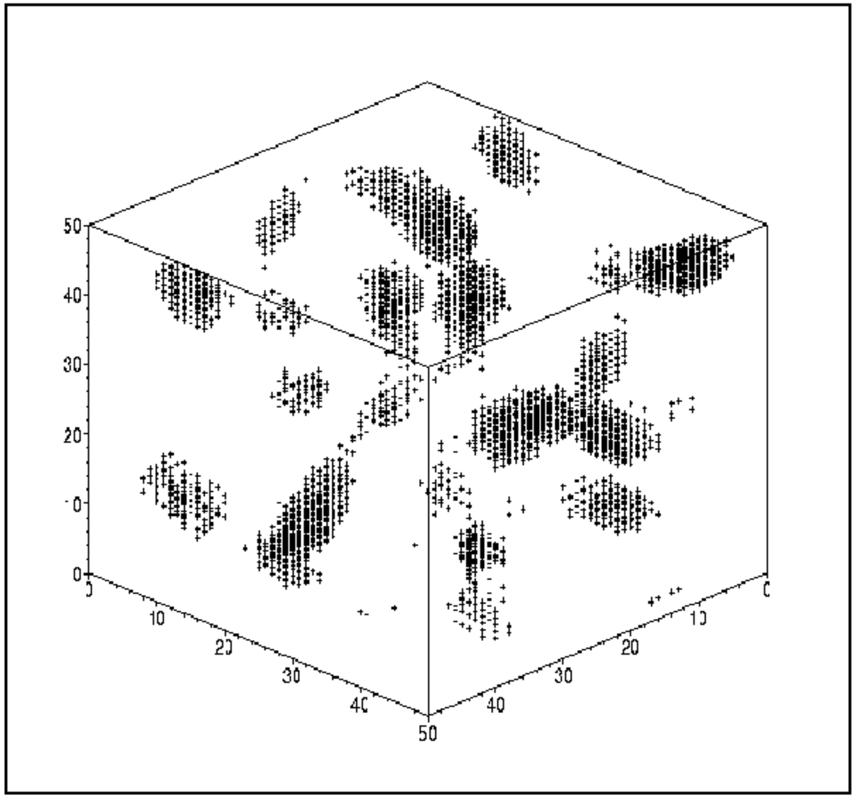
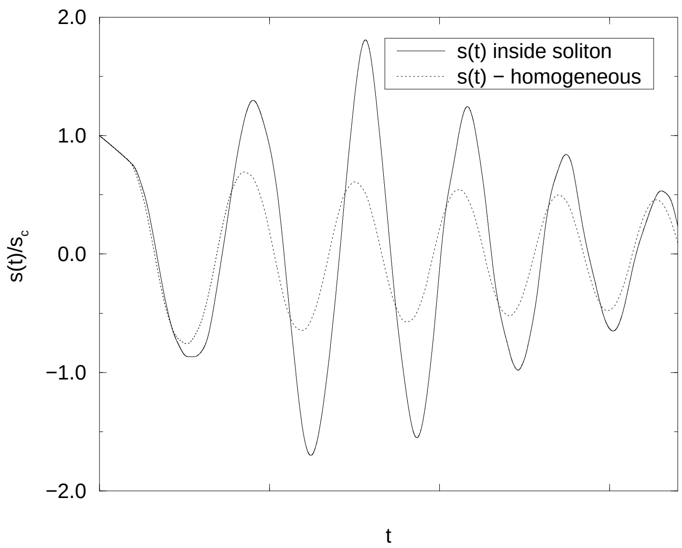
[Matt Broadhead and JMcD, hep-ph/0503081, PRD 72 (2005) 043519]

3-D Simulation: 50^3 lattice, $\lambda/g = 0.14$, $\rho > 4\overline{\rho}$. $t = 5\tau_S$:



Inflaton field at the centre of one of the lumps:



- Essentially a droplet of coherently oscillating inflaton field $\Rightarrow$ Oscillon
- Oscillons are known to be very long-lived, quasi-stable objects

  $\tau > 10^3 m_S^{-1}$ for '$S^2 - S^4$' effective theory oscillons (I-Balls) *[Kasuya et al, hep-ph/0209358 PLB 559 (2003) 99]*

=> Oscillon-dominated post-inflation era. ['Inflaton Condensate Lumps']

Real inflaton => Oscillon-dominated post-inflation era.
Complex inflaton => Q-ball-dominated post-inflation era

Q-Ball $\equiv$ Miniminum energy configuration for fixed global $U(1)$ charge.
Global charge conservation => stable.

SUSY Hybrid Inflation $\Rightarrow$ Global $U(1)$ under which $S \rightarrow e^{i\alpha} S$.

SUSY hybrid inflation Q-balls exist $\longrightarrow$ Two-field Q-balls with non-zero S charge, Q_S
[MB and JMcD, Phys. Rev. D69 (2004) 063510, hep-ph/0309298]

Q-Ball Solutions: $S = s(r) e^{i\omega t}/\sqrt{2}$, $\Phi_- = \phi_-(r)/\sqrt{2}$

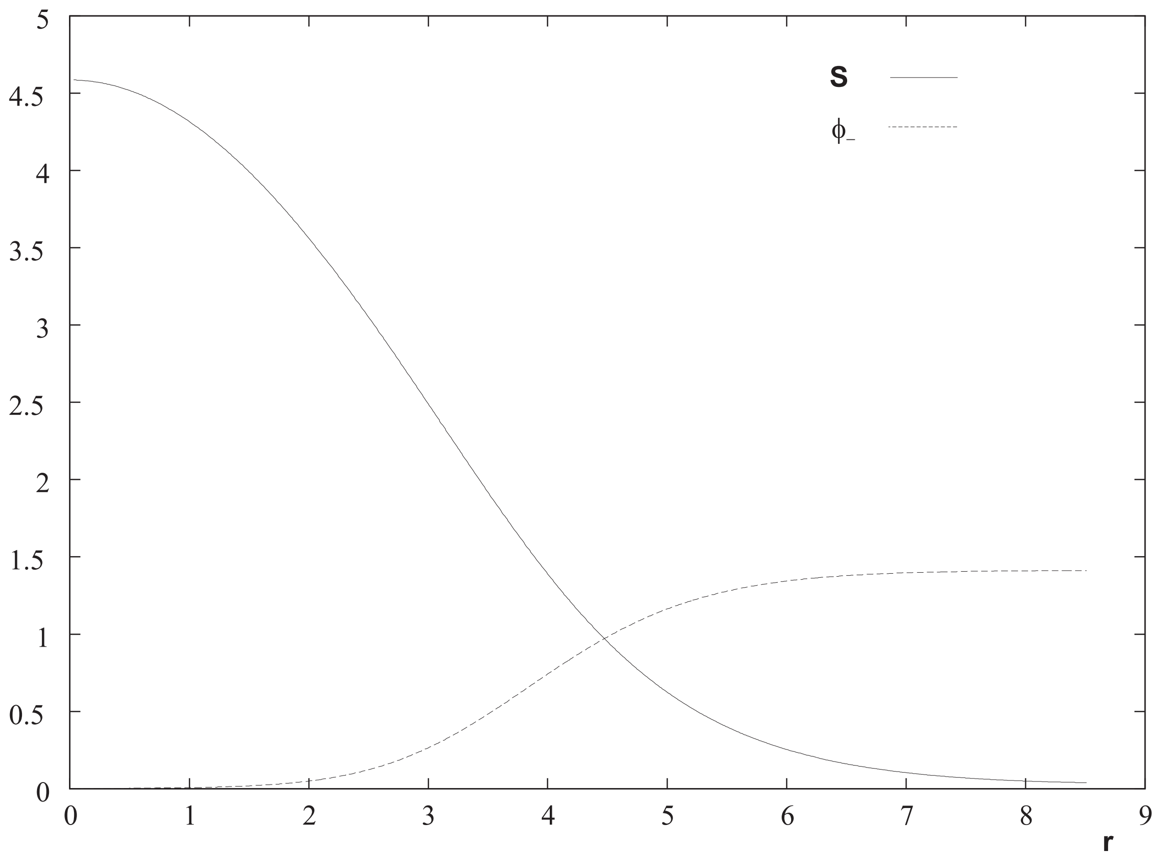
$s(r)$, $\phi_-(r)$ satisfy equations of motion + vacuum boundary conditions at $r \rightarrow \infty$:

$$\frac{\partial^2 s}{\partial r^2} + \frac{2}{r}\frac{\partial s}{\partial r} = \frac{\lambda^2}{2}\phi_-^2 s - \omega^2 s$$

$$\frac{\partial^2 \phi_-}{\partial r^2} + \frac{2}{r}\frac{\partial \phi_-}{\partial r} = \left(\frac{\lambda^2}{2}s^2 - g^2\xi\right)\phi_- + \frac{g^2}{2}\phi_-^3$$

=> Solutions with a different Q_S for each ω

Two-field Q-ball profile $s(r)$, $\phi_-(r)$ for $\omega = 0.6$, $\lambda/g = 1$: $E/Q_S = 0.76 m_S$ => stable



[Matt Broadhead and JMcD, Phys. Rev. D69 (2004) 063510, hep-ph/0309298]

Oscillons appear to form at the end of D- and F-term Hybrid Inflation

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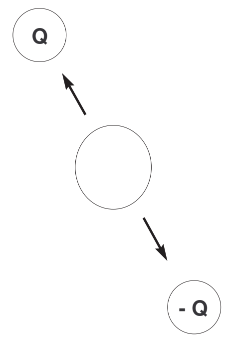
Stable Q-ball solutions of the SUSY hybrid inflation field equations exist

=>

Unstable neutral oscillons are likely to break-up into pairs of stable + and - charged Q-balls via growth of perturbations of the phase of the complex inflaton => lowest energy configuration



(Oscillon decay to Q-ball pairs observed in simulations of oscillon evolution in the context MSSM flat direction scalars *[Kasuya and Kawasaki, PR D61 (2000) 041301, D62 (2000) 023512; Enqvist et al PR D66 (2002) 043505]*)

∴ If oscillons form, a Q-ball dominated era following SUSY hybrid inflation is likely.

If the post-inflation era is dominated by oscillons or hybrid inflation Q-balls then post-inflation SUSY cosmology must be reconsidered:

- Reheating via oscillon/Q-ball decay
  Enhanced preheating: no dilution of the inflaton oscillation amplitude inside an oscillon
- Dynamics of flat direction scalars (MSSM flat direction field, SUSY curvaton, 'moduli') is different in an oscillon/Q-ball dominated background: No order H^2 corrections to scalar mass^2 terms

Order H^2 corrections come from Planck-suppressed interactions

$$\mathcal{L}_{int} = \frac{\lambda}{M_{Pl}^2} \int d^4\theta S^\dagger S \Phi^\dagger \Phi \rightarrow \frac{\lambda}{M_{Pl}^2} |F_S|^2 \Phi^\dagger \Phi \;\; ; \;\;\; |F_S|^2 = \partial_\mu S^\dagger \partial^\mu S + \left| \frac{\partial W}{\partial S} \right|^2$$

For a conventional homogeneous inflaton condensate, $|F_S|^2 = \rho_S \Rightarrow \mathcal{L}_{int} = \lambda \rho_S / M_{Pl}^2 \sim H^2 |\Phi|^2$

Q-balls typically have a Gaussian profile

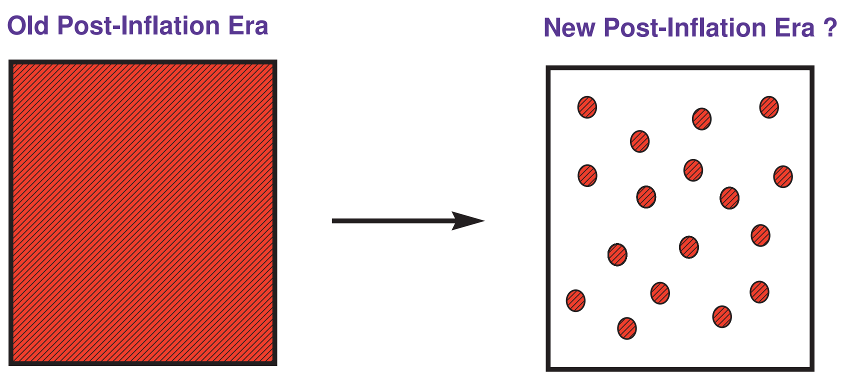
$$S(r,t) = s_o e^{-\frac{r^2}{R^2}} e^{i\omega t}$$

=> highly suppressed field outside the Q-balls => no H^2 correction outside the Q-balls. Near Q-balls, moduli fields deform slightly => gradient energy cancels effective mass^2 term => effectively no H^2 correction

• Important for combined D-term inflation/curvaton scenario

- Numerical simulations indicate an oscillon-dominated post-inflation era in D-term inflation for $\lambda/g \gtrsim 0.1$

  Likely to be a general feature of F-term inflation models ($\sim$ D-term with $\lambda/g \approx 1$)
- SUSY hybrid inflation models have Q-ball solutions => Q-ball-dominated era likely *if* oscillons form
- Post-inflation cosmology must be reconsidered:
  - Reheating via oscillon/Q-ball decay
  - SUSY scalar fields (moduli, curvatons, MSSM flat direction scalars ...) have no H^2 corrections to mass squared terms
  - Other consequences?
- Oscillon formation may also be a feature of non-SUSY hybrid inflation models
- More detailed numerical simulations are necessary to confirm or exclude a non-topological soliton-dominated post-inflation era of SUSY hybrid inflation models



In the vicinity of the Q-balls, $\rho_S \gg \overline{\rho}$ => very large ($\gg H^2$) mass^2 correction. How does this affect a coherently oscillating flat direction condensate?

Ans: The flat direction condensate acquires a very small ($\delta\Phi/\Phi \ll 1$) spatial dependence in the vicinity of the Q-ball such that its gradient energy cancels the effect of the Planck-suppressed interaction with the Q-ball
=> flat direction dynamics are equivalent to no H^2 correction.

Flat direction scalar $\Phi(\mathbf{x}, t)$:

$$\ddot{\Phi} - \nabla^2 \Phi = -\left(m_\phi^2 \Phi - \frac{\lambda}{M_{Pl}^2} f(S) \Phi \right) \quad ; \quad f(S) = |\dot{S}|^2 + |\nabla S|^2 \quad ; \quad S = s_o e^{-\frac{r^2}{R^2}} e^{i\omega t}$$

Solution for oscillating condensate in the vicinity of a Q-ball: $\Phi(r,t) = \frac{\phi(r)}{\sqrt{2}} \sin(m_\phi t)$

$$\phi(r) = \phi_o \exp\left(A e^{-\frac{2r^2}{R^2}} \right) \quad ; \quad A = \frac{\lambda \omega^2 R^2 s_o^2}{24 M_{Pl}^2}$$

$s_o/M_{Pl} \lesssim \sqrt{24/\lambda}\,\omega R \gtrsim 1$ => $A \ll 1$
=> Q-ball does not significantly alter the Φ evolution from the case with no H^2 correction

$\Rightarrow$ Scalar fields evolve as if no order H^2 corrections from SUSY-breaking background energy density *[JMcD, JCAP 08 (2004) 002, hep-ph/0308295]*