Polarisation on the CMB in Anisotropic Cosmological Model

by

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References:

S. S. Moskaliuk, A. V. Nesteruk, I. I. Sokolov, Hadronic J. **18**, 392 (1995).

S. S. Moskaliuk, A. V. Nesteruk, Phys. At. Nucl. **58**, 1744 (1995).

S. S. Moskaliuk, Hadronic J. **24**, 435 (2001); *ibid.* **24**, 531 (2001); *ibid.* **24**, 669 (2001).

S. S. Moskaliuk, Phys. Rev. D **68**, (2003) 084023.

In this report we will show that the CMB polarization arises not only from the Thompson scattering by electrons of a radiation field with a local quadrupole moment but from the quantum photon effectsthe too.

In this way, CMB polarization directly probes the dynamics from the epoch when the anisotropy of matter arose till the epoch of decoupling.

A determination of the CMB polarization would therefore provide a critical test of the underlying theoretical framework and therefore of the validity of cosmological parameters derived from CMB measurments.

Polarization meassuments also offer the potential to triple the number of observed CMB quantities and to enhance our ability to constrain cosmological parameters.

The first limit to the degree of polarization of the CMB was set by Penzias & Wilson (1965) who stated that the new radiation that they had discovered was isotropic and unpolarized within the limits of their observations.

Over the following 20 years, dedicated polarimeters have been used to set much more stringent upper limits on angular scales of order several degrees and larger (Caderni et al. 1978; Nanos 1979; Lubin & Smoot 1979, 1981; Lubin, Melese, & Smoot 1983; Sironi et al. 1997). The current best upper limits for the linesr and circular polarizations on large angular scales are 10 μK at 95% confidence for the multipole range $2 \leq l \leq 20$, set by the POLAR experiment (Keating et al. 2001).

In (Kovac et al. 2002) it was stated the first detection of polarized anisotropy in the CMB radiation with the Degree Angular Scale Interferometer (DASI), located at the Amundsen-Scott South Pole research station.

Here we consider the generation of the polarization of EMF under the supposition that the medium start be partially transparent. The result of this report is the assertion that the quantum effects of EMF in the external gravitational field in the anisotropic Bianchi I model give a contribution to the degree of polarization of EMF in the quadrupole harmonics.

It being known that the quantity of this effect parametrically depends on the moment of time starting from which the vacuum of EMF became unstable. On the assumption of the observational limits on the quantity of the degree of polarization of the CMB, one can determine the limits of the quantity of the red shift under which the quantum effects to play part.

In this report we present some results of the papers (Moskaliuk, & Nesteruk 1992, 1995; Moskaliuk, 2001-2003; etc.).

II. Formalism

A useful way to characterize the polarization properties of the CMB is to use the Stokes parameters formalism (Chandrasekhar 1960). For a nearly monochromatic plane electromagnetic wave propagating in the z direction,

$$E_x = a_x(t)\cos\left[\omega_0 t - \theta_x(t)\right], \qquad (1)$$
$$E_y = a_y(t)\cos\left[\omega_0 t - \theta_y(t)\right],$$

the Stokes parameters are defined by:

$$I \equiv \langle a_x^2\rangle + \langle a_y^2\rangle, \quad Q \equiv \langle a_x^2\rangle - \langle a_y^2\rangle, \qquad (2)$$
$$U \equiv \langle 2a_x a_y \cos(\theta_x - \theta_y)\rangle, \quad V \equiv \langle 2a_x a_y \sin(\theta_x - \theta_y)\rangle,$$

where the brackets $\langle\rangle$ represent time averages.

The parameter I is simply the average intensity of the radiation. The polarisation properties are described by the remaining parameters: Q and U describe linear polarisation, while V describes circular polarisation.

Unpolarised radiation (or natural light) is characterised by having $Q = U = V = 0$. CMB polarisation is produced through the photon quantum effects and Thomson scattering which cannot generate circular polarisation. Then, we can write $V = 0$ always.

The Stokes parameters Q and U are not scalar quantities. If we rotate the reference frame by an angle ϕ around the direction of observation, Q and U transform as:

$$\begin{aligned} Q' &= Q\cos(2\phi) + U\sin(2\phi), \\ U' &= -Q\sin(2\phi) + U\cos(2\phi). \end{aligned} \tag{3}$$

We can define a polarisation vector $\mathbf{P}$ having:

$$|\mathbf{P}| = \left(Q^2 + U^2\right)^{1/2}, \qquad \alpha = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right). \tag{4}$$

Although $\mathbf{P}$ is a good way to visualise polarisation, it is not properly a vector, since it remains identical after a rotation of π around z, thus defining an orientation but not a direction.

Mathematically, Q and U can be thought as the components of the second-rank symmetric trace-free tensor:

$$\mathbf{P}_{ab} = \frac{1}{2}\begin{pmatrix} Q & -U\sin\theta \\ -U\sin\theta & -Q\sin^2\theta \end{pmatrix}, \qquad (5)$$

where the trigonometric functions come from having adopted a spherical coordinate system.
We also describe the polarization effects of electromagnetic radiation with the aid of the polarization matrix J^{ab} defined in the plane orthogonal to the direction of propagation of the electromagnetic waves and expressed in terms of the Stokes parameters as follows:

$$J^{ab}_{+} = \frac{1}{2}\begin{pmatrix} I+Q & U \\ U & I-Q \end{pmatrix}, J^{ab}_{-} = \frac{1}{2}\begin{pmatrix} 0 & -iV \\ iV & 0 \end{pmatrix}. (6)$$

Here, I is the total intensity of radiation, the parameters Q and U are related to the degree of linear polarization and V determines the degree of circular polarization, correspondingly, via the relations

$$P_L = \sqrt{\frac{U^2+Q^2}{I}}, \quad P_C = V/I. \qquad (7)$$

III. The CMB polarization effects in anisotropic space-time

In the Bianchi I metrics

$$ds^2 = dt^2 - \sum_{i=1}^{3} A_i^2(t) \left(dx^i\right)^2 \,, \tag{8}$$

Maxwell equations for the free EMF can be written as

$$\nabla_\mu F^{\mu\nu} = 0 \,, \qquad \nabla_\mu (*F)^{\mu\nu} = 0 \,, \tag{9}$$

where $F_{\mu\nu}$ is electro-magnetic-field tensor and $(*F)^{\mu\nu}$ is adjoint magnitude, defined by the relation

$$(*F)^{\alpha\beta} = \frac{1}{\sqrt{-g}}[\alpha\beta\gamma\eta]F_{\gamma\eta}, \tag{10}$$

and $[\alpha, \beta, \gamma, \eta]$ is completely antisymmetric tensor [0123]=1.

The solutions of these equations can be represented in the form of electric- and magnetic-field vectors

$$\mathbf{E}(t,\mathbf{x}) = \int d^3k e^{i\mathbf{kx}} \left[\mathcal{E}^\theta(t,\mathbf{k})\mathbf{e}_\theta(t,\mathbf{k}) + \right. \\ \left. + \, \mathcal{E}^\varphi(t,\mathbf{k})\mathbf{e}_\varphi(t,\mathbf{k})\right], \tag{11}$$

$$\mathbf{H}(t,\mathbf{x}) = \int d^3k e^{i\mathbf{kx}} \left[\mathcal{H}^\theta(t,\mathbf{k})\mathbf{e}_\theta(t,\mathbf{k}) + \right. \\ \left. + \, \mathcal{H}^\varphi(t,\mathbf{k})\mathbf{e}_\varphi(t,\mathbf{k})\right], \tag{12}$$

The orthogonal vectors

$$\mathbf{e}_\theta = \cos\theta_t \, \cos\varphi_t \frac{\mathbf{e}_1}{A_1} + \cos\theta_t \, \sin\varphi_t \frac{\mathbf{e}_2}{A_2} - \sin\theta_t \frac{\mathbf{e}_3}{A_3}, \quad (13)$$

$$\mathbf{e}_\varphi = -\sin\varphi_t \frac{\mathbf{e}_1}{A_1} + \cos\varphi_t \frac{\mathbf{e}_2}{A_2}, \quad (14)$$

$$\mathbf{e}_k = \sin\theta_t \, \cos\varphi_t \frac{\mathbf{e}_1}{A_1} + \sin\theta_t \, \sin\varphi_t \frac{\mathbf{e}_2}{A_2} + \cos\theta_t \frac{\mathbf{e}_3}{A_3} \quad (15)$$

forming the tetrad unit basis in the momentum space.

The angles θ_t and φ_t are related with the spherical coordinates in the momentum space introduced via the relation

$$(k_1, k_2, k_3) = k \, (\sin\theta \, \cos\varphi, \, \sin\theta \, \sin\varphi, \, \cos\theta) \ , \quad (16)$$

using the formula

$$(\sin\theta_t \, \cos\varphi_t, \ \sin\theta_t \, \sin\varphi_t, \ \cos\theta_t) = \\ = \mu^{-1} \left(\frac{\sin\theta \, \cos\varphi}{A_1}, \ \frac{\sin\theta \, \sin\varphi}{A_2} \, , \ \frac{\cos\theta}{A_3} \right) , \quad (17)$$

where

$$\mu^2 = \frac{\sin^2\theta \, \cos^2\varphi}{A_1^2} + \frac{\sin^2\theta \, \sin^2\varphi}{A_2^2} + \frac{\cos^2\theta}{A_3^2} \, . \quad (18)$$

The components $\mathcal{E}^\theta$, $\mathcal{E}^\varphi$, $\mathcal{H}^\vartheta$, $\mathcal{H}^\varphi$ can be written as follows:

$$\begin{aligned}
\mathcal{E}^\theta(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^+ + y^-)\,,\\
\mathcal{E}^\varphi(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^+ - y^-)\,,\\
\mathcal{H}^\theta(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^+ - y^-)\,, \qquad (19)\\
\mathcal{H}^\varphi(t,\mathbf{k}) &= -\frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^+ + y^-)\,,
\end{aligned}$$

where

$$b = \frac{1}{\sqrt{-g}}\left(A_2^2\cos^2\varphi + A_1^2\sin^2\varphi\right)\,, \qquad (20)$$

and the functions $y^\pm = y^r$ satisfy the equation

$$\ddot{y}^r - \frac{\dot{b}}{b}\dot{y}^r + \left[k^2\mu^2 + rk\Delta\right]y^r = 0\,, \qquad (21)$$

$$\Delta = b\frac{d}{dt}(a/b)\,; \qquad a = \frac{\cos\theta\sin 2\varphi}{2\sqrt{-g}}\left(A_2^2 - A_1^2\right)\,. \qquad (22)$$

The density matrix can be represented in the form

$$J^{ab} = J_{+}^{ab} + J_{-}^{ab}, \quad a, b = \theta, \varphi,$$
$$J^{ab} = \frac{1}{2}\mathcal{E}^{\{a}(t, \mathbf{k}), \quad \overset{*}{E}{}^{b\}}(t, \mathbf{k}),$$
$$J_{-}^{ab} = \frac{1}{2}\mathcal{E}^{[a}(t, \mathbf{k}), \quad \overset{*}{E}{}^{b]}(t, \mathbf{k}), \qquad (23)$$

where the symbol $\{,\}$ ([,]) denotes symmetrization (antisymmetrization) with respect to the corresponding superscripts.

Substitution of the electric-field components (19) into expression (23) yields

$$J_{+}^{\theta\theta} = \left(2(2\pi)^3(-g)^{1/2}b\right)^{-1}\mu^2|Y^{+}|^2,$$
$$J_{+}^{\varphi\varphi} = \left(2(2\pi)^3(-g)^{1/2}bk^2\right)^{-1}|\dot{Y}^{-}|^2,$$
$$J_{+}^{\theta\varphi} = -\frac{\mu}{2}\left(2(2\pi)^3(-g)^{1/2}bk\right)^{-1} 2\mathrm{Re}\dot{Y}^{-}\,\overset{*}{Y}{}^{+},$$
$$J_{-}^{\theta\varphi} = \frac{i}{2}\frac{\mu}{2(2\pi)^3(-g)^{1/2}bk}2\mathrm{Im}\dot{Y}^{-}\,\overset{*}{Y}{}^{+}, \qquad (24)$$

where

$$Y^{\pm} = y^{+} \pm y^{-}.$$

Comparing equations (24) and (6), we find that the Stokes parameters are given by

$$
\begin{aligned}
I &= \frac{1}{2(2\pi)^3(-g)^{1/2}bk^2}\left[|\dot{Y}^-|^2 + K_0^2|Y^+|^2\right],\\
Q &= -\frac{1}{2(2\pi)^3(-g)^{1/2}bk^2}\left[|\dot{Y}^-|^2 - K_0^2|Y^+|^2\right],\\
U &= -\frac{\mu}{(2\pi)^3(-g)^{1/2}bk}\mathrm{Re}\dot{Y}^- \overset{*}{Y}{}^+, \qquad (25)\\
V &= -\frac{\mu}{(2\pi)^3(-g)^{1/2}bk}\mathrm{Im}\dot{Y}^- \overset{*}{Y}{}^+, \quad K_0 = k\mu.
\end{aligned}
$$

To pursue the investigation of polarization effects further, we assume that the propagation of waves in an anisotropic space-time can be described in the shortwave approximation; that is, we retain the first terms of the asymptotic expansion of the solutions of equation (21) for y^r in the limit $k \to \infty (\lambda \to 0)$.

Such an expansion was constructed by Sagnotti and Zwiebach (1981).

From their results, it follows that, in the leading approximation in k^{-1} for $k \to \infty$, the required solutions can be represented as

$$
\begin{aligned}
Y_0^+ &= \left(\frac{b\mu_0}{b_0\mu}\right)^{1/2} \left(C_0^+ e^{i\lambda} + C_0^- e^{-i\lambda}\right) e^{i\Omega}, \\
Y_0^- &= \left(\frac{b\mu_0}{b_0\mu}\right)^{1/2} \left(C_0^+ e^{i\lambda} - C_0^- e^{-i\lambda}\right) e^{i\Omega}, \\
\dot{Y}^+ &= iK_0 Y^+, \\
\dot{Y}^- &= iK_0 Y^-,
\end{aligned}
\qquad (26)
$$

where

$$
\lambda = \int_{t_0}^{t} \frac{\Delta}{2\mu}\, dt', \quad \Omega = \int_{t_0}^{t} k\mu dt', \qquad (27)
$$

t_0 corresponds to an arbitrary initial instant of propagation, and $C_0^\pm$ are the values of the functions $y^\pm(t)$ at the point t_0.

The substitution of the WKB solutions (26) into expressions (25) for the Stokes parameters yields

$$I = \frac{\mu\mu_0}{\sqrt{-g}(2\pi)^3 b_0}\left[|C_0^+|^2 + |C_0^-|^2\right], \quad (28)$$

$$Q = \frac{\mu\mu_0}{(2\pi)^3\sqrt{-g}b_0} 2\mathrm{Re}\left(C_0^+ \overset{*}{C}{}_0^- e^{2i\lambda}\right), (29)$$

$$U = \frac{\mu\mu_0}{(2\pi)^3\sqrt{-g}b_0} 2\mathrm{Im}\left(C_0^+ C_0^- e^{2i\lambda}\right), \quad (30)$$

$$V = -\frac{\mu\mu_0}{\sqrt{-g}b_0}\left[|C_0^+|^2 - |C_0^-|^2\right]. \quad (31)$$

Eliminating the constants from these relations, we can find the Stokes parameters as functions of time. These are given by

$$I(t) = \frac{\mu(t)}{\mu(t_0)}\sqrt{\frac{-g(t_0)}{-g(t)}}\, I(t_0), \quad (32)$$

$$Q(t) = \frac{\mu(t)}{\mu(t_0)}\sqrt{\frac{-g(t_0)}{-g(t)}}\, [Q(t_0)\cos 2\lambda(t) - \quad (33)$$

$$- U(t_0)\sin 2\lambda(t)], \quad (34)$$

$$U(t) = \frac{\mu(t)}{\mu(t_0)}\sqrt{\frac{-g(t_0)}{-g(t)}}\, [Q(t_0)\sin 2\lambda(t) + \quad (35)$$

$$+ U(t_0)\cos 2\lambda(t)], V(t) = \frac{\mu(t)}{\mu(t_0)}\sqrt{\frac{-g(t_0)}{-g(t)}}\, V(t_1).$$

It immediately follows that

$$P_L(t) = P_L(t_0), \quad P_C(t) = P_C(t_0); \tag{36}$$

that is, the degree of linear polarization and the degree of circular polarization do not vary with time as the electromagnetic wave propagates in an anisotropic space-time.

Under such conditions, the rotation of the polarization plane is the only nontrivial polarization effect. The angle τ of this rotation in the plane orthogonal to the direction of wave propagation is determined by the relation

$$\tan 2\tau = \frac{U}{Q}. \tag{37}$$

It follows that

$$\tan 2\tau(t) = \frac{\sin 2\lambda + \tan 2\tau(t_0)\cos 2\lambda}{\cos 2\lambda - \tan 2\tau(t_0)\sin 2\lambda}. \tag{38}$$

Differentiating this last relation, we find that the variable $y \equiv \tan 2\tau$ satisfies the equation

$$\frac{d}{dt}y = 2\lambda(1 + y^2). \tag{39}$$

Solving this equation, we obtain

$$\tau(t) = \lambda(t) + \tau_0$$

or

$$\Delta\tau = \int\limits_{t_0}^{t} \frac{\Delta}{2\mu}\, dt'. \tag{40}$$

Taking into account expression (22) for Δ and using the linear approximation in the anisotropy parameter $\Delta\bar{A} = A_2 - A_1$ [the latter is reduced to setting $b = \mu = 1/A$, where $A = (A_1 A_2 A_3)^{1/3}$], we obtain

$$\Delta\tau(t) = \frac{1}{2}\cos\theta \sin 2\varphi \left(A(t)\Delta A(t) - A(t_0)\Delta A(t_0)\right).$$

We can see that the polarization plane undergoes rotation only if the wave propagates in a direction other than that specified by the coordinate values $\theta = \frac{\pi}{2}(2k+1)$ and $\varphi = \frac{\pi}{2}k\,(k = 0, 1, 2, \ldots)$ and if $\Delta A \neq 0$, that is, if the anisotropic model under study is not axially symmetric.

IV. The CMB linear polarization effects due to the quantum generation of photons

Assuming that at the time moment t_{in} the Universe start be partially transparent on the one hand and the metrics can be represented on the other hand as the Bianchi I metrics

$$ds^2 = dt^2 - \sum_{i=1}^{3} A_i^2(t) \left(dx^i\right)^2 \ ,$$

Let us assume also that at $t < t_{\text{in}}$ the state of the EM field can be described with the density matrix with non-zero occupation number of the photons in the mode $n_0(\nu_0)$ corresponding to the black-body radiation.

The latter is strictly constant at $t < t_{\text{in}}$ and constant in the zeroth in respect to the anisotropy parameters approximation at $t > t_{\text{in}}$:

$$\frac{\partial}{\partial t} n_0(\nu_0) = 0.$$

The frequency ν_0 is considered to be independent of time and equal to the radiation frequency in the current epoch.

With the frequency at any time moment t it is related as follows

$$\nu_0 A(t_0) = \nu(t) A(t), \tag{41}$$

and $A^3 = (A_1 A_2 A_3)$ at $t > t_{\text{in}}$.
The particle number in the mode at the time moment $t > t_{\text{in}}$ satisfies the relation

$$n\,(t, \nu_0, \theta, \varphi) = n_0(\nu_0) + \\ + n_1\,(t, \nu_0, \theta, \varphi) + n_q\,(t, \nu_0, \theta, \varphi)\,. \tag{42}$$

$$n_1\,(t, \nu_0, \theta, \varphi) = n_0(\nu_0)\delta\,(t, \nu_0, \theta, \varphi)\,, \tag{43}$$

where $|\delta| \ll 1$ is the correction, describing the anisotropic distribution over momenta which arises due to the anisotropic expansion of the photons already existing to the time moment t_{in}.

The quantity

$$n_q\,(t,\nu_0,\theta,\varphi)=2\left(\sum_{r-\pm 1}|\beta^r\,(t,\nu_0,\theta,\varphi)\,|^2\right)\times$$
$$\times\;(2n_0(\nu_0)+1)\,, \qquad (44)$$

is the additional number of photons which arose due to their generation in the non-stationary gravitational field.

β^r is the coefficient of the Bogoliubov transformation at the transition to the time-independent operators of the generation-annihilation of photons bringing to the diagonal form the instantaneous Hamiltonian of the quantised EMF at the time moment t on the operators of the generation-annihilation, in terms of which the Hamiltonian has the diagonal form at the initial time moment t_{in} ;
$|\beta^r(t,\nu_0,\theta,\varphi)|^2$ is the density of the probability of the generation of a photon with a certain frequency ν_0, direction of the wave vector θ,φ, and the spin projection r on the direction of the wave vector.

Let us generalise the relation (42) – (44) to the case when the matter of interest is the particle number in the mode, which polarisation vector is oriented along a certain direction in the coordinate frame connected with the wave vector of the photon. Then we can introduce the symbolic vector

$$\mathbf{n}\left(t, \nu_0, \theta, \varphi\right) = \frac{1}{2}\frac{c^3}{h\nu^3(t)} \times \qquad (45)$$
$$\times \quad \left(\begin{array}{c} I\left(t, \nu_0, \theta, \varphi\right) + Q\left(t, \nu_0, \theta, \varphi\right) \\ I\left(t, \nu_0, \theta, \varphi\right) - Q\left(t, \nu_0, \theta, \varphi\right) \\ U\left(t, \nu_0, \theta, \varphi\right) \end{array} \right),$$

where I, Q, U are the Stokes parameters of the EM radiation. By analogy with (42), $\mathbf{n}$ can be represented as

$$\mathbf{n}\left(t, \nu_0, \theta, \varphi\right) = \mathbf{n}_0(\nu_0) + \qquad (46)$$
$$+ \mathbf{n}_1\left(t, \nu_0, \theta, \varphi\right) + \mathbf{n}_q\left(t, \nu_0, \theta, \varphi\right),$$

where

$$\mathbf{n}_0(\nu_0) = n_0(\nu_0) \left(\begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right),$$

which corresponds to the isotropic nonpolarised radiation.

In the case when there is no scattering of the photons on the electrons of the cosmic plasma, $\mathbf{n}_1$ can be represented as

$$\mathbf{n}_1\,(t,\nu_0,\theta,\varphi) = n_0(\nu_0)\,(\alpha(t,\nu_0)\mathbf{a}(\theta,\varphi)+ \\ +\,\bar{\alpha}(t,\nu_0)\bar{\mathbf{a}}(\theta,\varphi))\,,$$

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ o \end{pmatrix} \left(\cos^2\theta - \frac{1}{3}\right), \qquad (47)$$

$$\bar{\mathbf{a}} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \left(1 - \cos^2\theta\right)\cos 2\varphi.$$

This corresponds to the start of the dependence on angles, i.e. anisotropy, in the distribution of the photons over momenta.

The coefficients α and $\bar{\alpha}$ characterise the degree of the quadrupole anisotropy of the CMB.

The quantity $\mathbf{n}_q$ describes the contribution of the quantum effects.

In the linear in respect to the anisotropy parameters approximation of the metrics (8) it can be represented in the form:

$$\mathbf{n}_q\,(t,\nu_0,\theta,\varphi) = \frac{2n\alpha(\nu_0)+1}{2}\,\times \qquad (48)$$
$$\times\ \frac{c^3}{h\nu^3(t)}\begin{pmatrix} Q^{\text{vac}}\,(t,\nu_0,\theta,\varphi) \\ -Q^{\text{vac}}\,(t,\nu_0,\theta,\varphi) \\ 2U^{\text{vac}}\,(t,\nu_0,\theta,\varphi) \end{pmatrix}.$$

The quantities I^{vac}, Q^{vac}, U^{vac} are the vacuum Stokes parameters (VSP), evaluated for the case when the initial state of the EM field at the time moment t_{in} was vacuum. In general (i.e. not in the linear approximation) the Stokes parameters are related with the polarisation density matrix of the quantum effects of the EM field as a new characteristics of the latter.

As usually:

$$J^{ab} = \frac{1}{2}\left(\begin{array}{cc} I^{\mathsf{vac}} + Q^{\mathsf{vac}} & U^{\mathsf{vac}} - iV^{\mathsf{vac}} \\ U^{\mathsf{vac}} + iV^{\mathsf{vac}} & I^{\mathsf{vac}} - Q^{\mathsf{vac}} \end{array} \right), \qquad (49)$$

where

$$J^{ab\,vac} = J_{+}^{ab\,vac} + J_{-}^{ab\,vac},$$

$$J_{\pm}^{ab\,vac}(t, \mathbf{k}) = \frac{1}{2}\frac{hk}{\mu(t,\theta,\varphi)} \times$$
$$\times \ \langle 0_{t_{\mathsf{in}}}| N_t \left[\widehat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k}), \widehat{\mathcal{E}}^b(t,\mathbf{x},\mathbf{k}) \right] |0_{t_{\mathsf{in}}}\rangle.$$

Here $\widehat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k})$ are the components of the spectral component of the vector of electric field (19), multiplied by $\exp(i\mathbf{kx})$.
The calculations have shown that $J^{ab\,vac} = 0$, i.e. according to the common interpretation, the generating photons do not have an admixture of the circular polarisation.

The symmetric part of the polarisation tensor could be expressed via spectral components of the vacuum averages of the operator of energy-momentum tensor

$$T_{\mu\nu}^{\mathsf{vac}}(t) = \int d\varphi d\theta \sin\theta \int dK_0\left(t,k,\theta,\varphi\right) \times$$
$$\times \ \tilde{T}_{\mu\nu}^{\mathsf{vac}}\left(t,k,\theta,\varphi\right),$$
$$K_0\left(t,k,\theta,\varphi\right) = ck\mu\left(t,\theta,\varphi\right) \ ,$$

as follows

$$J_{+}^{ab} = G_{\mu\nu}^{ab}\tilde{T}_{\mu\nu}^{\mathsf{vac}}.$$

The specific form of the components $G_{\mu\nu}^{ab}$ has been evaluated by Grib A.A. and Nesteruk A.V. (1983).

Where also is given the explicit form of the spectral components $\tilde{T}_{\mu\nu}^{\mathsf{vac}}$.

So the VSP can be presented as :

$$\left(I^{\mathsf{vac}}, Q^{\mathsf{vac}}, U^{\mathsf{vac}}, V^{\mathsf{vac}}\right) = \tag{50}$$
$$= \frac{hk^3}{V} \sum_{r=\pm 1} \left(2s^r, u^r, r\tau^r, 0\right),$$

The functions s^r, u^r, τ^r satisfy the set of the equations

$$\begin{cases} \dot{s}^r = \frac{W}{2} u^r + r\frac{\overline{W}}{2}\tau^r, \\ \dot{u}^r = W\left(2s^r+1\right) - \left(r\overline{W} + 2cK_0\right)\tau^r, \\ \dot{\tau}^r = r\overline{W}\left(2s^r+1\right) + \left(r\overline{W} + 2cK_0\right)u^r, \end{cases} \quad (51)$$

$$\left(u^r\right)^2 + \left(\tau^r\right)^2 = 4s^r\left(s^r+1\right)$$

with the initial values
$s^r(t_{\text{in}}) = u^r(t_{\text{in}}) = \tau^r(t_{\text{in}}) = 0$.

The quantities W and $\overline{W}$ in the metrics, linear in respect to the anisotropy parameters of metrics (8) are as follows:

$$W = \left(1-\cos^2\theta\right)\Delta H + \frac{\overline{\Delta H}}{2}\left(1+\cos^2\theta\right)\cos 2\varphi,$$
$$\overline{W} = -\cos\theta\,\sin 2\varphi\overline{\Delta H}, \qquad (52)$$

where

$$\Delta H = H - \frac{1}{2}\left(H_1 + H_2\right), \qquad \overline{\Delta H} = H_1 - H_2,$$
$$H^3 = (H_1 H_2 H_3), \qquad H_i = \dot{A}_i / A_i$$

(the parameters ΔA, $\overline{\Delta A}$, A could be introduced similarly).

The set of the equations (51) plays the part of the transfer equations for VSP.

Let us analyse the expressions (50), solving (51) via expansion of the functions in a power series in respect to small parameter $\tilde{h}$ which is introduced as

$$\Delta H \to \tilde{h}\Delta H, \quad \overline{\Delta H} \to \tilde{h}\overline{\Delta H},$$
$$\Delta A \to \tilde{h}\Delta A, \quad \overline{\Delta A} \to \tilde{h}\overline{\Delta A}.$$

In doing so,

$$s^r = \sum_{n=0} \tilde{h}^n s^r_n, \; u^r = \sum_{n=0} \tilde{h}^n u^r_n, \; \tau^r = \sum_{n=0} \tilde{h}^n \tau^r_n. \tag{53}$$

The expansions of W and $\overline{W}$ are given with (52). Further we shall keep in the expansion (53) only linear terms in respect to $\tilde{h}$. In the zeroth approximation in respect to $\tilde{h}$ it follows from (51), (52), taking into the account the initial values, that

$$s^r_0 = u^r_0 = \tau^r_0 = 0.$$

This means that there are no photon quantum effects in isotropic case.

In the linear approximation the set of the equations for s_1^r, u_1^r, τ_1^r is

$$\dot{s}_1^r = 0, \quad \dot{u}_1^r = W - 2\nu\tau_1^r, \quad \dot{\tau}_1^r = r\overline{W} + 2\nu u_1^r$$

(in the zeroth approximation in respect to $\tilde{h}$ $K_0(t,k,\theta,\varphi) = k_0/A(t) \equiv \nu(t)$).

Solving this set, one can obtain the expressions for VSP in the linear approximation:

$$\left(I^{\text{vac}},\, Q^{\text{vac}},\, U^{\text{vac}}\right) = \tag{54}$$

$$= \frac{2h\nu^3}{c^3} \int\limits_{t_{\text{in}}}^{t} \left(0, W(t'), \overline{W}(t')\right) \cos\left(2(\Omega(t) - \Omega(t'))\right) dt',$$

$$\Omega(t) = \int dt \nu(t).$$

Such a distribution of the VSP quantum effects in the anisotropic gravitational field is unusual from the viewpoint of the classical electrodynamics.

The reason is the structure of the vacuum energy-momentum tensor of the EMF in the external gravitational field.

(Zeldovich, & Starobinsky (1971) have remarked that quantum effects of the material field in the external anisotropic gravitational field bring about the breaking of the condition of the energy dominance of the energy-momentum tensor (EMT) of these fields.)

This fact shows itself in different dependence of the EMT components on the anisotropy parameters of the metrics (8), namely:

$$T_0^0 \sim \tilde{h}^2, \qquad T^{ik} \sim \tilde{h}, \qquad i,k = 1,2,3.$$

The fact that EMT does not satisfy the condition of the energy dominance means that it contains both the contribution from the really generated particles and the contribution of the vacuum polarisation of the EM field.

Let us bring $\mathbf{n}_q$ to the form analogous that of $\mathbf{n}_1$, singling out explicitly the dependence on the angles θ and φ. To this end we use (52) and (54), then

$$\mathbf{n}_q(t,\nu_0,\theta,\varphi) = \frac{2n_0(\nu_0)+1}{2} \times \\ \times \left(\beta_q(t,\nu_0)\mathbf{b}(\theta,\varphi) + \bar{\beta}_q(t,\nu_0)\bar{\mathbf{b}}(\theta,\varphi)\right), \tag{55}$$

here

$$\beta_q(t) = \int_{t_{\text{in}}}^{t} \Delta H(t')\cos\left(2(\Omega(t)-\Omega(t'))\right)dt'. \tag{56}$$

The expression for $\bar{\beta}_q$ can be obtained from (56), substituting $\overline{\Delta H}$ for ΔH. Vectors $\mathbf{b}$ and $\bar{\mathbf{b}}$ are defined via the relations:

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} (1-\cos^2\theta),$$

$$\bar{\mathbf{b}} = \frac{1}{2}\begin{pmatrix} (1+\cos^2\theta)\cos 2\varphi \\ -(1+\cos^2\theta)\cos 2\varphi \\ 4\cos\theta\sin 2\varphi \end{pmatrix}.$$

Assuming that $n_0 \gg 1$ and substituting (47), (55) into (56), we obtain

$$\mathbf{n} = \mathbf{n}_0 + n_0\left[\alpha\mathbf{a} + \beta_q\mathbf{b} + \bar{\alpha}\bar{\mathbf{a}} + \bar{\beta}_q\bar{\mathbf{b}}\right]. \tag{57}$$

The quantities β_q and $\bar{\beta}_q$ are related with the degree of the linear polarisation of the CMB.

The relation (57) is similar to that derived by Basco M.M. and Polnarev A.G. (1980) under the solution the radiation transfer equation taking into the account Thomson scattering of the photons on the electrons of the cosmic plasma.

The quantities which undergo the measurement in the course of experiment are the Stokes parameters I, Q and U.

Let us evaluate, for example, the Stokes parameter Q in the Heckman-Schüking model. According to (45), (57)

$$Q(t, \nu_0, \theta, \varphi) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \left(1 - \cos^2\theta\right) \beta_q(t, \nu_0).$$

The dependence of Q on the time t is determined with the quantity β_q (56). Let us transform it to more clear form.

To this end let us temporarily change in the integral (56) the integration variable X as follows:

$$
\begin{aligned}
X &= (1+\chi)^{-1/2}, \\
\Delta H &= \Delta H_0 (1+\chi)^3 = \Delta H_0 / X^6, \\
dt &= -\frac{1}{H_0} \frac{d\chi}{(1+\chi)^{5/2}} = \frac{2}{H_0} X^2 dX \, , \\
\nu(t) &= \nu_0 (1+\chi) = \nu_0 X^{-2},
\end{aligned}
$$

where ΔH_0, H_0, ν_0 are the current values of the corresponding parameters. By this change of variable the integral is brought to the form

$$
\beta_q = \frac{\Delta H_0}{H_0} \int\limits_{X_{\mathrm{in}}}^{X} \frac{\cos \lambda (X - X')}{X'^4} dX', \qquad (58)
$$

where $\lambda = 4\nu_0 / H_0 \approx 10^{30}$ is a large parameter.

Estimating (58) asymptotically in λ, we obtain

$$
\beta_q = \frac{\Delta H_0}{H_0} \frac{1}{\lambda X_{\mathrm{in}}^4} \sin \left(\lambda (X - X_{\mathrm{in}}) \right) .
$$

Let us change in the last formula from the current variable X to the synchronous time t according to the relation $X = (3H_0 t/2)^{1/3}$.

The time t is synchronous cosmological time, counted from the singularity.

Let us divide it into two summands as follows:

$$t = t_0 + t', \qquad t' \ll t_0,$$

where $t_0 = \frac{2}{3H_0}$ is the time counted from the beginning of the expansion, corresponding to the current epoch, and t' is the current time, for example, the period, during which the observations take place.

Then

$$Q(t,\nu_0,\theta,\varphi) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0)(1 - \cos^2\theta) \times \\ \times \ \frac{\Delta H_0}{H_0}\frac{\chi_{\rm in}}{\lambda}\sin\Big(2\nu_0 t' + \lambda(1 - X_{\rm in})\Big).$$

Let us discuss the possibility of experimental measuring of Q. The power of the polarised component of interest for us of the EM radiation, hitting the aerial of the radio-telescope, with the directivity diagram $P_n(\theta, \varphi)$ and the effective area of the surface A_{eff} per unit frequency band is defined as:

$$W(t) = \frac{1}{2} A_{\text{eff}} \left| \int\int_{\Omega} Q(t, \nu_0, \theta, \varphi) P_n(\theta, \varphi) d\Omega \right| . \quad (59)$$

Separating the dependence on angle, we bring (51) to the form

$$W(t) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \Omega_{\text{eff}}^{Q} \left| \beta_q(t, \nu_0) \right| .$$

The quantity $W(t)$ is the instantaneous power of the signal in the aerial. On the load of the aerial the voltage is induced, the square of which is proportional to the instantaneous power, so that the current at the output of the detector is as follows:

$$i_{\text{det}}^{Q}(t) = k' \widehat{U}^2(t) = k' W(t) =$$
$$= k' \Omega_{\text{eff}}^{Q} \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \left| \beta_q(t, \nu_0) \right| .$$

After averaging over time, we obtain

$$\overline{i^{Q}_{\text{det}}} = k'\left(\Omega^{Q}_{\text{eff}}\right)\left(\frac{2h\nu_0^3}{c^3}n_0(\nu_0)\right)\frac{\Delta H_0}{H_0}\frac{\chi_{\text{in}}^2}{\lambda}\frac{2}{\pi}\,.$$

Similarly, measuring the Stokes parameter I in the zeroth approximation with respect to the anisotropy parameter ΔH, we arrive at

$$\overline{i^{I}_{\text{det}}} = k'\left(\Omega^{T}_{\text{eff}}\right)\left(\frac{2h\nu_0^3}{c^3}n_0(\nu_0)\right).$$

The observable degree of polarisation turns out to be

$$P = \frac{\overline{i^{Q}_{\text{det}}}}{\overline{i^{I}_{\text{det}}}} = \frac{2}{\pi}\frac{\Omega^{Q}_{\text{eff}}}{\Omega^{I}_{\text{eff}}}\frac{\Delta H_0}{H_0}\frac{\chi_{\text{in}}^2}{\lambda}\,. \tag{60}$$

Thus, the degree of polarisation of the CMB due to the quantum effects turns out to be of essence only in the case, when the anisotropy of the metrics has manifested itself sufficiently early, i.e. under the red shifts χ_{in}, which are determined by the order of magnitude from the relation $\chi_{\text{in}}^2/\lambda \sim 1$. For the characteristic value $\lambda \approx 10^{30}$ at $\nu_0 \approx 10^{12}_{Hz}$ and $H_0 \approx 10^{-18}$ we conclude that the effect is important if $\chi_{\text{in}} \approx 10^{15}$.

V. Conclusions

- The background of this report constitutes the idea, which attempts to explain some matter in the Universe as produced by quantum creation from vacuum.

- Thus, it turns out that in the case of the transparent medium when there is no scattering of the photons, the CMB in the homogeneous anisotropic and non-stationary Universe becomes linearly polarised due to the quantum effects of the photon generation and polarisation of the vacuum of the EM field. It is remarkable that the angular dependence of the photon number is quadrupole and completely coincides with that arising under the scattering of the photons on electrons in the epoch of the recombination or the secondary ionization.

- The peculiar feature of (57) is that β_q is defined according to (56) independently of the degree of anisotropy in contrast to Basco M.M. and Polnarev A.G. (1980), where both characteristics of the radiation could be determined from the general transfer equation.

- The importance of CMB polarisation concerning the physics of the early universe relies on the fact that scalar fluctuations and the photon quantum effects can produce only linear polarisation and no circular polarisation. A measurement of the circular polarisation could therefore be interpreted as the detection of primordial gravitational waves.

- Anisotropy in a cosmological photon distribution usually refers to the cosmic background radiation after the decoupling of photons from matter.

- Such anisotropy is predominantly caused by density perturbations in the matter at the epoch of last scattering. Here we investigated another kind of anisotropy, not connected with density perturbations or gravity wave perturbations, and not arising from scalar field fluctuations in the inflationary epoch, but caused by vacuum-polarisation type quantum effects of the electro-magnetic field itself. These early-universe effects would be partially washed out by the interaction of photons with matter during the recombination epoch. As such, our results are a contribution to the early-universe dynamics. The new frontier of cosmological exploration will then shift towards observations of the CMB linear polarisation, for which the rotation of the polarization plane is the second nontrivial polarization effect, if the anisotropic model under study is not axially symmetric .