Multiple Inflation
&
the String Landscape

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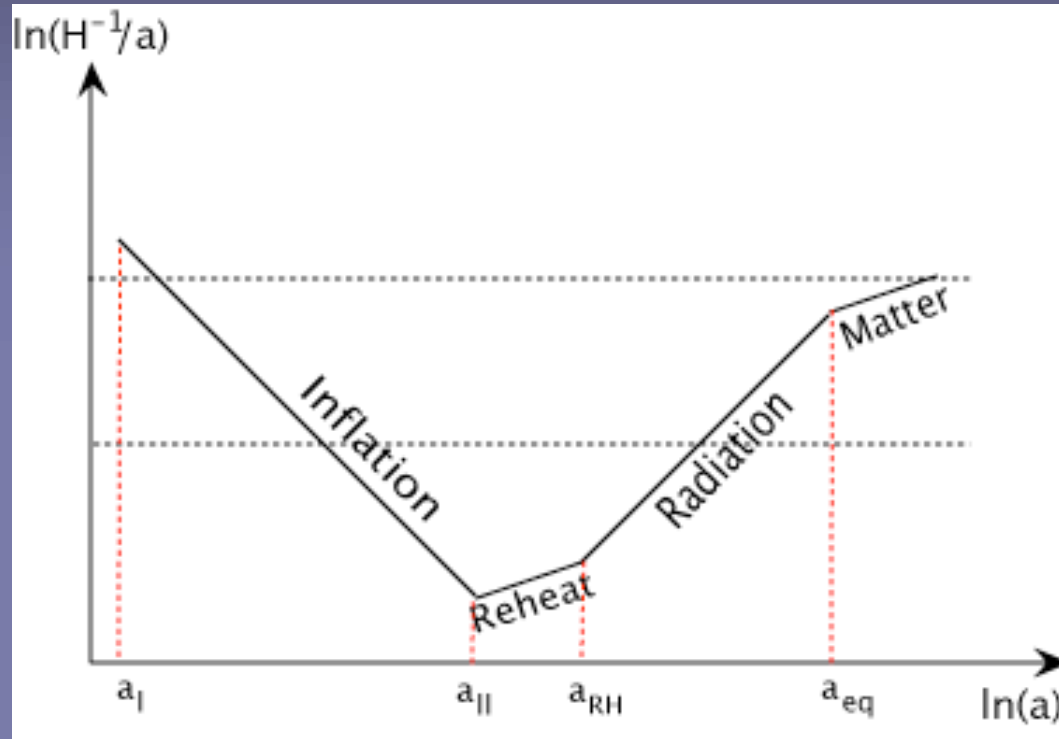
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With: C. Burgess, R. Easther, A. Mazumdar and D. Mota

Inflation: Standard Lore

- Modes are generated and leave the horizon to re-enter later
- Existence of large scale fluctuations requires (mode entering now):

$$N_{tot} \approx 70 + \frac{1}{4}\ln\left(\frac{V}{M_{Pl}^4}\right)$$

Inflation: Observations

- Inflationary predictions are probed observationally by observing imprints of the primordial perturbation spectrum
  - CMB sky
  - Galaxy surveys
- Existence of large-scale power is strong evidence for inflation (or for some other process of creating perturbations larger than the Hubble radius)
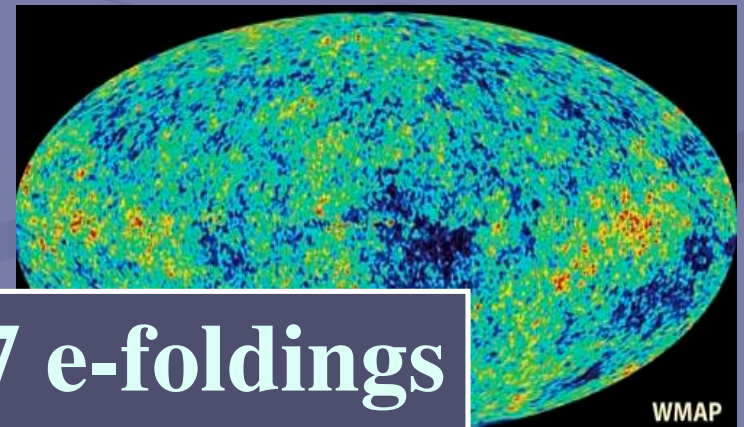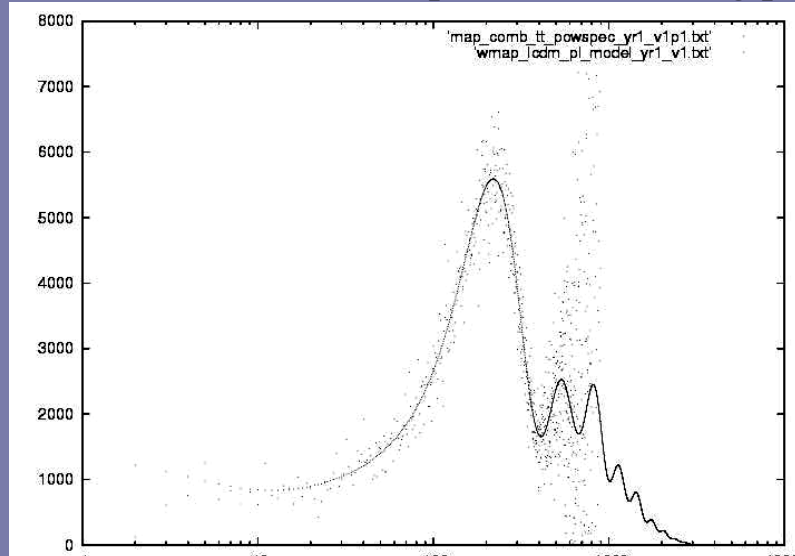
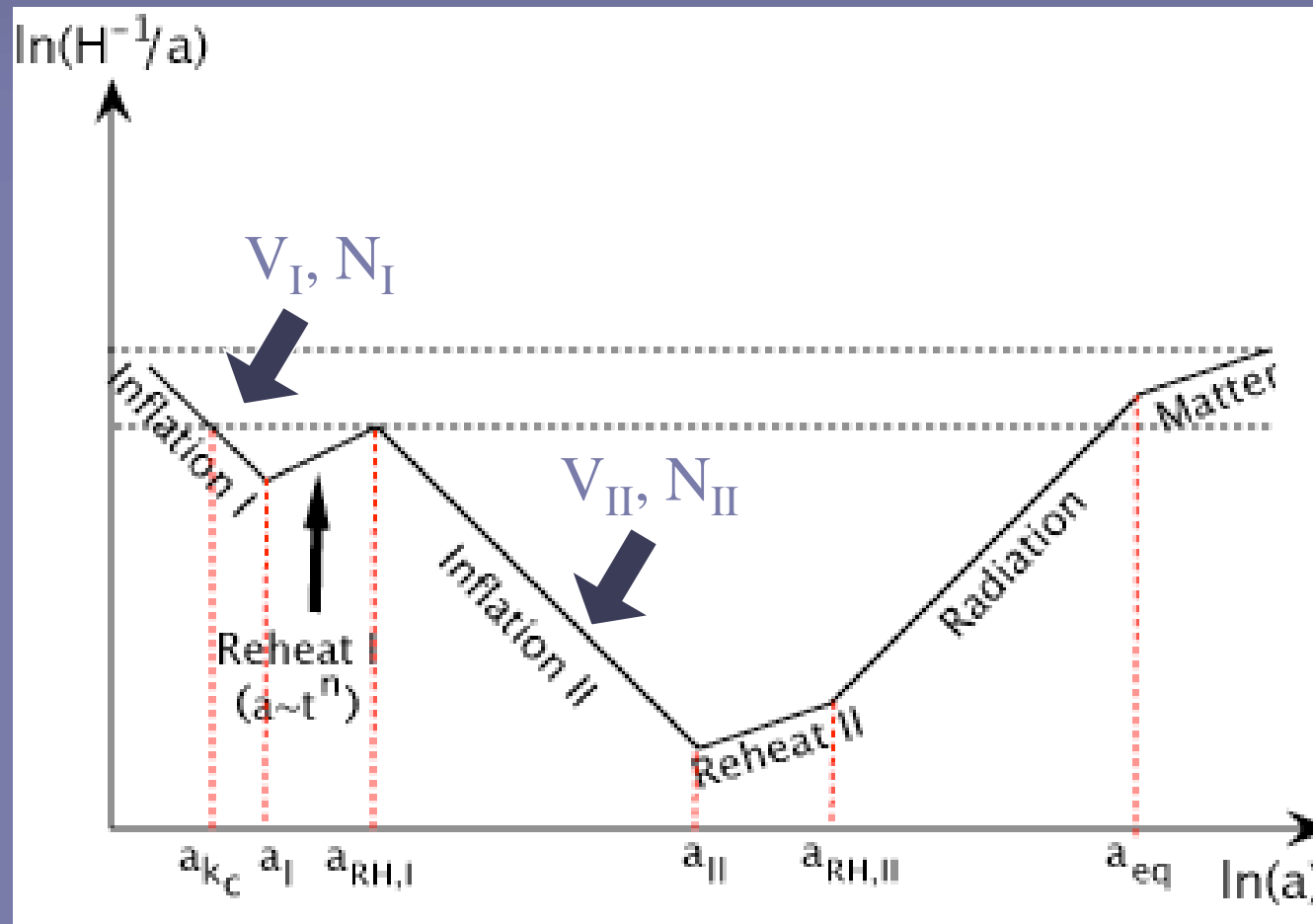**We only "see" about 7 e-foldings**

Inflation: Multiple stages
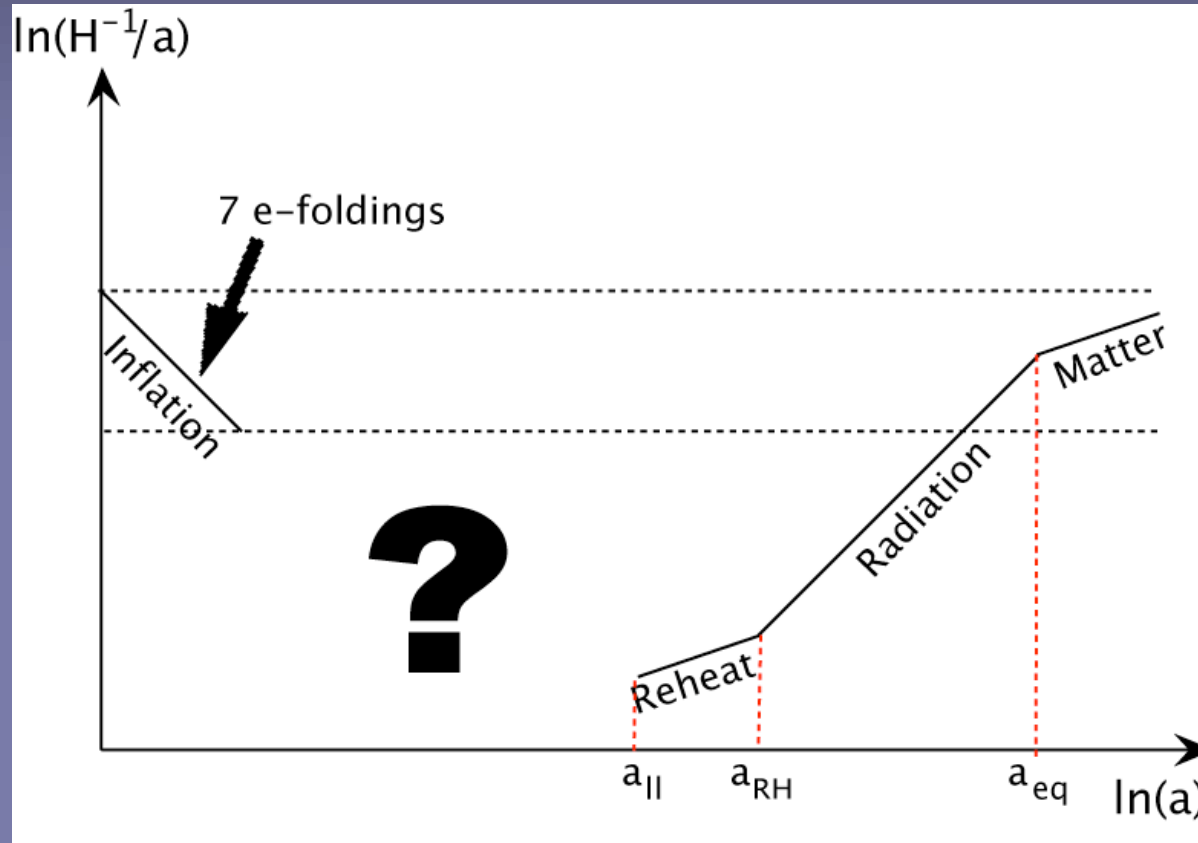
Wish List
- 7 e-foldings of flat spectrum
- existence of large scale modes
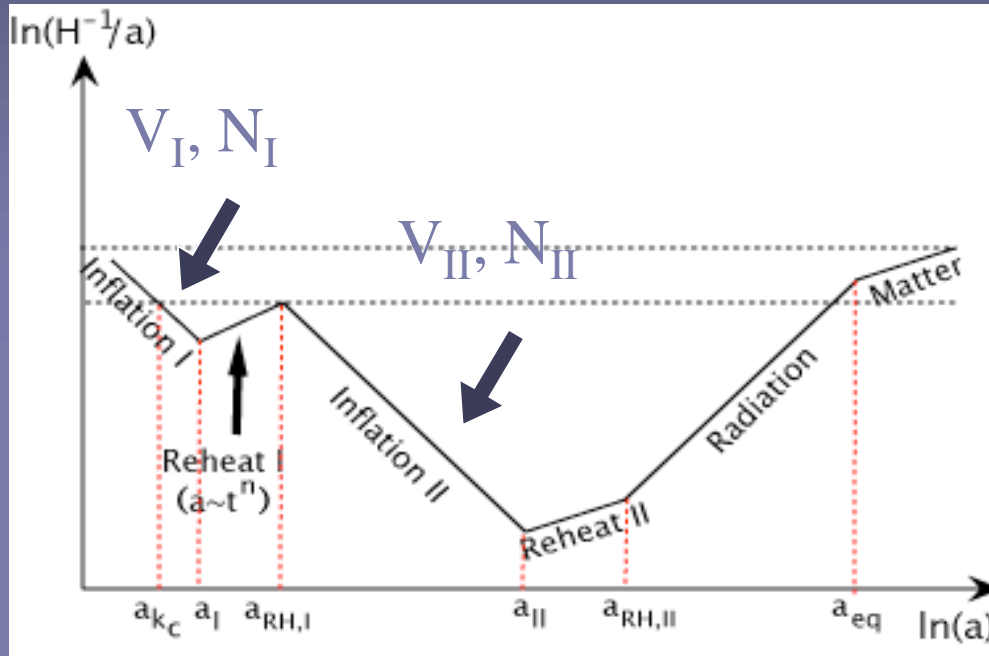- (flatness)

Multiple Inflation: Basics

A more generic picture of inflation has a number of different stages of inflation

Two inflationary stages, something in between (matter n=2/3, radiation n=1/2)

Multiple Inflation

To avoid reprocessing of modes:

$$N_I > 7 + \frac{1-n}{2} \ln\left(\frac{V_I}{V_{II}}\right)$$
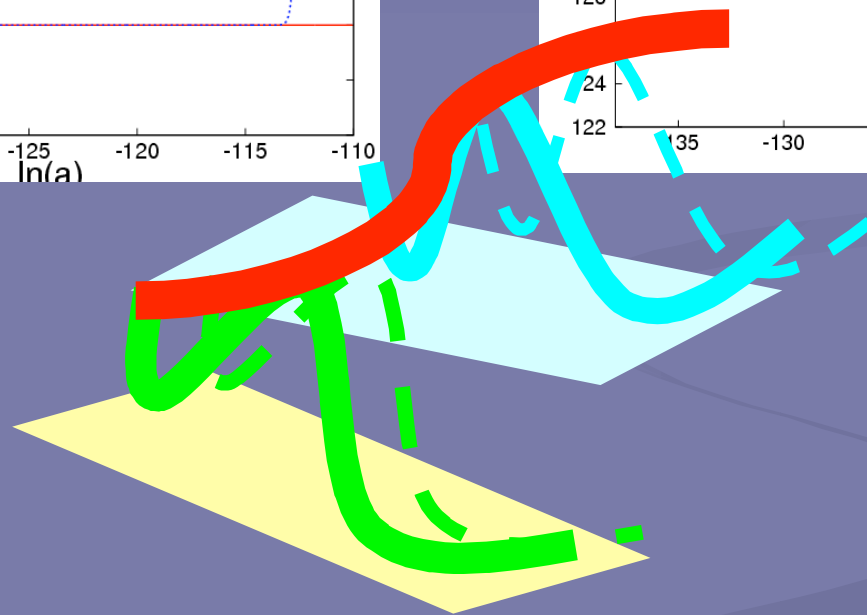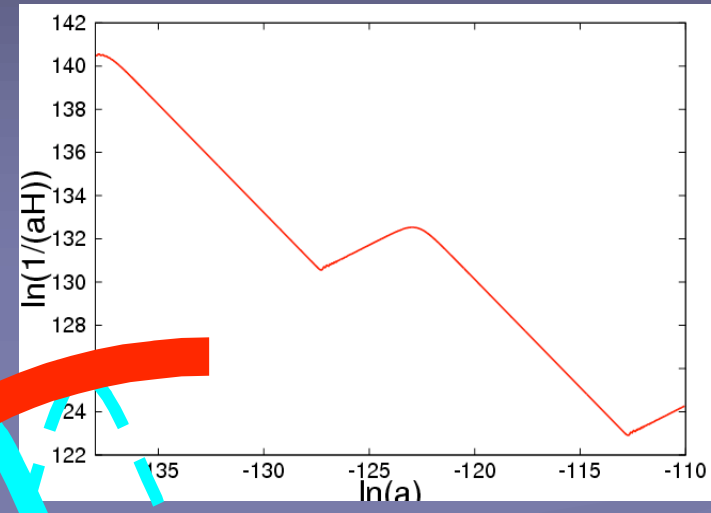
Mode entering now requires:

$$N_I + N_{II} \approx 70 + \frac{1}{4} \ln\left[\frac{V_{II}}{M_{Pl}^4}\left(\frac{V_I}{V_{II}}\right)^{2(1-n)}\right]$$

Eg. $V_I \sim (10^{16}\ \text{GeV})^4$, $V_{II} \sim (10^3 \text{GeV})^4$ $n=2/3$:

$N_{tot}=63$, $N_I>27$

(matter $n=2/3$, radiation $n=1/2$)
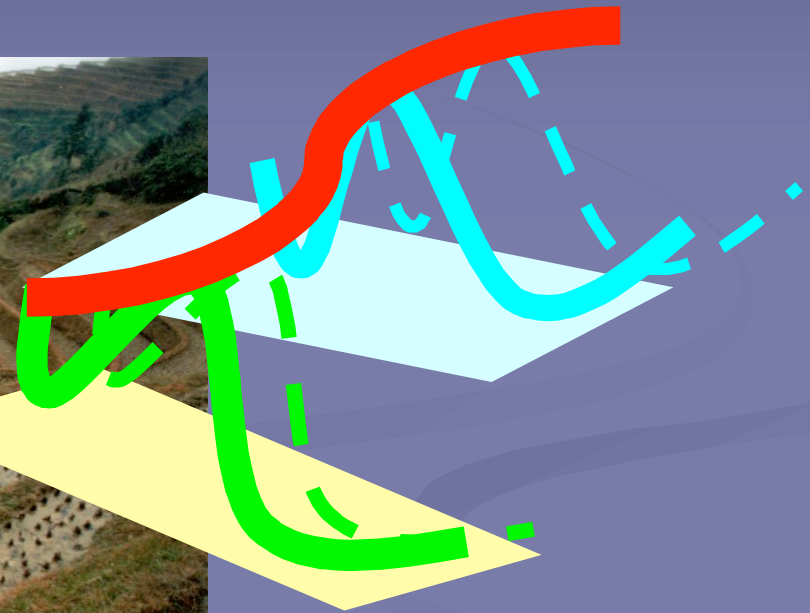
Multiple inflation: toy model
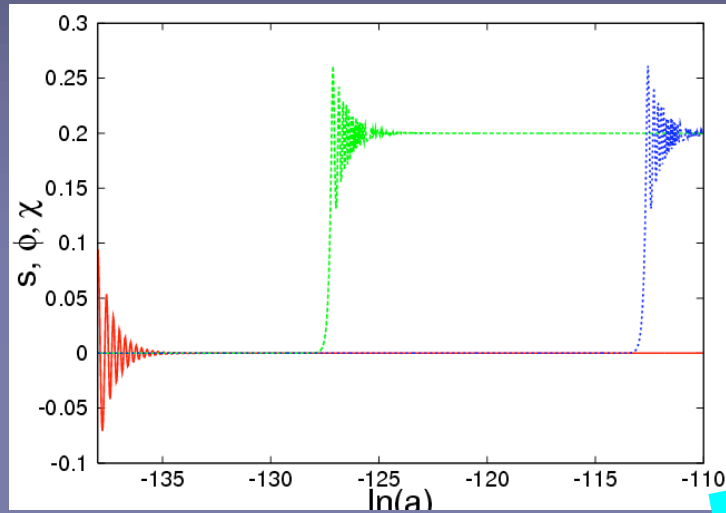
$$V = \lambda_1^2(\phi^2 - \phi_c^2)^2 + \lambda_2 S^2\phi^2 + \frac{1}{2}m^2 S^2 + g_1^2(\chi^2 - \chi_c^2)^2 + g_2^2 S^2\chi^2 + \ldots$$

Inflationary Landscape

- 60+ e-folding requires fine-tuning
- moduli fields are abundant

Solution?: Inflation is a multi-step process caused by the moduli rolling in a complicated landscape

Cosmic String Networks

$$N_I > 7 + \frac{1-n}{2}\ln\left(\frac{V_I}{V_{II}}\right)$$
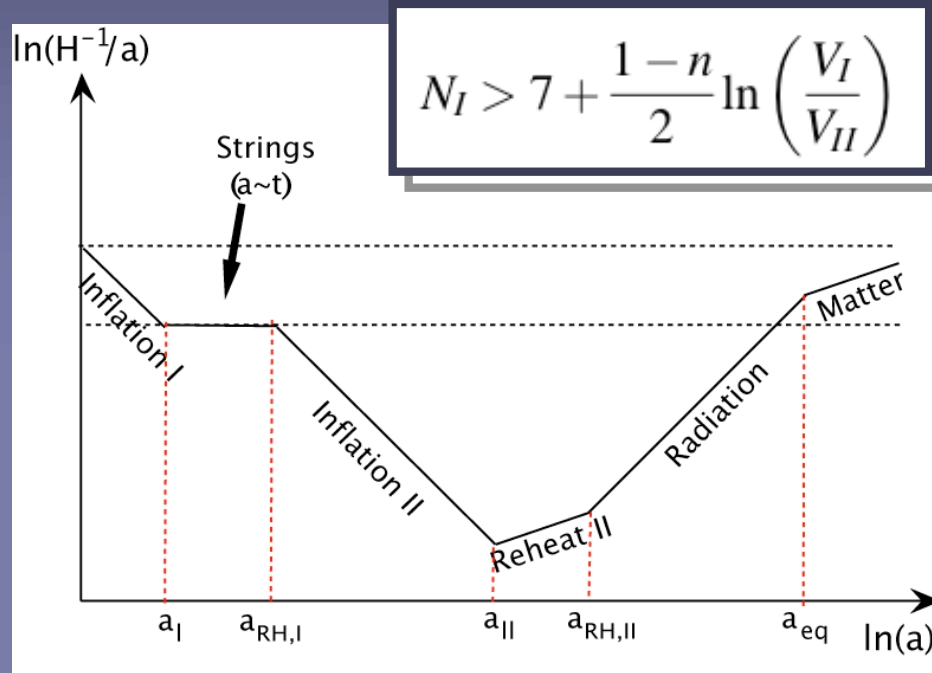
- Special case: n=1, w=-1/3
- If n=1, the least amount of e-foldings are required from the first stage
- Cosmic string network has the right property
- Can be produced by brane-anti brane annihilation

Brane   Brane
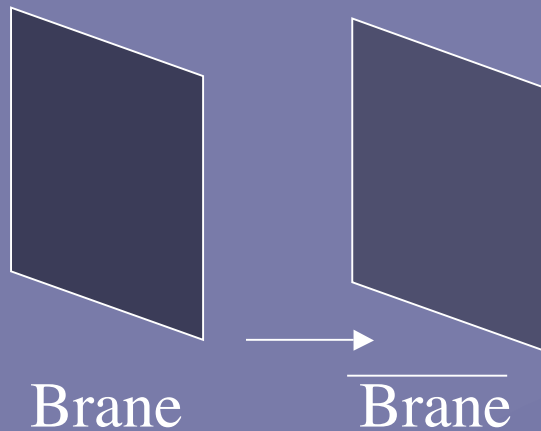
Multiple Inflation: mode evolution

Mode evolution of the curvature perturbation:

$$\partial_\tau u_k + \left(k^2 - \frac{\partial_\tau^2 z}{z} \right) u_k = 0$$

$$u_k = |z\mathcal{R}_k|, \; z \equiv a\dot{\phi}/H$$

Power spectrum:

$$\mathcal{P}_{\mathcal{R}}^{1/2} = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{z} \right|$$

Deep inside the Hubble radius we have a Bunch-Davies vacuum:

$$u_k \propto \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

Multiple Inflation: mode evolution

$$' \equiv \frac{d}{d\eta}, \quad \eta = \ln(a)$$

Curvature perturbation:

$$u_k'' - \frac{1}{2}(1+3w)u_k' + \left[\frac{k^2}{H_0^2}a^{1+3w} - \frac{1}{2}(1-3w)\right]u_k = 0$$
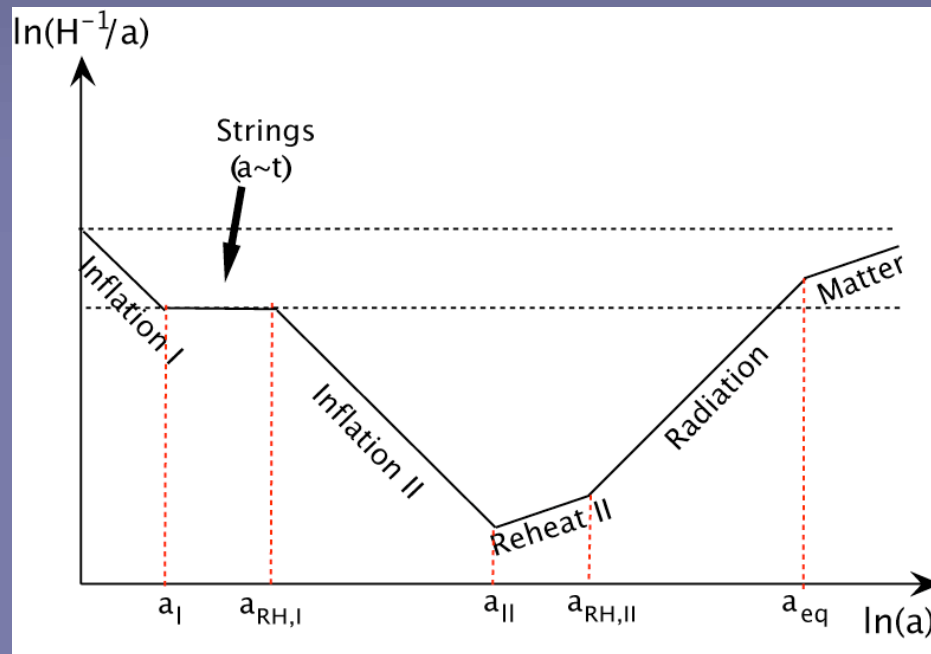
$$u_k = \left|a\mathcal{R}_k\sqrt{1+w}\right|$$

A special case: String network (w=-1/3)

$$u_k'' + \left[\frac{k^2}{H_0^2} - 1\right]u_k = 0$$

Spectrum is affected (k=1 is horizon size):

$$\mathcal{P}_{\mathcal{R}}(k) \sim \left(\frac{a_{RH,I}}{a_I}\right)^{2\sqrt{1-k^2}-4}$$

Multiple Inflation: minimum no. of e-foldings

• Modes "hovering" just outside the Hubble radius are affected
• The reprocessing of modes causes bending of the spectrum
• In order to preserve a flat spcetrum, observable modes must leave the horizon at an earlier moment to avoid this reprocessing
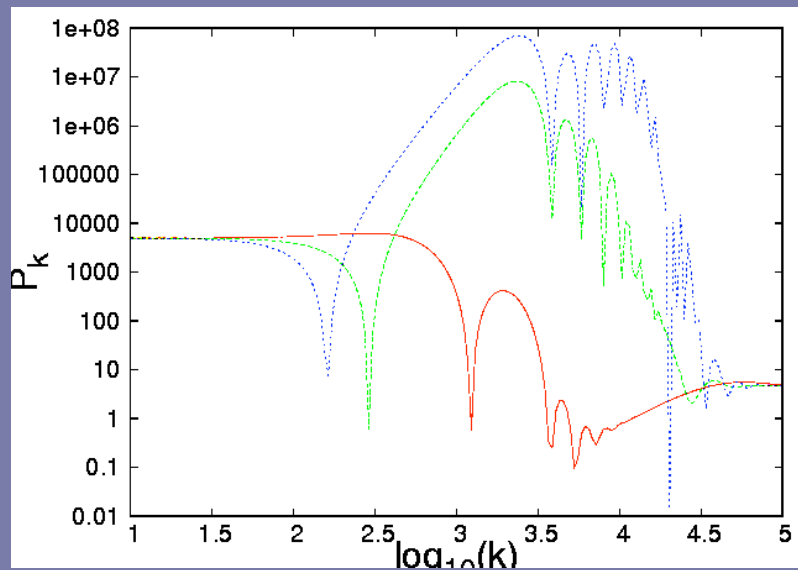
Eg. $V_I \sim (10^{16} \text{ GeV})^4$, $V_{II} \sim (10^3 \text{GeV})^4$, n=1:

Max 1% deviation from flat spectrum ⟹ $N_I > 12$

Perturbation spectrum: detailed calculations

- Changing effective equation of state:

$$u_k'' - \frac{1}{2}(1+3w)u_k' + \left[\left(\frac{k}{aH}\right)^2 - \frac{1}{2}(1-3w) - \frac{3}{4}\frac{1-w}{1+w}w' - \frac{1}{2}\frac{w''}{1+w} + \frac{1}{4}\left(\frac{w'}{1+w}\right)^2 \right]u_k = 0$$
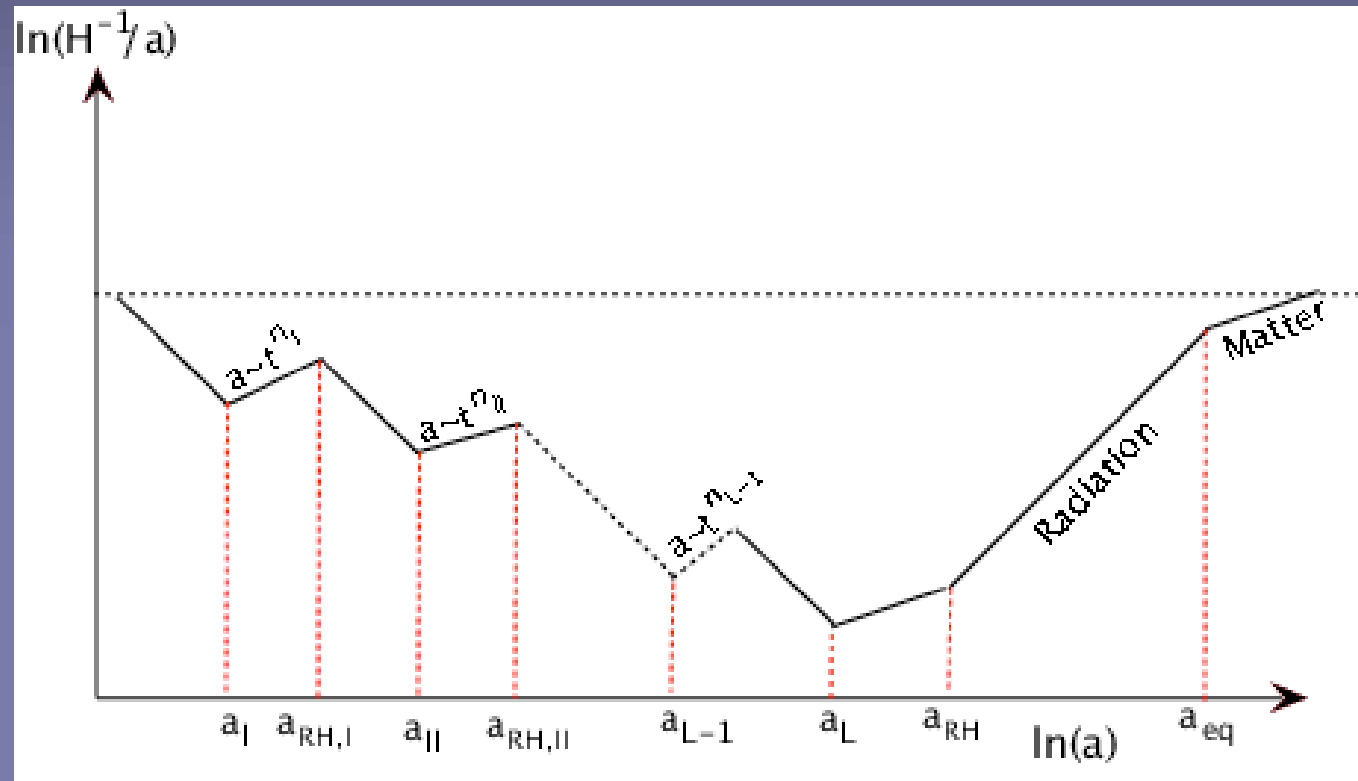

(horizon at k~3.5)

- Large deviations from flat spectrum
- Now observable modes must leave 4-5 e-foldings before string domination

$$' \equiv \frac{d}{d\eta}, \quad \eta = \ln(a)$$

N-Inflation

$$N_I > 7 + \frac{1-n_1}{2} \ln\left(\frac{V_I}{V_{II}}\right)$$

$$\sum_{j=2}^{i} N_j > \frac{1}{2} \ln\left(\frac{V_{II}}{V_{i+1}}\right) - \sum_{j=2}^{i} \frac{n_j}{2} \ln\left(\frac{V_i}{V_{i+1}}\right), \ i = 2, ..., L-1$$

Conclusions

- Getting 60+ e-foldings of inflation in string theory typically requires fine-tuning

- Only ~7 e-foldings are seen in the observations

- A multi step inflationary process is observationally allowed if modes are not reprocessed during the inter inflationary epoch

  - Inflationary landscape

- The first stage of inflation must be at least 12 e-foldings

  - Larger if the inter-inflationary period is not dominated by a cosmic string network