Dynamics of extra dimensions

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Outline

- Perturbations of extra dims
- Instability of dS compactifications
- Coupling of the perturbatios to brane fields
- Changing the shape of gravity

Nature as we observe it is 4 dimensional. Still, many theoretical extensions of the SM have extra dimensions.

(1) KK compactification: they are too small, (traditionally, $R \sim 10^{-32}$ m) to be excited in lab. (2) Large, but fermions / gauge fields live on 4d branes.

New dynamical degrees of freedom; probing extra dims relies on their excitation

- Early universe
- Accelerator experiments
- Modifications of Newton law

Perturbations in 4d cosmology

Bardeen '80

- Classify perturbations into irreducible SO(3) representations (do not mix at linear level)

$$ds^2 = a^2(t)\left[(1+2\Phi)\,dt^2 - 2\left(E_{;i} + \tilde{E}_i\right) dt\,dx^i + \right.$$
$$\left. - \left(\delta_{ij} - 2\,\psi\delta_{ij} - B_{;ij} - \tilde{B}_{(i;j)} - h_{ij}\right) dx^i\,dx^j\right]$$

tensor mode (gravitational waves) only affected by the background expansion $a(t)$

vector mode damped away during inflation

scalar mode(s) $\leftrightarrow \delta\phi\ ,\ \delta\rho_{\text{matter}}\ ,\ \delta\rho_\gamma\ldots$

- Analogous classification for braneworlds,

van de Bruck, Dorca, Brandenberger, Lukas, '00

Minkowski branes: RS1 vs RS2

- Only tensor mode if only one brane present (RS2). Standard $4d$ gravity at "large" distances, modified at $r \lesssim l$ (AdS radius) from the KK modes, Randall Sundrum '99; Giddings, Katz, Randall '00; Garriga, Tanaka '00

- Additional scalar mode (radion: interbrane distance) with more branes (RS1). 0−mode truncation: Brans–Dicke gravity if no stabilization; Einstein gravity if radion is stabilized Garriga, Tanaka '00; Tanaka, Montes '00

Expanding branes:

- Modes coupled for FRW branes; equation for the perturbations not separable in $t - y$. Decoupling (and exact analytical solutions) only for exact dS branes.

Langlois, Maartens, Wands '00

Tensor modes

- Mass gap $m_{\mathrm{KK}} \geq 3\,H/2$. Only 0−mode amplified during inflation. Garriga, Sassaki '00; Langlois, Maartens, Wands '00; Frolov, Kofman '02

- In RS2, due to the modified expansion

$$\mathcal{P}_T = \frac{2}{M_p^2}\left(\frac{H}{2\pi}\right)^2 C^2\,(l\,H)\;, \quad \begin{array}{l} \simeq 1,\; l\,H \ll 1 \left(H^2 \propto \rho\right) \\ \simeq \frac{3Hl}{2},\; l\,H \gg 1 \left(H^2 \propto \rho^2\right) \end{array}$$

Langlois, Maartens, Wands '00; Kobayashi, Kudoh, Tanaka '03; Kobayashi, Tanaka '05

- Mode mixing during following FRW era; studied either numerically + analytical approx. Most significant effects for modes re-entering the horizon in the high energy regime ($H^2 \propto \rho^2$) Hiramatsu, Koyama, Taruya '03, '04; Ichiki, Nakamura '04; Easther, Langlois, Maartens, Wands '03; Battye, van de Druck, Mennim '03

Standard 4d result, up to $\mathrm{O}\left(l^2/\lambda_0^2\right)$ corrections, if $l\,H \ll 1$, $l \ll \lambda$ Kobayashi, Tanaka '04

Vector modes

Bridgman, Malik, Wands '01

- Two gauge invariant modes. For FRW, coupled equations in the bulk. For dS branes, equations decoupled (and factorizable).

- For a perfect fluid on the brane (vanishing anisotropic pressure) one of the mode vanishes at the brane.

- Master variable (Mukoyama '00), with effective potential

$$V(z) = \frac{3}{4} l^2 A^2 + \frac{9}{4} H^2 + 3H\delta(y)$$

Analogously to tensor modes, $m_{\mathrm{KK}} > 3H/2$. Opposite sign for δ term. No normalizable $0-$mode

- Possible presence of $0-$mode for a sourcing anisotropic stress contribution on the brane

Bridgman, Malik, Wands '01
Ringeval, Boehm, Durrer '03

Scalar modes

- Analogous complications arise for scalar modes in FRW. Not reviewed here.

Maartens '04

- Restrict to pure dS branes, and discuss the effect of the cosmological expansion on the compactification of the internal space.

- Volume / interbrane distance not fixed by gravity. Require additional fields in the bulk.

E.g. scalar field ϕ with bulk + brane potentials Goldberger, Wise '99. $\delta\phi$ coupled to scalar metric perturbations ($\sim$ scalar field inflation)

- In particular, the radion for a system of two dS branes with no stabilization is tachyonic,

$$m_r^2 = -4\, H^2$$

Gen, Sasaki '00; Frolov, Kofman '03

- Identify physical modes from $\delta g_{\mu\nu} \leftrightarrow \delta\phi$

$$v = A^{3/2}\left(\delta\phi + \frac{A\,\phi'}{2A'}\Phi\right)$$

A warp factor
Φ metric fluct.

5d counterpart of Mukhanov–Sasaki variable

$$\left[\Box_4 + \frac{d^2}{dy^2} + \mathcal{F}\left[A,\phi\right]\right] v = 0 \qquad + \text{b.c.}$$

- Separate $v = \sum_n \tilde{v}_n\left(y\right)\, Q_n\left(x\right)$

$$\begin{cases} \left(\Box_4 - m_n^2\right) Q_n = 0 \\ \left[\frac{d^2}{dy^2} + \mathcal{F} + m_n^2\right] \tilde{v}_n = 0 \end{cases}$$

- General upper bound on the radion mass,

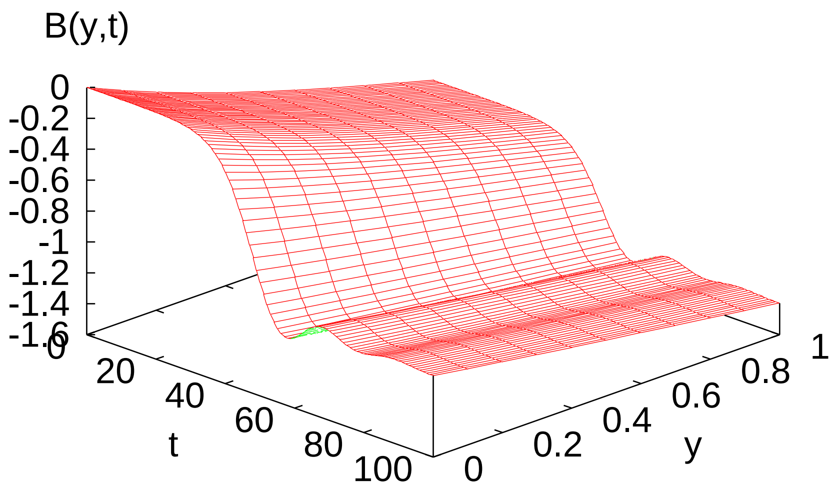
Frolov, Kofman '03

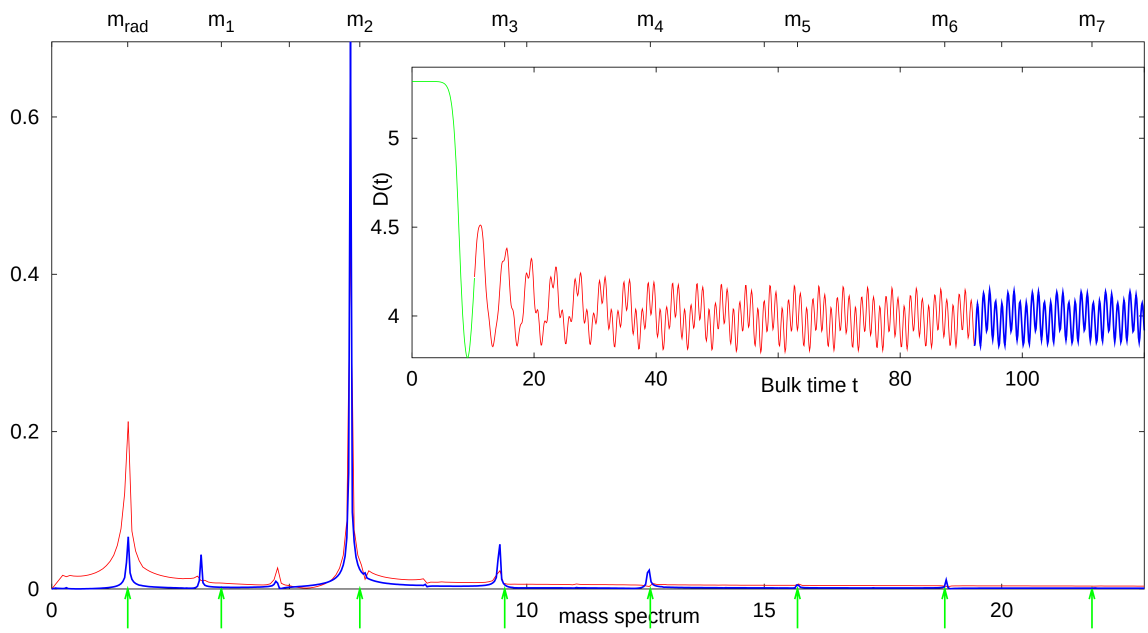
$$m^2 \leq -4H^2 + \frac{2\int dy/A}{3\int dy/\left(A\,\phi'^2\right)}$$

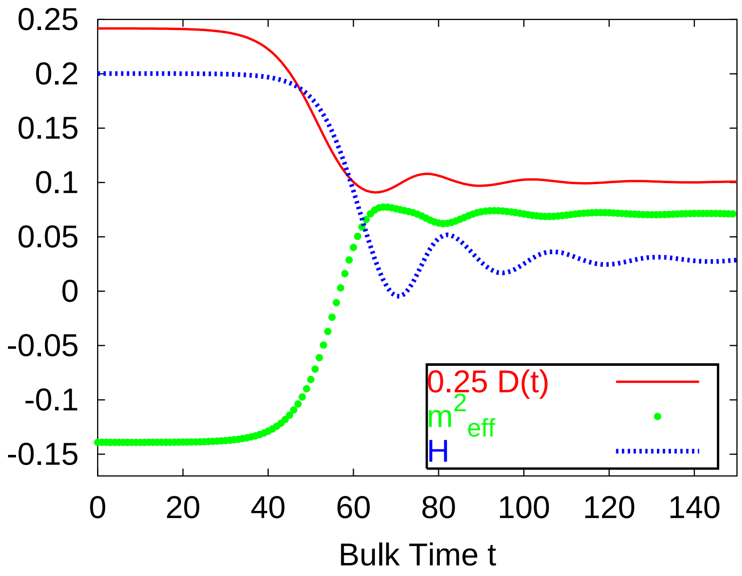
High H $\rightarrow$ instability of the internal space.

Transitions high $H \to$ low H observed numerically, Martin, Felder, Frolov, M.P., Kofman '03

$$ds^2 = e^{2B}\left(-dt^2 + dy^2\right) + e^{2A}d\mathbf{x}^2$$

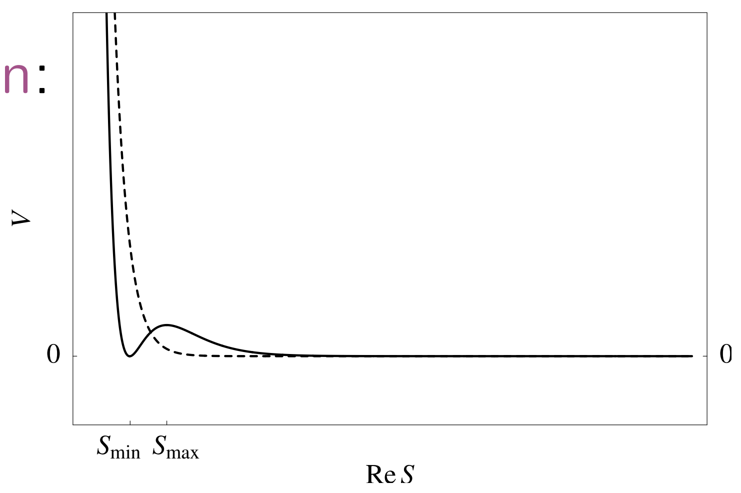






Analogous situation:

Multiple gaugino condensation



Destabilization at finite temperature

Buchmuller, Hamaguchi, Lebedev, Ratz '04

1−Loop free energy in susy SU (N_c)

$$\mathcal{F} = -\frac{\pi^2 T^4}{24}\left\{\alpha_0 - \alpha_2\, g^2 + \ldots\right\} \quad , \quad g^2 = 1/\mathrm{Re}\, S$$

Modulated perturbations

Φ appears as a conformal perturbation in the 4d induced geometry

$$\gamma_{\mu\nu} = A^2\,(y)\,[1 + 2\,\Phi\,(x, y)]\,g_{\mu\nu}$$

The conformal redefinition which sets the physical scales in 4d has a small $x-$dependency

Φ excited during inflation if $m_\Phi < H$. Inflaton mass and decay rate (4d inflaton) m_ϕ , Γ_ϕ are $x-$dependent. The inflaton will decay at slightly different time in different places $\Rightarrow$ adiabatic perturbations at the decay

Idea of $\delta\Gamma_\phi \rightarrow \delta g_{\mu\nu}$ proposed by Dvali, Gruzinov, Zaldarriga '03; Kofman '03. Γ_ϕ related to the vev of some generic MSSM or string modulus. This is a concrete (and rather general) realization

de Sitter compactifications $dS_4 \times \mathcal{M}_d$

- General effect, not limited to codimension one braneworlds, Contaldi, Kofman, M.P. '04

- Consider (dS) expansion of the noncompact coordinates. Analysis as before. Solve the eigenvalue equation for $\tilde{\Psi}_n$, and look at the equations for Q_n. $\Box_4$ always enters in the combination

$$-m_0^2\, Q_n = -\left[\Box_4 + \frac{12\, d}{d+2} H^2\right] Q_n\,(x) =$$

$$\left[\partial_t^2 + 3\, H\, \partial_t - \mathrm{e}^{-2Ht} \partial_i^2 - \frac{12\, d\, H^2}{d+2}\right] Q_n\,(x)$$

Hence

$$m_{\mathrm{phys}}^2 = m_0^2 - \frac{12\, d\, H^2}{d+2}$$

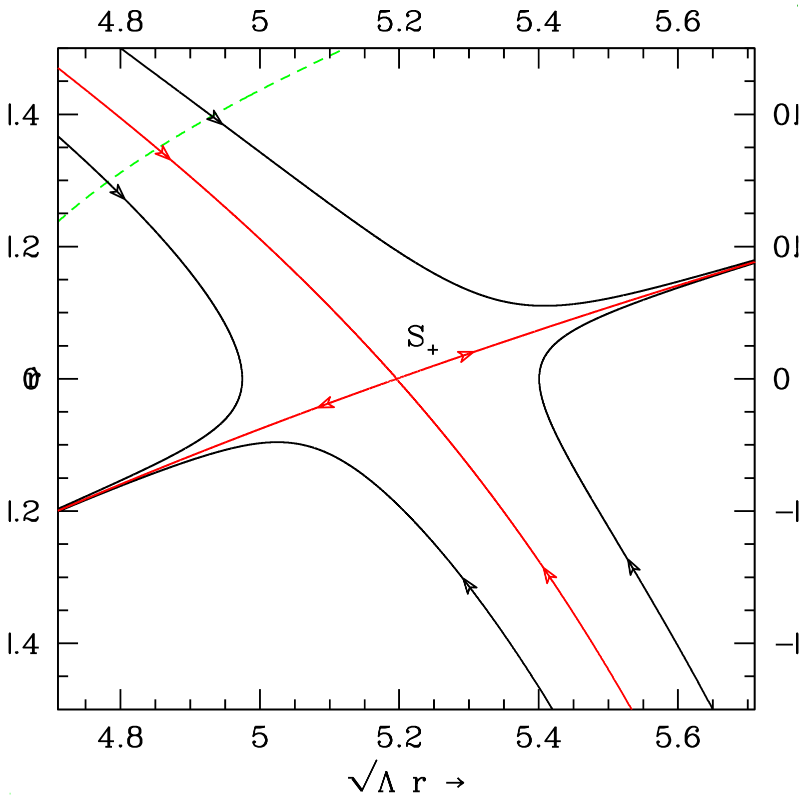
m_0 (from the compactification) must be sufficiently high to accomodate for H

Endpoint of the instability ?

$dS_4 \times S_d$; $\Lambda > 0$, no stabilization mechanism;

(flux compactification, Bousso, DeWolfe, Myers, '02; Martin '04)

$$-\Box_4 - \frac{12\,d}{d+2} H^2 + \frac{4\,(d-1)}{d\,(d+2)} R_d = -\Box_4 - 6\,H^2$$



- Up-right: instability towards dS_{4+d}
- Down-left: S_d crunches with Kasner asymp.

Coupling to brane fields and accelerator experiments

$$\mathcal{L}_{\text{int}} \;\sim\; \Phi\, T_{\text{br}}{}^{\mu}_{\mu} \;\sim\; \tilde{v}_n\,(y_{\text{br}})\, Q_n\, T_{\text{br}}$$

- Linearized Einstein equations determine the wave functions $\tilde{v}_n$ only up to an arbitrary normalization

Normalization can be extracted only form the quadratic action for the perturbations

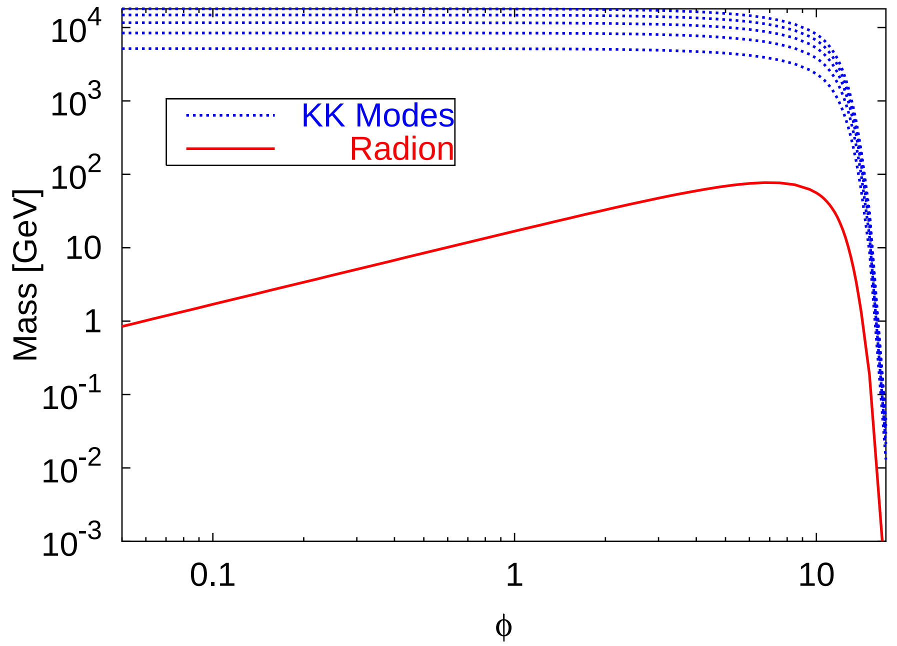
Kofman, Martin, M.P. '04

$$S \;=\; \sum_n S_n = \sum_n C_n \int d^4 Q_n \left[\Box - m_n^2\right] Q_n$$

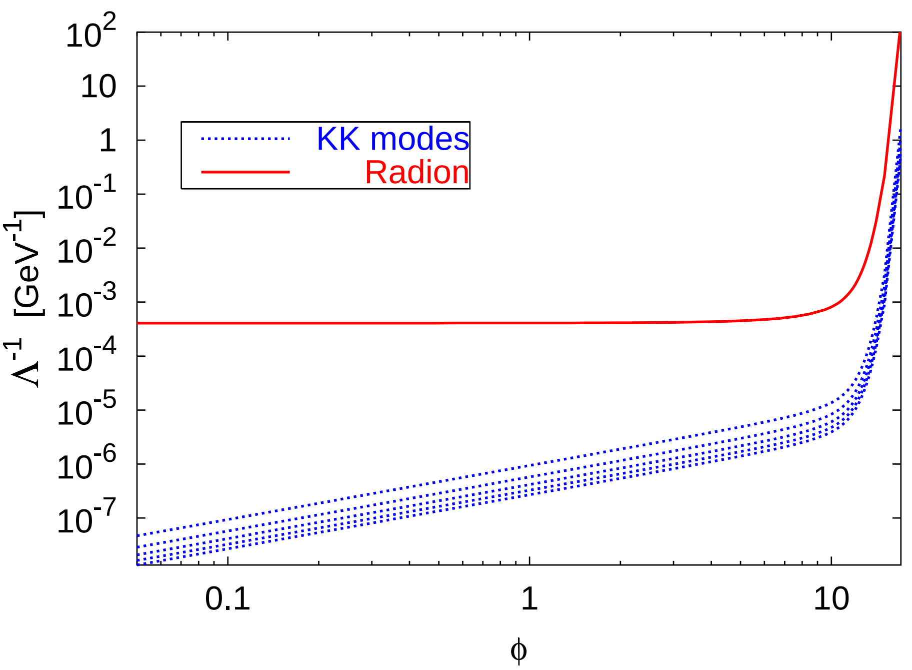
$$C_n \;=\; \frac{3M^3}{2} \int_0^{y_1} \frac{dy}{A} \left[\frac{3M^3}{2\phi'^2} \left(A^2 \tilde{\Phi}_n\right)'^2 + \left(A^2\, \tilde{\Phi}_n\right)^2 \right]$$

$C_n \equiv 1/2 \;\rightarrow\;$ normalization $\;\rightarrow\;$ coupling

Results previously known only for negligible backreaction ($\phi \to 0$), Csaki, Graesser, Kribs '00



Radion massless without bulk scalar ϕ



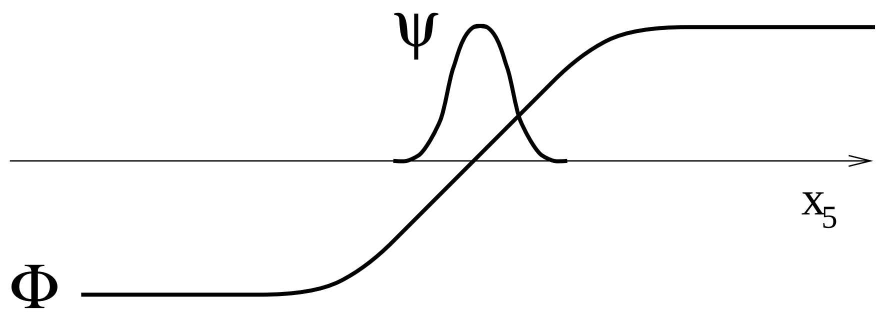
Changing the shape of gravity

Fermions on a kink, Rubakov, Shaposhnikov, '83

$$\mathcal{L} = \bar{\Psi}\,[i\partial\!\!\!/ \; - (m_5 + g\,\Phi)]\,\Psi$$

Fermions localized where $m_5 + g\,\phi = 0$

5d "mass" parameter $\rightarrow$ localization



- Fermion / scalar fields on AdS + branes. Also in this case, bulk profile determined by bulk / brane masses, Pomarol, Gherghetta '00

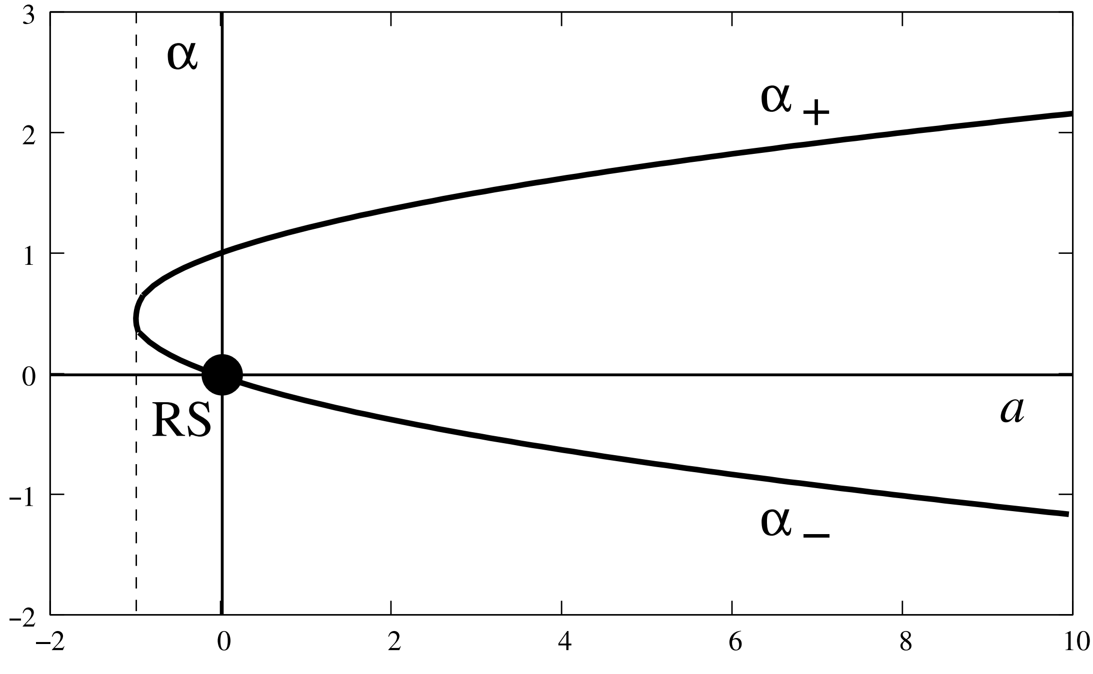
- Gauge fields, Ghoroku, Nakamura, '01

- Gravity, Gherghetta, M.P., Poppitz, '05

$$
\begin{aligned}
S_{\text{mass}} &= -M^3k^2\int d^5x\sqrt{-g}\, a\left[h^{MN}h_{MN} - h^M_M h^N_N\right] \\
&\pm \sum_i M^3k\int d^4x\sqrt{-\gamma_i}\,\alpha\left[h^{\mu\nu}h_{\mu\nu} - h^\mu_\mu h^\nu_\nu\right]_i
\end{aligned}
$$

- "Fierz–Pauli" mass terms to avoid ghosts at the linearized level.

- Massless tensor and vector modes for $a \leftrightarrow \alpha$. No scalar modes.



$f_T^{(0)} \propto e^{-(1-4\alpha)ky}$

$\alpha < 1/4$ UV localization

$\alpha > 1/4$ IR localization

- Matter on the UV brane

$$M_P^2 \simeq \begin{cases} \frac{M^3}{k(1-4\alpha)} & , \quad \alpha < 1/4\,, \quad (\sim \text{RS}) \\ M^3\, 2\,\pi\, R & , \quad \alpha = 1/4\,, \quad (\text{ADD}) \\ \frac{M^3}{k(4\alpha-1)}\, \mathrm{e}^{2(4\alpha-1)\pi k R} & , \quad \alpha > 1/4 \end{cases}$$

Gravity exponentially suppressed if graviton towards IR brane. Can choose $M \sim k \sim$ TeV

Newtonian potential

- For $\alpha < 1/4$,

$$V \simeq -\frac{\mu}{M_p^2\, r}\left[1 + \ldots \left(\frac{r_{\text{UV}}}{r}\right)^{2-8\alpha}\right] \quad , \quad r_{\text{UV}} \equiv \frac{1}{k} \sim \frac{1}{\text{TeV}}$$

- For $\alpha > 1/4$, real power at the IR scale

$$V \simeq -\frac{\mu}{M_p^2\, r} \quad , \quad r > r_{\text{IR}} \equiv \frac{\mathrm{e}^{\pi k R}}{k} \sim \frac{1}{\text{TeV}}$$

$$V \propto -\frac{\mu}{M_p^2\, r}\left(\frac{r_{\text{IR}}}{r}\right)^{8\alpha-2} \quad , \quad r < r_{\text{IR}}$$

CFT dual

- IR brane associated to C$\not{F}$T . Fields on IR brane as composite CFT states. Graviton emerging from strongly coupled CFT

- Properties CFT from

$$\langle \mathcal{O}\mathcal{O} \rangle (p) \subset \Sigma (p) \propto \frac{1}{G_p (z_0, z_0)}$$

- Pole $1/p^2$ only when graviton at the IR brane, and for $p \ll 1/r_{\rm IR}$

Analytic part of $\Sigma \to$ massive graviton in dual theory. Integrating it out, and keeping CFT as propagating d.o.f., one recovers the Newtonian potential with precise M_p.

- Analogous to “fat” graviton of Sundrum, ’04, designed to turn off gravity at short distance. However, gravity becomes stronger in our case (all KK contributions attractive)

Conclusions

Perturbations of extra dimensions can be studied through known techniques in cosmology. Same formalism for collider phenomenology as well as the early time dynamics

- General instability effect for dS compactifications. To be accounted for in model of inflation with extra dimensions.

- Quantization of perturbations $\rightarrow$ coupling to brane (SM) fields

- Gravity towards the IR brane (emerging gravity in the CFT)

Bulk action (bulk + IR b.c.)

$$S = \frac{M^3}{4} \int \frac{d^4p}{(2\pi)^4} \left[A^3 \ \widehat{H}(p,z)(\widehat{H}'(-p,z) - 4\alpha A k \widehat{H}(-p,z)) \right] \Big|_{z=z_0}$$

Correlator

$$\begin{aligned} \Sigma(p) &= \int d^4x e^{-ip\cdot x} \frac{\delta^2 S}{\delta(A_0^2 \widehat{h}(x,z_0))\delta(A_0^2 \widehat{h}(0,z_0))} \\ &= \frac{M^3}{2A_0^4} \frac{1}{G_p(z_0,z_0)} \end{aligned}$$

A_0, A_1 warp factor at UV, IR brane.

$q_0 = \frac{p}{kA_0}$, $q_1 = \frac{p}{kA_1}$, $\nu = 4\alpha - 1$

Take first large $A_0 \to$ limit, and identify the first term dependent on q_1 (the larger terms are cancelled by counter terms). Then expand this term for $q_1 \ll 1$ Rattazzi, Zaffaroni '01; Contino Pomarol '02

$$\Sigma(p)_{IR} \simeq -\left(\frac{M}{k}\right)^3 k^4 \Big[(\nu-1) + q_0^2 \frac{1}{4(\nu-2)} -4\nu(\nu-1)^2 \, \frac{A_1^{2\nu}}{A_0^{2\nu}} \, \frac{1}{q_0^2}\Big]$$

Mass (UV) and kinetic term for the graviton, plus CFT pole

$$\mathcal{L}_{IR} = \frac{1}{4} h_{\mu\rho}(\Box - m_h^2)h^{\mu\rho} + \frac{\chi}{k}\, h_{\mu\rho}T^{\mu\rho}_{CFT} + \frac{\chi}{k}\, h_{\mu\rho}T^{\mu\rho}_{matter} + \mathcal{L}_{CFT}$$

$$\chi = (\nu_+ - 2)^{1/2}(M/k)^{-3/2} \ , \ \ m_h^2 = 4(\nu-1)(\nu-2)k^2$$

$$\langle \mathcal{O}\mathcal{O}\rangle \simeq (Mk)^3\, 16\nu_+(\nu_+-1)^2 A_1^{2\nu_+}\frac{1}{p^2}$$

Large distance

$$\begin{aligned} V(r) &\simeq -\mu\frac{\chi^2}{k^2}\int \frac{d^3p}{2\pi^2} \ e^{ip\cdot x} \ \frac{\chi^2}{k^2}\frac{\langle \mathcal{O}\mathcal{O}\rangle(p)}{m_h^4} \\ &= -\mu\frac{\chi^4}{m_h^4}\frac{M^3}{k}16\nu_+(\nu_+-1)^2 A_1^{2\nu_+}\int \frac{d^3p}{2\pi^2} \ e^{ip\cdot x}\frac{1}{p^2} \\ &= -\frac{\mu}{M_P^2 r} \end{aligned}$$

$$
\begin{aligned}
S &= \int d^5x\sqrt{|g|}\left\{\frac{1}{2}R - \Lambda - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right\} \\
&- \sum_{i=1}^{2}\int d^4\xi\sqrt{|\gamma|}\left\{\,[K]_i + \Lambda_i + U_i(\phi)\right\} \\
V &= -6\,k^2 + \left(2\,k\,u + \frac{u^2}{2}\right)\phi^2 - \frac{u^2}{6}\phi^4
\end{aligned}
$$

De Wolfe, Freedman, Gubser, Karch '99

Background solutions

$$
\begin{aligned}
\phi &= \phi_0\,\mathrm{e}^{-u\,y} \\
A &= \exp\left[-k\,y + \frac{\phi_0^2}{12}\left(1 - \mathrm{e}^{-2\,u\,y}\right)\right]
\end{aligned}
$$

Choose brane potentials to satisfy the junction conditions.

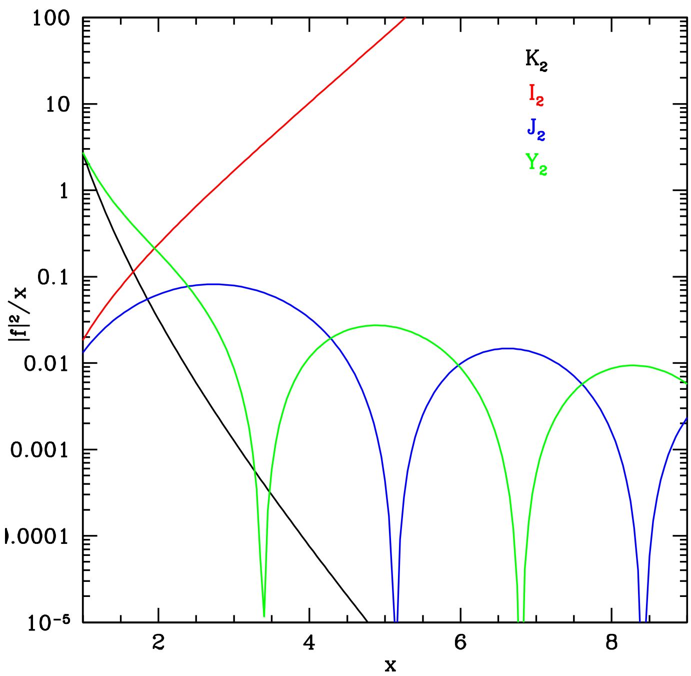
For any given ϕ_0, choose parameters to reproduce M_p and TeV at the second brane. In particular, the interbrane distance has to be increased when ϕ_0 increases.

Tachyonic “K” modes

Cartier, Durrer ’05

- Equations for perturbations in AdS bulk admit in principle $m^2 < 0$ modes. E. g., eq. for tensor modes in RS2 has eigenmodes $\propto$

$$\propto J_2\,(my)\ ,\ J_2\,(my) \qquad m^2 > 0$$
$$\propto K_2\,(my)\ ,\ I_2\,(my) \qquad m^2 > 0$$



- K is normalizable. B. c. at the brane forbids it, unless suitable anisotropic tensor

$$\partial_y H_{ij|y_b} \propto \Pi_{ij}^{\text{brane}}$$

- Π from collisionless particles. No tachyonic modes for Minkowski (causality arguments)

- A possible causality argument

RS1 Cosmology with stabilization mechanism and brane fields. Radion of infinite mass (rigid extra dimension),

Lesgourgues, Pastor, M.P., Sorbo '00

$$H_1^2 \simeq \frac{1}{3M_p^2}\frac{\rho_1 + \rho_0 - \rho_0\rho_1/\left(10\, M_p^2\, \mathrm{TeV}^2\right)}{1 - \rho_0/\left(10\, M_p^2\, \mathrm{TeV}^2\right)}$$

(phisical quantities; $H_1 = H\mathrm{e}^{-A(y_1)}, \ldots$)

ρ_0 (hidden brane) dark matter/energy/radiation....
Intermediate scale $\bar{\rho} \sim \sqrt{M_p\, TeV}$?

$$\rho_0 > \bar{\rho} \quad \leftrightarrow \quad H > D^{-1}$$

No matter what the stabilization mechanism is, the bulk is not static when the size of the extra space is greater than H^{-1} ("different edges do not talk to each other")

$$S_2 = M^3 \int d^5x\, A^3 \left\{ \frac{\partial^\mu \delta\phi\, \partial_\nu \delta\phi}{-M^3} + 6\partial^\mu (\Phi + \Psi)\, \partial_\nu \Psi \right.$$
$$- \frac{1}{M^3} \delta\phi'^2 + 12\Psi'^2 + \frac{2}{M^3} \phi' \delta\phi' (\Phi - 4\Psi)$$
$$+ 48 \frac{A'}{A} \Psi' \left(\Psi - \frac{\Phi}{2} \right) - \left(\frac{\phi'^2}{M^3} - 12 \frac{A'^2}{A^2} \right) \left(\Phi^2 + 8\Psi^2 \right)$$
$$\left. - \left[\frac{1}{M^3} A^2 V'' \delta\phi^2 - \frac{2}{M^3} A^2 V' \delta\phi (\Phi + 4\Psi) \right] \right\}$$
$$- \sum_i \int d^4x A^4 \left[4U\Psi^2 + 4U' \delta\phi \Psi + \frac{1}{2} U'' \delta\phi^2 \right]$$

$$\Downarrow \qquad \Phi = \textstyle\sum_n \tilde{\Phi}_n(y)\, Q_n(x) \ \ldots$$

$$S = \sum_n C_n \int d^4x\, Q_n \left[\Box - m_n^2 \right] Q_n$$

$$C_n = \frac{3\, M^3}{2} \int_0^{y_0} \frac{dy}{A} \left[\frac{3\, M^3}{2\, \phi'^2} \left(A^2\, \tilde{\Phi}_n \right)'^2 + \left(A^2\, \tilde{\Phi}_n \right)^2 \right]$$
$$\equiv \frac{1}{2}$$

Kasner asymptotic in brane collisions

Endpoint of the singularity ?

(i) A stable minimum

(i) Decompactification limit (e.g. dS_{4+d})

(iii) Singularity

The last one has a Kasner asymptotics. E.g. Brane-collisions; example an example of strong-gravity regime, where the 4d theory (moduli approximation) is inadequate.

However, some expectations: gradients in y switch-off $E \propto \Delta y \, (\Delta\phi/\Delta y)^2$; sources become unimportant

$$\begin{aligned} ds^2 &= -dt^2 + (t_c - t)^{2p_y} \, dy^2 + (t_c - t)^{2p_x} \, d\mathbf{x}^2 \\ \phi &= q \ln (t_c - t) \end{aligned}$$

with $p_x + 3\, p_y = 1$, $p_y^2 + 3p^2 = 1 - q^2$

Analogy with scalar field inflation in $3+1$ d

3+1	4+1
$t-$dep. background	$y-$dep. background
$\phi(t)$ for inflation	$\phi(y)$ for stabilization
$a(t)$ scale factor	$A(y)$ warp factor
$\delta g_{\mu\nu} \leftrightarrow \delta\phi \Rightarrow$ CMB	$\delta g_{\mu\nu} \leftrightarrow \delta\phi \Rightarrow$ LHC

Linearized Einstein eqs. for perturbations:

Initial-value pbm	Boundary-value pbm

Action perturbations for the normalization:

$\longrightarrow$ initial conditions	$\longrightarrow$ coupling to branes