Superconducting Cosmic Strings in Supergravity

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Outline

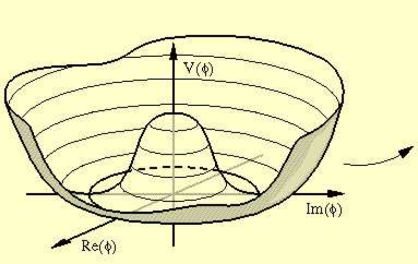
1. Introduction
   Strings, fermionic zero modes & vortons
2. D-term hybrid inflation in global SUSY
   One of zero modes is goldstino
3. D-term hybrid inflation in SUGRA
   Zero mode absent due to super Higgs effect
4. Conclusions

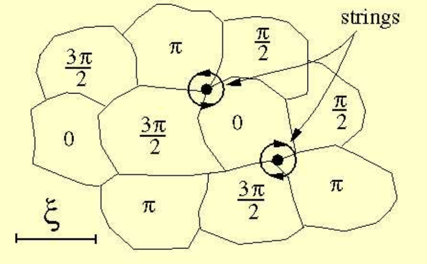
R. JEANNEROT & MP ('04)

BECKER, BECKER, STROMINGER ('95)

Kibble mechanism

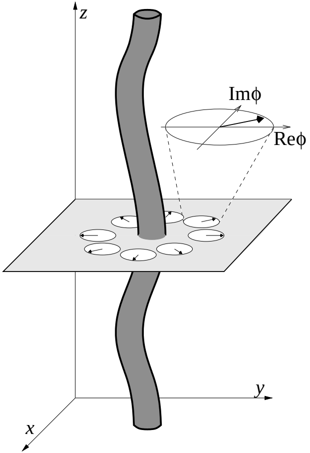
Phase transition in the early universe

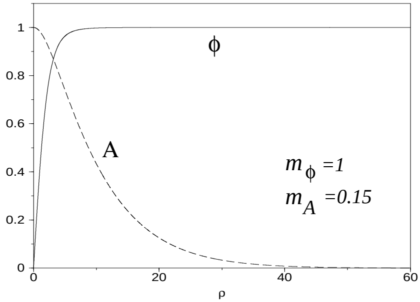




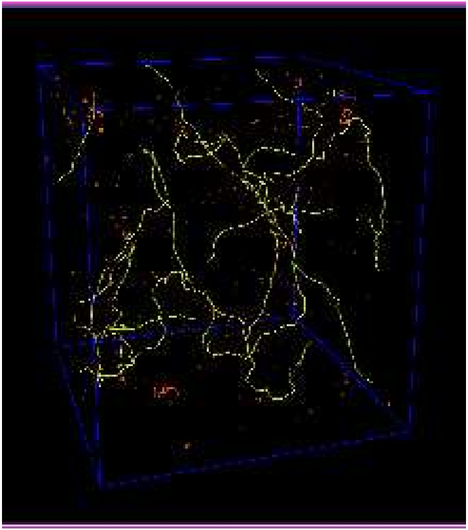
$$l_H = H^{-1} \text{ with } H = \dot{a}/a = \rho/3m_{\mathrm{p}}^2$$

Cosmic strings





String network

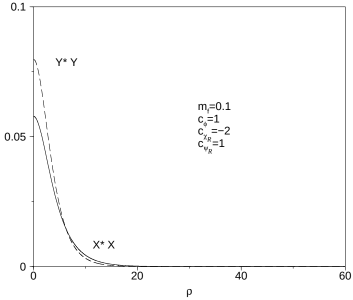


ALLEN, SHELLARD

Fermionic zero modes

$$\mathcal{L}_F = i\psi_i \sigma^\mu D_\mu \bar{\psi}_i - \lambda\phi\psi_i\psi_j + \mathrm{h.c.}$$

$m_\psi = \lambda\phi$ with $\phi = 0$ in string core, $\phi = v$ outside



zero mode solution:

$$\psi_i^{\mathrm{zm}} = \xi_i(r,\theta)\mathrm{e}^{\pm \mathrm{i}k(t-z)}$$

JACKIW & ROSSI

Index theorems

$$\mathcal{L}_y = -\lambda\phi\psi_i\psi_j - \lambda\phi^\dagger\chi_i\chi_j + \text{h.c.} \qquad \text{with Higgs } \phi = vf(r)e^{in\theta}$$

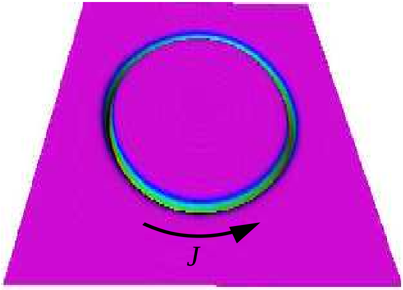
Index theorem

There are n left-moving zero modes for each coupling to ϕ, and n rigth-moving zero modes for each coupling to $\phi^\dagger$.

E.WEINBERG

String loop stabilized by angular momentum of the fermion current

R.L. DAVIS, SHELLARD



- chiral strings
- CDM: constraints

A.C. DAVIS, TRODDEN, BRANDENBERGER, CARTER

R. JEANNEROT & MP, JHEP 0412:032,2004
R. JEANNEROT & MP, JHEP 0412:043,2004

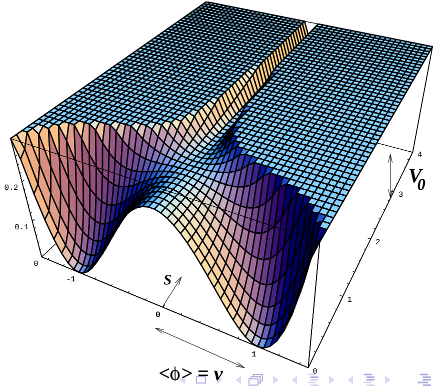
- FI-term $\Leftrightarrow$ $U(1)$
- $W = \kappa S \phi_+ \phi_-$

$$V = \kappa^2 \left(|S|^2|\phi_+|^2 + |S|^2|\phi_-|^2 + |\phi_+|^2|\phi_-|^2\right) + \frac{g^2}{2}(|\phi_+|^2 - |\phi_-|^2 - \xi)^2$$

Vacuum

$$|\phi_+| = \sqrt{\xi}$$

$$|S| = |\phi_-| = 0$$



$$V_D = \frac{g^2}{2}(|\phi_+|^2 - \xi)^2$$

string: $\phi_+ = \sqrt{\xi} f(r) e^{in\theta}$

$$A_\theta = -\frac{n}{g} a(r)$$

with $f(0) = a(0) = 0$ and $f(\infty) = a(\infty) = 1$

BOGOMOLNY

$$\begin{aligned} \mu &= \int r\,\mathrm{d}r\,\mathrm{d}\theta \left(|D_\mu \phi_+|^2 + \frac{1}{4}F^2 + \frac{1}{2}D^2 \right) \\ &= \int r\,\mathrm{d}r\,\mathrm{d}\theta \left(|(D_r \mp \frac{i}{r} D_\theta)\phi|^2 + \frac{1}{2}|F_{12} \mp D|^2 \right) \pm 2\pi n \xi \end{aligned}$$

D-term inflation — global SUSY

$$V_D = \frac{g^2}{2}(|\phi_+|^2 - \xi)^2$$

string: $\phi_+ = \sqrt{\xi} f(r) e^{in\theta}$ $\qquad f' = n\frac{1-a}{r}f$

$A_\theta = -\frac{n}{g}a(r)$ $\qquad n\frac{a'}{r} = g^2\xi(1-f^2)$

with $f(0) = a(0) = 0$ and $f(\infty) = a(\infty) = 1$

BOGOMOLNY

$$\begin{aligned}\mu &= \int r\,\mathrm{d}r\,\mathrm{d}\theta \left(|D_\mu \phi_+|^2 + \frac{1}{4}F^2 + \frac{1}{2}D^2 \right) \\ &= \int r\,\mathrm{d}r\,\mathrm{d}\theta \left(|(D_r \mp \frac{i}{r}D_\theta)\phi|^2 + \frac{1}{2}|F_{12} \mp D|^2 \right) \pm 2\pi n \xi\end{aligned}$$

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with $f(0) = a(0) = 0$ and $f(\infty) = a(\infty) = 1$

BPS

$$\delta\psi_+ = \sqrt{2}(\epsilon_- F_+ + iD_\mu \phi_+ \sigma^\mu \bar{\epsilon}_-) = 0$$

$$\delta\lambda = -(iD + \frac{1}{2}\sigma^\mu \bar{\sigma}^\nu F_{\mu\nu})\epsilon_- = 0$$

with $\sigma^0 \sigma^3 \epsilon_\pm = \pm \epsilon_- \quad \Rightarrow \quad \epsilon_- = \binom{0}{1}$

$$\mathcal{L}_Y = -ig\sqrt{2}\phi_+^\dagger \psi_+ \lambda - \kappa \phi_+ \psi_S \psi_- + \text{h.c.}$$

Index theorem:

- *n* left-moving zero modes (ψ_+, λ)
- *n* right-moving zero modes (ψ_S, ψ_-)

Susy trafo with $\sigma^0\sigma^3\epsilon_+ = +\epsilon_+ \;\Rightarrow\; \epsilon_+ = \binom{1}{0}$

- $[Q, H] = 0$
- satisfy eq. of motion

DAVIS, DAVIS, TRODDEN

$$\begin{aligned} \delta\lambda(r,\theta) &= -i2g\xi(1-f^2)\epsilon_+ \\ \delta\psi_+(r,\theta) &= i2\sqrt{2}\sqrt{\xi}\frac{n}{r}(1-a)fe^{i(n-1)\theta}\epsilon_+^* \end{aligned}$$

zero mode:

$$\psi_{\mathrm{zm}}(t,z,r,\theta) = \alpha(t,z)\Big(\delta\lambda(r,\theta) + \delta\psi_+(r,\theta)\Big)$$

$$\left[\sigma^0\partial_t - \sigma^3\partial_z\right]\alpha(t,z) = 0 \quad\Rightarrow\quad \alpha(t,z) = e^{\pm ik(t+z)}$$

zero mode is goldstino

- SUSY broken in string:

$$\phi_+ = \sqrt{\xi} f(r) \mathrm{e}^{in\theta}$$

$$A_\theta = -\frac{n}{g} a(r)$$

$$D = g\xi(1 - f(r)^2) \qquad \text{with } f(0) = 0 \ \& \ f(\infty) = 1$$

- Goldstino transforms non-linearly:

$$\delta\psi_+ = \sqrt{2}(\epsilon_+ F_+ + iD_\mu \phi_+ \sigma^\mu \bar{\epsilon}_+)$$

$$\delta\lambda = -(iD + \frac{1}{2}\sigma^\mu \bar{\sigma}^\nu F_{\mu\nu})\epsilon_+$$

D-term inflation — SUGRA

FI-term in SUGRA

$U_\xi(1) = U_V(1) \oplus U_R(1)$

BARBIERI, FERRARA, NANOPOULOS, STELLE
STELLE, WEST; FREEDMAN

BINETRUY, DVALI, KALLOSH, VAN PROEYEN

$$K = \sum |\phi_i|^2$$
$$W = \kappa S \phi_+ \phi_i$$
$$f_{\alpha\beta} = 1$$

$$V = V_F + \frac{g^2}{2}\left(q_+|\phi_+|^2 + q_-|\phi_-|^2 - \xi\right)^2$$

vacuum:

$$\phi_- = S = 0, \quad \phi_+ = \xi/\sqrt{q_+}$$

BINÉTRY, DVALI, KALLOSH, VAN PROEYEN

$$V_D = \frac{g^2}{2}(|\phi_+|^2 - \xi)^2$$

string: $\phi_+ = \sqrt{\xi} f(r) e^{in\theta}$

$$A_\theta = -\frac{n}{g} a(r)$$

$$ds^2 = dt^2 - dz^2 - dr^2 - C(r)^2 d\theta^2$$

BPS

$$\delta\psi_+ = \sqrt{2}(\epsilon'_- F_+ + iD_\mu \phi_+ \sigma^\mu \bar{\epsilon}'_-) = 0$$

$$\delta\lambda = -(iD + \frac{1}{2}\sigma^\mu \bar{\sigma}^\nu F_{\mu\nu})\bar{\epsilon}'_- = 0$$

$$\delta\psi_\mu = 2(\partial_\mu + \frac{1}{4} w_\mu^{ab}\sigma_{ab} + \frac{1}{2} i A_\mu^B)\epsilon'_- = 0$$

with $\epsilon'_\pm = e^{\pm\frac{1}{2}i\theta}\epsilon_\pm$ and $\sigma^0\sigma^3\epsilon_\pm = \pm\epsilon_\pm$

Binétry, Dvali, Kallosh, van Proeyen

$$V_D = \frac{g^2}{2}(|\phi_+|^2 - \xi)^2$$

string: $\phi_+ = \sqrt{\xi} f(r) e^{in\theta}$

$$A_\theta = -\frac{n}{g} a(r)$$

$$ds^2 = dt^2 - dz^2 - dr^2 - C(r)^2 d\theta^2$$

$$f' = n\frac{1-a}{C} f$$

$$n\frac{a'}{C} = g^2 \xi (1 - f^2)$$

$$(1 - C') = A_\theta^B(C, a)$$

BPS

$$\delta\psi_+ = \sqrt{2}(\epsilon'_- F_+ + iD_\mu \phi_+ \sigma^\mu \bar{\epsilon}'_-) = 0$$

$$\delta\lambda = -(iD + \frac{1}{2}\sigma^\mu \bar{\sigma}^\nu F_{\mu\nu})\bar{\epsilon}'_- = 0$$

$$\delta\psi_\mu = 2(\partial_\mu + \frac{1}{4} w_\mu^{ab} \sigma_{ab} + \frac{1}{2} i A_\mu^B)\epsilon'_- = 0$$

with $\epsilon'_\pm = e^{\pm\frac{1}{2}i\theta}\epsilon_\pm$ and $\sigma^0\sigma^3\epsilon_\pm = \pm\epsilon_\pm$

BINÉTRY, DVALI, KALLOSH, VAN PROEYEN

$$V_D = \frac{g^2}{2}(|\phi_+|^2 - \xi)^2$$

string: $\phi_+ = \sqrt{\xi} f(r) \mathrm{e}^{in\theta}$ $\qquad f' = n\frac{1-a}{C}f$

$A_\theta = -\frac{n}{g}a(r)$ $\qquad n\frac{a'}{C} = g^2\xi(1-f^2)$

$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}r^2 - C(r)^2\mathrm{d}\theta^2$ $\qquad (1-C') = A_\theta^B(C,a)$

Asymptotic solutions:

$r \to 0$: $\quad f = 0, \quad a = 0, \quad D = g\xi, \quad A_\theta^B = 0, \quad C = r$

$r \to \infty$: $\quad f = \sqrt{\xi}, \quad a = 1, \quad D = 0, \quad A_\theta^B = \frac{n\xi}{m_\mathrm{p}^2}, \quad C = r\left(1 - \frac{n\xi}{m_\mathrm{p}^2}\right)$

SUSY trafo with $\epsilon'_+ = \mathrm{e}^{+\frac{1}{2}i\theta}\binom{1}{0}$

$$\delta\lambda = -i2g\xi(1-f^2)\epsilon'_+$$
$$\delta\psi_+ = i2\sqrt{2}\sqrt{\xi}\,\frac{n}{C}(1-a)f\,\mathrm{e}^{i(n-1)\theta}\epsilon'^*_+$$

$$\delta\psi_\theta = 2iA^B_\theta = \begin{cases} 0 & (r\to 0) \\ \frac{2in\xi}{m_\mathrm{p}^2} & (r\to\infty) \end{cases}$$

Non-renormalizable!

Super-Higgs effect:

$$e^{-1}\mathcal{L}_\mathrm{mix} = \frac{i}{2}D\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu + \frac{i}{\sqrt{2}}D\bar{\psi}_\mu\gamma^\mu(\delta\lambda+\delta\psi_+)$$
$$= \frac{i}{2}D\bar{\psi}'_\mu\sigma^{\mu\nu}\psi'_\nu$$

- gravitino not confined —no equivalent of SUSY goldstino zero mode

  $m_{3/2} = D = g\xi$ in core and $m_{3/2} = 0$ outside

- n right-moving zero modes (ψ_S, ψ_-)

  $\mathcal{L}_Y = -\kappa\phi_+\psi_S\,\psi_- + \text{h.c.}$

  $m = 0$ in core and $m = \kappa\sqrt{\xi}$ outside

Conclusions

Cosmic strings break SUSY in their core

global SUSY

- n independent zero modes for each breaking —goldstinos
- possible additional zero modes from superpotential

SUGRA

- goldstino zero modes eaten in super-Higgs effect, absent
- additional zero modes from superpotential unaffected

SUGRA D term inflation: n zero modes

SUGRA F term inflation: no zero modes

$\Rightarrow$ Implications for vorton formation

Confusion about number of zero modes

- S. C. Davis, A. C. Davis and M. Trodden, Phys. Lett. B **405** (1997) 257 ,Phys. Rev. D **57** (1998) 5184.
- B. Carter and P. Peter, Phys. Lett. B **466** (1999) 41
- B. Carter and A. C. Davis, Phys. Rev. D **61** (2000) 123501
- A. C. Davis, T. W. B. Kibble, M. Pickles and D. A. Steer, Phys. Rev. D **62** (2000) 083516
- R. Cordero and E. Rojas, Int. J. Mod. Phys. A **17** (2002) 73
- C. N. Ferreira, M. B. D. Porto and J. A. Helayel-Neto, Nucl. Phys. B **620** (2002) 181
- M. Pickles and A. C. Davis, Phys. Lett. B **520** (2001) 345
- A. Achucarro, A. C. Davis, M. Pickles and J. Urrestilla, Phys. Rev. D **66** (2002) 105013, Phys. Rev. D **68** (2003) 065006
- S. C. Davis, P. Binetruy and A. C. Davis, arXiv:hep-th/0501200.

F-term inflation

string: SUSY broken, non BPS

$$W = \kappa S(\phi_+\phi_- - v^2)$$

$$\phi_+ = \phi_-^\dagger = e^{in\theta} v f$$

$$A_\theta = -\frac{n}{g} a$$

$$F_0 = \kappa v^2 (1 - f^2)$$

Global SUSY: two goldstinos from SUSY trafo with $\epsilon_\pm$

index theorem:

$$\mathcal{L}_Y = ig\sqrt{2}\phi_+^\dagger \psi_+ \lambda - \kappa \phi_+ \psi_- \psi_S - ig\sqrt{2}\phi_-^\dagger \psi_- \lambda - \kappa \phi_- \psi_+ \psi_S$$

$$\sim -\phi\psi_-(\psi_S + \lambda) - \phi^*\psi_+(\psi_S - \lambda)$$

SUGRA: no zero modes

Chiral strings possible via Majorana term:

$$W = h\phi_+ \chi^2_{-1/2}$$

NB. RH neutrino zero mode decays at EW phase transition

Susy trafo with $\sigma^0\sigma^3\epsilon_+ = +\epsilon_+ \;\Rightarrow\; \epsilon_+ = \binom{0}{1}$

- $[Q, H] = 0$
- satisfy eq. of motion

DAVIS, DAVIS, TRODDEN

$$\begin{aligned} \delta\lambda(r,\theta) &= -i2g\xi(1-f^2)\epsilon_+ \\ \delta\psi_+(r,\theta) &= i2\sqrt{2}\sqrt{\xi}\frac{n}{r}(1-a)f e^{i(n-1)\theta}\epsilon_+^* \end{aligned}$$

How about $n > 1$?

Lagrangian invariant under trafo with $\zeta(x)$ with $\bar\sigma^\mu\partial_\mu\zeta(x) = 0$

since $\delta\mathcal{L} = \partial_\mu\bar\zeta j^\mu = \ldots(\bar\sigma\partial_\mu\zeta)^\dagger\ldots = 0$

SUSY trafo with $\zeta_+ = e^{i(n-1)\theta} r^{(n-1)}\epsilon_+$

BARBIERI, FERRARA, NANOPOULOS, STELLE
STELLE, WEST; FREEDMAN

FI-term

global SUSY

$$\int \mathrm{d}^4x\mathrm{d}^4\theta\xi V$$

$$V \to V + \frac{1}{g}(\Lambda - \bar{\Lambda})$$

SUGRA

$$\int \mathrm{d}^4x\mathrm{d}^4E\theta\mathrm{e}^{\xi V/m_\mathrm{P}^2}$$

$$V \to V + \frac{1}{g}(\Lambda - \bar{\Lambda})$$

$$E \to E\mathrm{e}^{\Sigma+\bar{\Sigma}}$$

R-symmetry: $[Q,R] = -iQ$

$$\theta \to \mathrm{e}^{-i\alpha}\theta \ \ \& \ \ \bar{\theta} \to \mathrm{e}^{i\alpha}\bar{\theta}$$

$$V \to V, \quad \lambda \to \mathrm{e}^{i\alpha}\lambda$$

$$\phi \to \mathrm{e}^{ir\alpha}\phi, \ \ \psi \to \mathrm{e}^{i(r-1)\alpha}\psi$$

$$e_\mu^m \to e_\mu^m, \ \ \psi_\mu \to \mathrm{e}^{i\alpha}\psi_\mu$$