# The Cosmology of the Nonsymmetric Theory of Gravitation (NGT)

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**Based on astro-ph/0503289 and on unpublished work with Wessel Valkenburg** (master's student)

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# Introduction: Historical remarks

- NGT: metric tensor is not symmetric

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu}$$

$$\longrightarrow \quad g_{\mu\nu} = \bar{g}_{(\mu\nu)}, \qquad B_{\mu\nu} = \bar{g}_{[\mu\nu]} = \frac{1}{2}\left(\bar{g}_{\mu\nu} - \bar{g}_{\nu\mu}\right)$$

In 1925 Einstein proposed it as a unified theory of gravity and electromagnetism

- It does not work since

  (a) Geodesic equation does not reproduce Lorentz force

  (b) Equations of motion do not impose divergenceless magnetic field

- In 1979 Moffat proposed it as a generalised theory of gravitation:

  Nosymmetric Theory of Gravitation (NGT)

- Advantages of NGT

- Unlike Einstein's Theory of General Relativity, contains no singularities?

Moffat

# Introduction: Absence of singularities

- E.g. most general static spherically symmetric solution

$$ds^2 = \left(1 - \frac{2m}{r}\right)\left(1 - \frac{l^4}{r^4}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

- From geodesic equation: when $l > 2m$ a test particle motion stops at $r = l > 2m$ before it reaches event horizon

- Problems with Nosymmetric Theory of Gravitation (NGT)

  (a) When quantised, in its simplest disguise, NGT contains ghosts

  (b) There is instability (growing mode) even in Minkowski background

- Fix: make the nonsymmetric field massive

  ⟹ That way one gets rid of ghosts and unstable mode

# Introduction: Modified Newton Law

- Moffat (2004) has proposed that NGT can explain the flat rotation curves of galaxies without invoking dark matter

$$a = -\frac{G_0 M}{r^2}\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp\left(-\frac{r}{r_0}\right)\left(1 + \frac{r}{r_0}\right)\right]\right\}$$

- such that at short distances

$$\Longrightarrow \quad a = -\frac{G_0 M}{r^2} \qquad (r \ll r_0)$$

- and at large distances

$$\Longrightarrow \quad a = -\frac{G_\infty M}{r^2}, \quad G_\infty = G_0\left(1 + \sqrt{\frac{M_0}{M}}\right) \qquad (r \gg r_0)$$

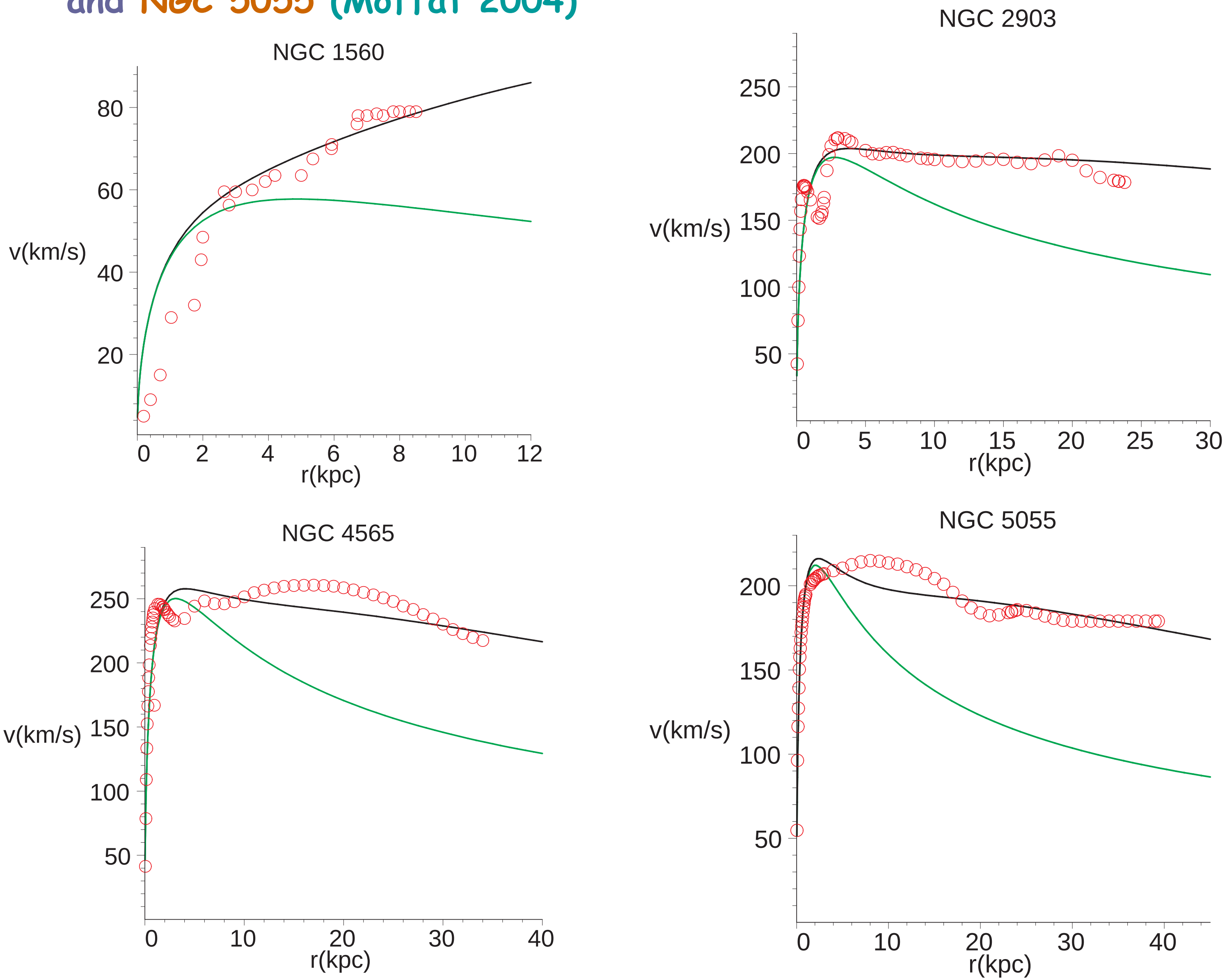
- Moffat takes $r_0 \approx 14\,\text{kpc}$, $M_0 \approx 10^{12} M_{Sun}$ to explain flat rotation curves

→ **To explain light lensing in clusters, Moffat takes a different $M_0 \approx 10^{15} M_{Sun}$**

- Also: Pioneer 10 & 11 acceleration anomaly (Moffat 2004)

# Introduction: Galaxy rotation curves

- Fits to the rotation curves for the galaxies: NGC1560, NGC 2903, NGC 4565 and NGC 5055 (Moffat 2004)




$r_0 \approx 14\,\mathrm{kpc}$,

$M_0 \approx 10^{12} M_{Sun}$

# NGT Action

- In the weak B-field limit, the NGT action can be written as

$$S = S_{HE} + S_{NGT} + S_{matter}$$

- The Hilbert-Einstein action

$$S_{HE} = -\int d^4x \sqrt{-g}\, \frac{1}{16\pi G_N}(R + 2\Lambda)$$

- The action of the Nonsymmetric Theory of Gravitation (to 2nd order)

$$S_{NGT} = \int d^4x \sqrt{-g}\left(\frac{1}{12} g^{\mu\alpha} g^{\nu\beta} g^{\rho\gamma} H_{\mu\nu\rho} H_{\alpha\beta\gamma} - \frac{1}{4} m_B^2 g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu} B_{\alpha\beta} - c\, R_{\mu\nu} B^{\mu\alpha} B_{\alpha}^{\ \nu} - c' R_{\mu\nu\alpha\beta} B^{\mu\alpha} B^{\nu\beta} + ..\right)$$

- Field strength

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

- mass is added for stability (Kunstater et al 1984; Damour, Deser, McCarthy 1993), but it also arises from a cosmological term $\Lambda$, since $\sqrt{-\bar{g}} = \sqrt{-g}\left(1 - \frac{1}{4} B^{\mu\alpha} B_{\mu\alpha} + ..\right)$
- At linearised order, first term is gauge invariant, while the whole action may be **diffeomorphism invariant** (with the appropriate choice of constants c, c')

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

# Conformal space-times

- The (symmetric part of) the metric tensor in a conformal space time is

$$g_{\mu\nu} = a^2\,\eta_{\mu\nu}, \quad \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

⟹ a = scale factor

- The NGT action is then

$$S_{NGT} \to \int d^4x \left( \frac{1}{12a^2}\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\rho\gamma}H_{\mu\nu\rho}H_{\alpha\beta\gamma} - \frac{1}{4}m_B^2\eta^{\mu\alpha}\eta^{\nu\beta}B_{\mu\nu}B_{\alpha\beta} \right)$$

⟹ Note that in the limit a -> ∞ **(late time inflation)**, the **kinetic term** drops out, and the field **fluctuations** can grow without a limit

- This is opposite of a scalar field action (kinetic term), for which fluctuations get frozen in

$$S_{scalar} \to \int d^4x \frac{1}{2} a^2 \eta^{\mu\alpha} (\partial_\mu \phi)(\partial_\alpha \phi)$$

# Physical Modes

Consider the electric-magnetic decomposition of the Kalb-Ramond B-field

$$B_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

Ei = spin 1, parity -

Bi = spin 1, parity +

- equations of motion

$$\left(\partial^2 + a^2 m_B^2\right)\vec{E} = 0$$

$$\left(\partial^2 + a^2 m_B^2\right)\vec{B} - \frac{2a'}{a}\left(\partial_\eta \vec{B} + \vec{\partial} x \vec{E}\right) = 0$$

- Lorentz "gauge" (consistency) condition $\eta^{\mu\nu}\partial_\mu B_{\nu\rho} = 0$ implies

$$\partial_\eta \vec{E} - \vec{\partial}\, x\, \vec{B} = 0, \qquad \vec{\partial} \cdot \vec{E} = 0$$

NB1: $\vec{\partial} \cdot \vec{B} = 0$ is missing $\rightarrow \vec{B}^T$ may be dynamical

NB2: $\vec{B}^T$ equation is not independent (given by the transverse electric field)

NB3: $\vec{E}^L = 0$

NB4: From $\partial_\eta \vec{E}^T - \vec{\partial}\, x\, \vec{B}^T = 0 \rightarrow \vec{B}^T$ is a function of $\vec{E}^T$

- Physical DOFs (massive case): pseudovector $\vec{B}$ (spin = 1, parity = +)
- Physical DOF (massless case): longitudinal magnetic field $\vec{B}^L$

# Canonical quantisation

**Impose canonical commutation relation on B-field and its canonical momentum**

$$\hat{\vec{B}}^L(\vec{x},\eta) = a(\eta)\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}}\vec{\varepsilon}^L(\vec{k})\left[B_{\vec{k}}(\eta)\hat{b}_{\vec{k}} + B^*_{-\vec{k}}(\eta)\hat{b}^\dagger_{-\vec{k}}\right]$$

$$\left[\hat{b}_{\vec{k}}, \hat{b}^\dagger_{\vec{k}'}\right] = (2\pi)^3\delta^3(\vec{k}-\vec{k}')$$

$$\vec{k}\times\vec{\varepsilon}^L(\vec{k}) = 0$$

- Momentum space equation of motion for the modes

$$\left[\partial_\eta^2 + \vec{k}^2 + \frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2\right]B_{\vec{k}}(\eta) = 0$$

- **NB: Contrary to a scalar field (which decays with the scale factor), the Kalb-Ramond B-field grows with the scale factor a**

# Vacuum fluctuations in de Sitter inflation



**In De Sitter inflation the scale factor is,** $a = -\frac{1}{H_I \eta} \quad (\eta < -1/H_I)$ **such that**

$$\frac{a''}{a} - 2\left(\frac{a'}{a}\right)^2 = 0$$

- mode equation of motion reduces to that of conformal vacuum

$$\left[\partial_\eta^2 + \vec{k}^2\right] B_{\vec{k}}(\eta) = 0$$

- The conformal rescaling of the longitudinal B-mode

$$\vec{B}^L(\vec{x},\eta) \rightarrow \vec{B}_c^L(\vec{x},\eta) = \frac{\vec{B}^L(\vec{x},\eta)}{a}$$

- The mode functions are those of conformal vacuum

$$B_{\vec{k}}(\eta) = \frac{1}{\sqrt{2k}} e^{\pm ik\eta}$$

NB: **In conformal vacuum there is very little particle production during inflation**

# Radiation era

**In radiation era,** $a = H_I \eta \quad (\eta > 1/H_I)$ **such that the mode equation**

$$\left[\partial_\eta^2 + \vec{k}^2 - \frac{2}{\eta^2}\right] B_{\vec{k}}(\eta) = 0$$

- which is Bessel's equation of index 3/2. The solution is (Bunch-Davies)

$$B_{\vec{k}}(\eta) = \frac{1}{\sqrt{2k}}\left[\alpha_{\vec{k}}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta} + \beta_{\vec{k}}\left(1 + \frac{i}{k\eta}\right)e^{+ik\eta}\right]$$

with the "Wronskian" condition

$$|\alpha_{\vec{k}}|^2 - |\beta_{\vec{k}}|^2 = 1$$

- The matching coefficients are

$$\alpha_{\vec{k}} = -\frac{1}{2}\frac{H_I^2}{k^2}\left(1 - 2i\frac{k}{H_I} - 2\frac{k^2}{H_I^2}\right)e^{2ik/H_I}, \qquad \beta_{\vec{k}} = -\frac{1}{2}\frac{H_I^2}{k^2}$$

NB: **For superhorizon modes at the end of inflation (a=1):** $k \ll H_I$

# Matter era

**In matter era,** $a = \frac{H_I}{4\eta_{eq}}(\eta+\eta_{eq})^2 \quad (\eta > \eta_{eq})$ **and the mode equation**

$$\left[\partial_\eta^2 + \vec{k}^2 - \frac{6}{(\eta+\eta_{eq})^2}\right] B_{\vec{k}}(\eta) = 0$$

- which is Bessel's equation of index 5/2, whose solution is

$$B_{\vec{k}}(\eta) = \frac{1}{\sqrt{2k}}\left[\gamma_{\vec{k}}\left(1 - \frac{3i}{k\bar{\eta}} - \frac{3}{(k\bar{\eta})^2}\right)e^{-ik\bar{\eta}} + \delta_{\vec{k}}\left(1 + \frac{3i}{k\bar{\eta}} - \frac{3}{(k\bar{\eta})^2}\right)e^{+ik\bar{\eta}}\right], \qquad \bar{\eta} = \eta + \eta_{eq}$$

with the "Wronskian" condition

$$|\gamma_{\vec{k}}|^2 - |\delta_{\vec{k}}|^2 = 1$$

and the matching coefficients

$$\gamma_{\vec{k}} e^{-2ik\eta_{eq}} = \alpha_{\vec{k}}\left(1 + \frac{1}{2}\frac{i}{k\eta_{eq}} - \frac{1}{8}\frac{1}{(k\eta_{eq})^2}\right)e^{-ik\eta_{eq}} + \beta_{\vec{k}}\frac{1}{8}\frac{1}{(k\eta_{eq})^2}e^{+ik\eta_{eq}}$$

$$\delta_{\vec{k}} e^{+2ik\eta_{eq}} = \alpha_{\vec{k}}\frac{1}{8}\frac{1}{(k\eta_{eq})^2}e^{-ik\eta_{eq}} + \beta_{\vec{k}}\left(1 - \frac{1}{2}\frac{i}{k\eta_{eq}} - \frac{1}{8}\frac{1}{(k\eta_{eq})^2}\right)e^{+ik\eta_{eq}}$$

# Spectrum of energy density

The stress energy tensor of a Kalb-Ramond field is standard,

$$T^{NGT}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{NGT}}{\delta g_{\mu\nu}}$$

- The energy density is

$$T_0^{0NGT} = \frac{1}{2a^6}\left[\left(\partial_\eta \vec{B} + \vec{\partial} \times \vec{E}\right)^2 + (\vec{\partial}\cdot\vec{B})^2 + a^2 m_B^2\left(\vec{E}^2 + \vec{B}^2\right)\right] \equiv \rho_{NGT}$$

- When $m_B^2 \to 0$ the only contribution to the **spectrum** comes from long. B-field

$$\left\langle 0 \,|\, \hat{T}_0^{0NGT} \,|\, 0 \right\rangle = \int \frac{dk}{k} P_{NGT}(\vec{k},\eta)$$

$$P_{NGT}(\vec{k},\eta) = \frac{k^3}{2\pi^2} T_0^{0NGT}(\vec{k},\eta)$$

$$P_{NGT}(\vec{k},\eta) = \frac{H_I^4}{4\pi^2 a^4}\left\{\left|\partial_\eta B^L_{\vec{k}}(\eta) + \frac{a'}{a}B^L_{\vec{k}}(\eta)\right|^2 + \left(k^2 + a^2 m_B^2\right)|B^L_{\vec{k}}(\eta)|^2\right\}$$

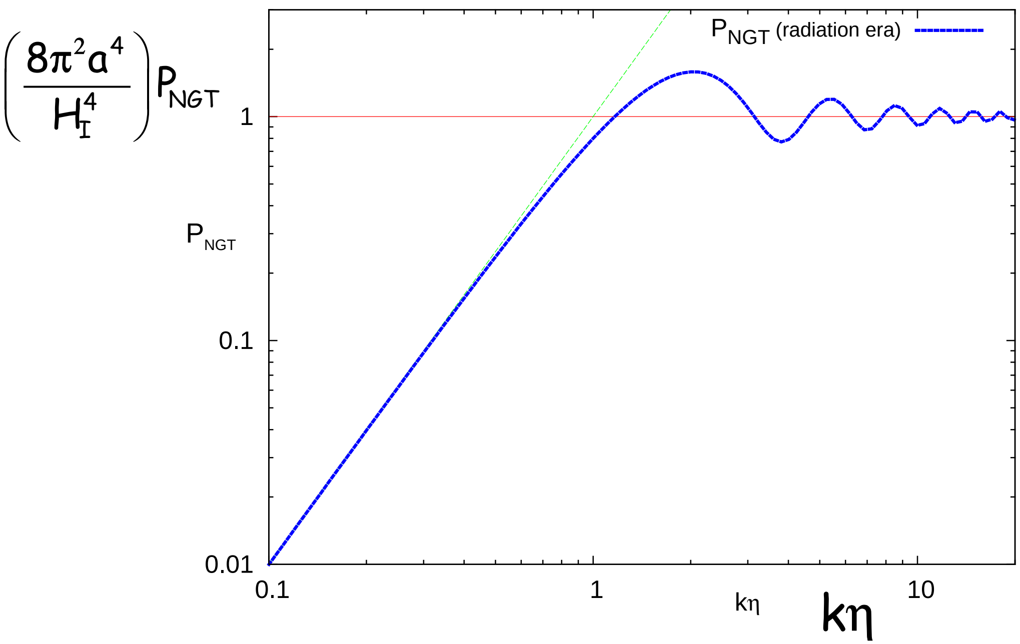
NB1: this spectrum is relevant for coupling to (Einstein) gravity

NB2: for coupling to (CMB) photons, $\left\langle 0 \,|\, \hat{\vec{B}}^L(\vec{x},\eta)^2 \,|\, 0 \right\rangle = \int \frac{dk}{k} P'_{NGT}(\vec{k},\eta)$ may be relevant

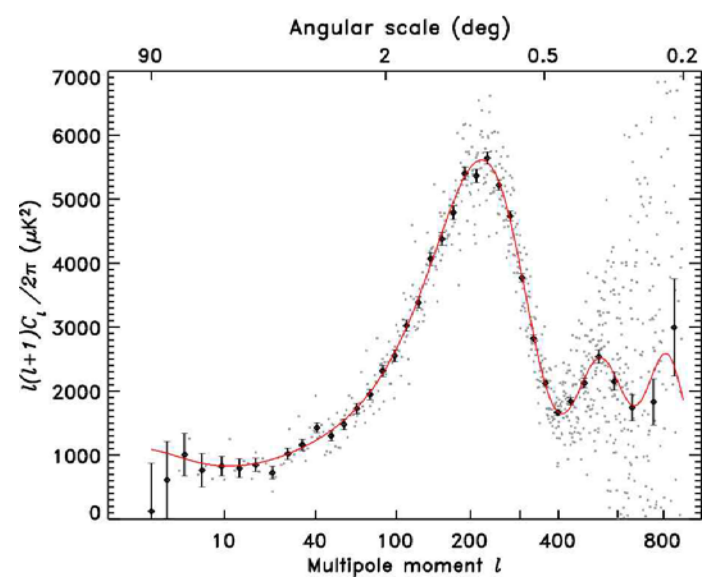
# Spectrum in radiation era

The spectrum in radiation era for a massless Kalb-Ramond field

$$P_{NGT}(\vec{k},\eta) = \frac{H_I^4}{8\pi^2 a^4}\left\{\left(1+\frac{1}{2}\frac{1}{(k\eta)^2}\right) - \frac{\sin(2k\eta)}{k\eta} - \frac{1}{2}\frac{\cos(2k\eta)}{(k\eta)^2}\right\}$$




- on superhorizon scales ($k\eta<1$) the energy density spectrum scales as $P \sim k^2$.
- on subhorizon scales ($k\eta>1$), $P \sim$ const. + small oscillations




To be compared with the 1 year WMAP CMB spectrum (2003)

NB: The energy density in the field, $\rho_{NGT} = \int dk\, P_{NGT}/k$ is dominated by the ultraviolet, where it exhibits a log divergence,

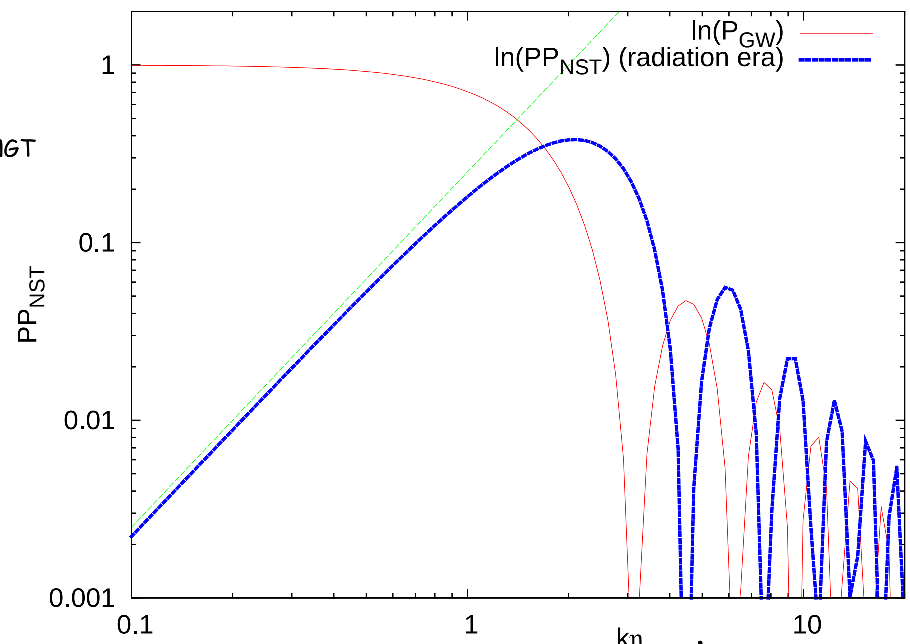
$$\rho_{NGT} \approx \frac{H_I^4}{8\pi^2 a^4}\ln(k_{max}\eta)$$

# Comparison with gravitational wave spectrum

- **massless NGT field spectrum:** $\langle 0 | [\hat{B}^L(\vec{x},\eta)]^2 | 0 \rangle = \int \frac{dk}{k} P'_{NGT}(\vec{k},\eta)$

$$P'_{NGT}(\vec{k},\eta) = \frac{a^2 H_I^4}{8\pi^2 k^2}\left\{\left(1+\frac{1}{(k\eta)^2}\right) - \frac{2}{k\eta}\sin(2k\eta) + \left(1-\frac{1}{(k\eta)^2}\right)\cos(2k\eta)\right\}$$

$$\left(\frac{8\pi^2 a^4}{H_I^4}\right) P'_{NGT}$$




kη

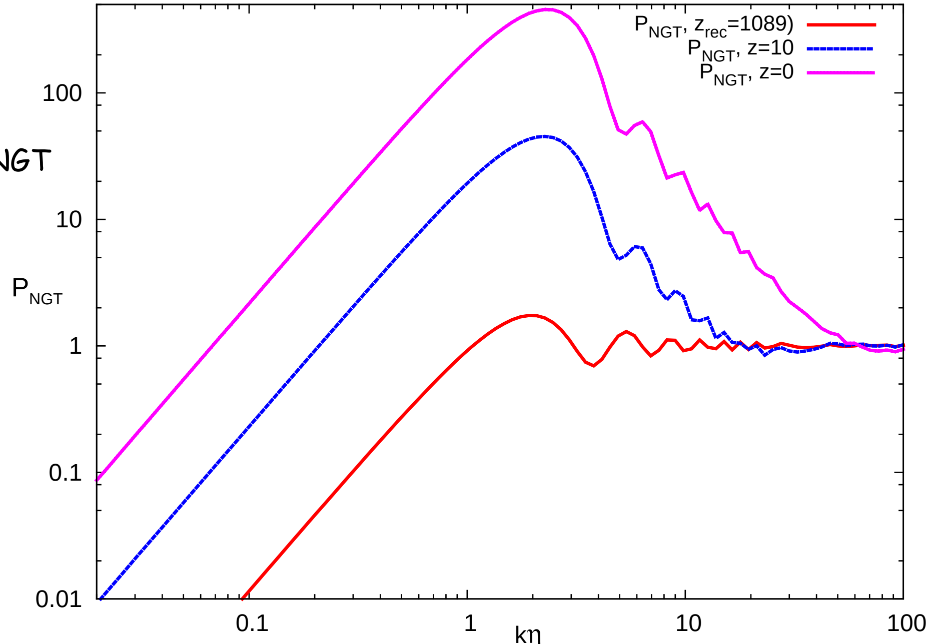
$$P'_{GW}(\vec{k},\eta) \propto \frac{\sin^2(k\eta)}{k^2}$$

- **NB: NGT field is important at horizon crossing, gravitational waves dominate on superhorizon scales**

# Spectrum in matter era

The spectrum in matter era is shown in figure (log-log plot)

$$\left(\frac{8\pi^2 a^4}{H_I^4}\right) P_{NGT}$$




kη

- on superhorizon scales ($k\eta<1$) the energy density spectrum scales as $P\sim k^2$.
- on subhorizon scales ($1<k\eta<a/a_{eq}$), $P\sim 1/k^2$
- on subhorizon scales ($k\eta>a/a_{eq}$), $P\sim$const.

NB1: The bump in the power spectrum in matter era is caused by the modes which are superhorizon at equality, and which after equality begin scaling as nonrelativistic matter

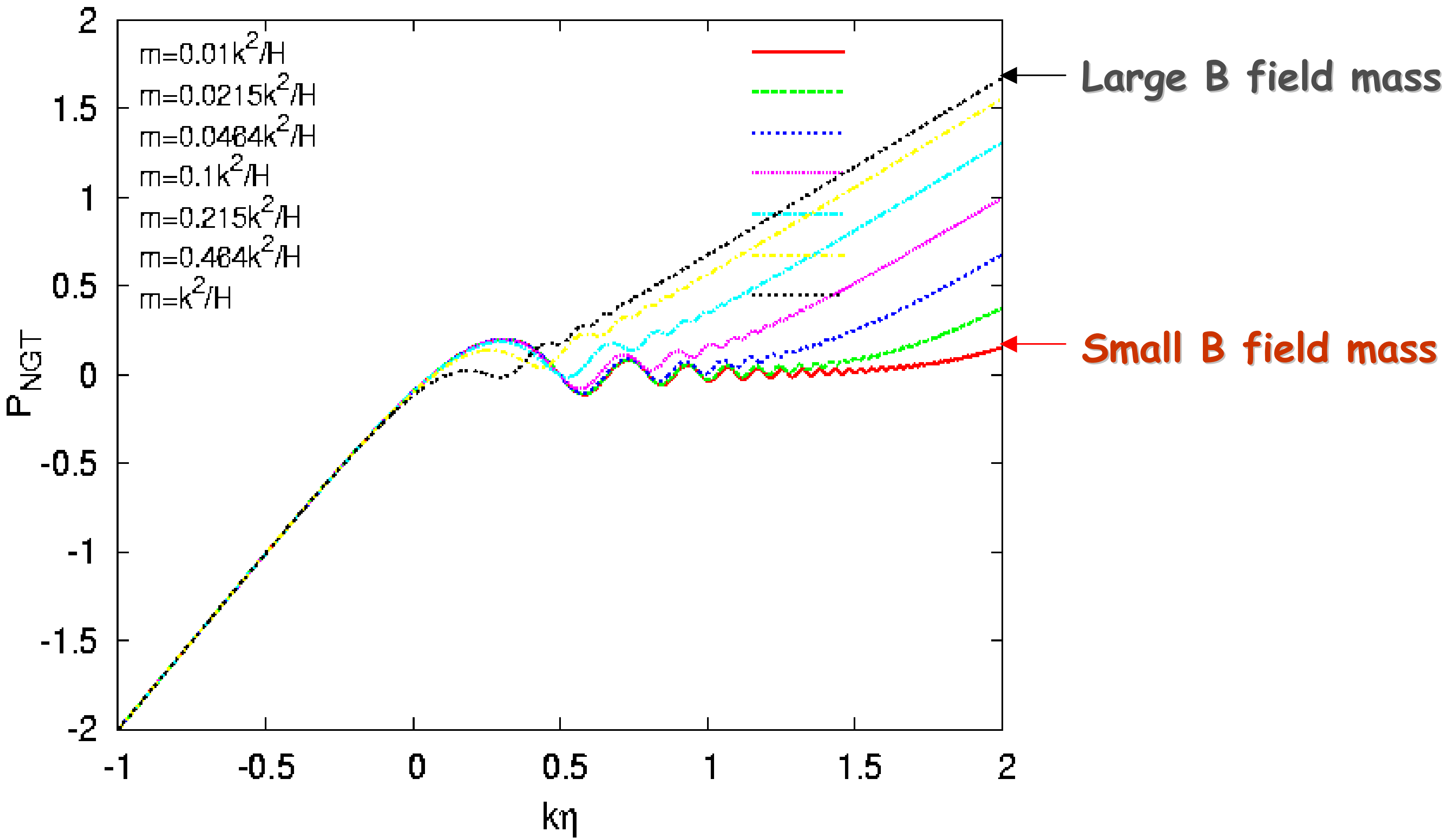
$$P_{NGT} \propto 1/a^3$$

NB2: The log divergent part of the spectrum continues scaling in matter era as, $P_{NGT} \propto \frac{1}{a^4}$ such that the energy density becomes eventually dominated by the "bump"

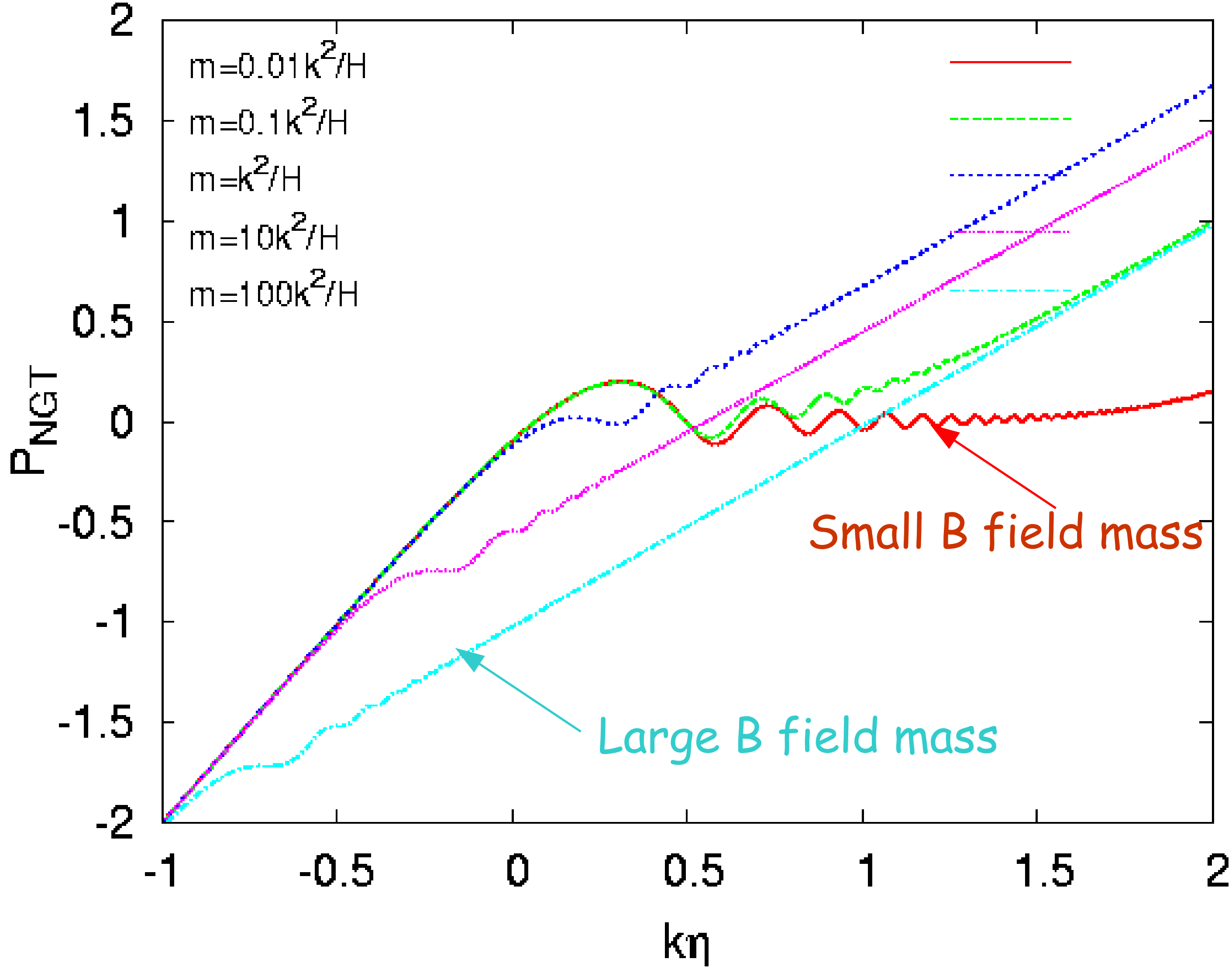
$$\rho_{NGT} \approx \frac{H_I^4}{8\pi^2 a^3 a_{eq}}$$

# Radiation era: massive B field spectrum

- $B_k^L \propto$ Hankel function (de Sitter inflation)
- $B_k^L \propto$ Whittaker function (radiation era)



# Radiation era: massive B field spectrum (2)



# Discussion

- Since during matter era, all momenta with $k\eta_{eq} < 1$, corresponding to today's (comoving) momentum, $k_{phys} < H_0\sqrt{z_{eq}}/2 \;\; (z_{eq} \approx 3230)$ $\left(l_{phys} \geq \pi/k_{phys} \sim \pi \times 150\,\mathrm{Mpc}\right)$ scale as nonrelativistic matter, the amplitude of their perturbations grow, which begs the question: Can the nonsymmetric field be a DARK MATTER CANDIDATE?

→ ANSWER: PROBABLY NOT (similarly as primoridial gravitational waves cannot) Basically, the NGT field lacks power on **small** scales)

- It has been suggested that NGT can provide an explanation for DARK ENERGY. **Our results for the energy density scaling show that this is not the case.**

- The simplest coupling to photons, $\delta L_{NGT-EM} \supset -\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}B_{\alpha\beta}$ breaks gauge invariance. The relevant spectrum is in this case, $P'_{NGT} \propto P_{NGT}/k^2$, which is scale invariant on superhorizon scales, and scales as $1/k^2$ on subhorizon scales

- Introducing a small mass $m_B$ (less than horizon) does not alter these conclusions dramatically (additional power on small scales)

- A more complete study of the spectra of geometric NGTs is a subject of a forthcoming publication