Post-Newtonian cosmology

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Motivation

i. Is there a **systematic** framework which allows us to **quantify** general relativistic (GR) effects in cosmology?

ii. Is there a **systematic** framework which allows us to **test** and **classify** different gravity theories by using cosmological tests?

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Approximation schemes (GR)

Post-Minkowskian
(Weak field / Fast motion)

$$g^{\mu\nu} = \eta^{\mu\nu} + \epsilon\gamma^{\mu\nu} + \dots$$

- Expand metric around Minkowski background
- Try to mimic electrodynamics
- Do not expand matter variables

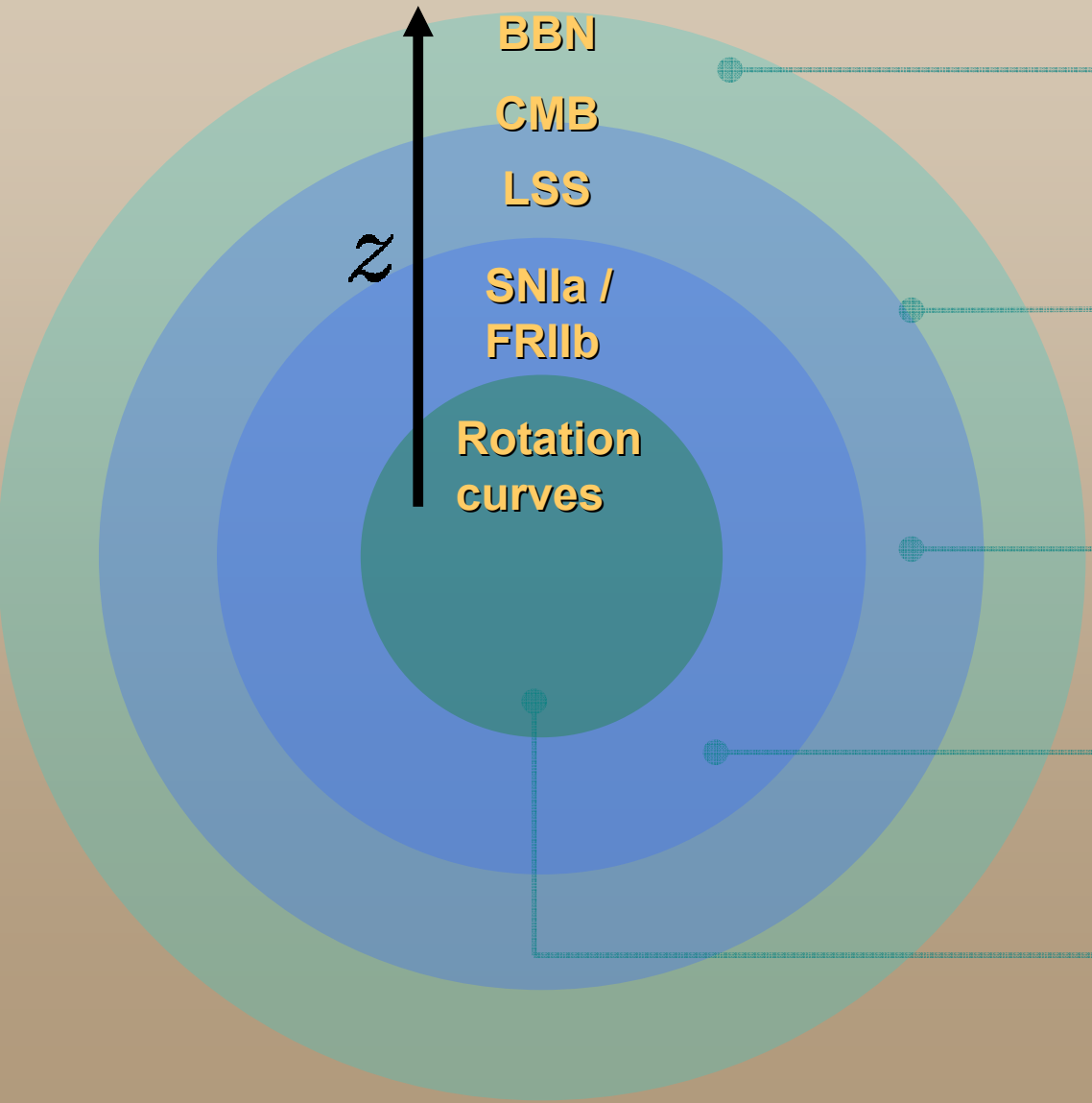
Post-Newtonian
(Weak field / Slow motion)

$$g^{\mu\nu} = \overset{0}{g}^{\mu\nu} + c^{-1}\,\overset{1}{g}^{\mu\nu} + \dots$$

$$u^{\mu} = \overset{0}{u}^{\mu} + c^{-1}\,\overset{1}{u}^{\mu} + \dots$$

- Start from Newtonian limit
- Expand metric and velocities in powers of c
- Try mimic Newtonian gravity
- Do not expand other matter variables

Cosmological tests and approximation methods



Expansion rate parametrization

Linearized field equations

Newtonian limit / numerical simulations

Expansion rate parametrization

Weak field approximation / Newtonian limit

Approximation schemes (cosmology)

Cosmological Perturbations
(Weak field / Fast motion)

$$g^{\mu\nu} = \eta^{\mu\nu} + \epsilon\gamma^{\mu\nu} + \dots$$

$$\eta^{\mu\nu} \rightarrow g^{\mu\nu}_{\text{FLRW}}$$

- Expand metric around **FLRW** background
- Try to mimic electrodynamics
- Do not expand matter variables

Cosmological Post-Newtonian
(Weak field / Slow motion)

$$g^{\mu\nu} = \overset{0}{g}{}^{\mu\nu} + c^{-1}\,\overset{1}{g}{}^{\mu\nu} + \dots$$

$$u^{\mu} = \overset{0}{u}{}^{\mu} + c^{-1}\,\overset{1}{u}{}^{\mu} + \dots$$

- Start from Newtonian limit (with additional cosmic **expansion**)
- Expand metric and velocities in powers of c
- Try mimic Newtonian gravity
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Approximation schemes (cosmology)

Cosmological Perturbations
(Weak field / Fast motion)

$$g^{\mu\nu} = \eta^{\mu\nu} + \epsilon\gamma^{\mu\nu} + \dots$$

$$g^{\mu\nu}_{\text{FLRW}}$$

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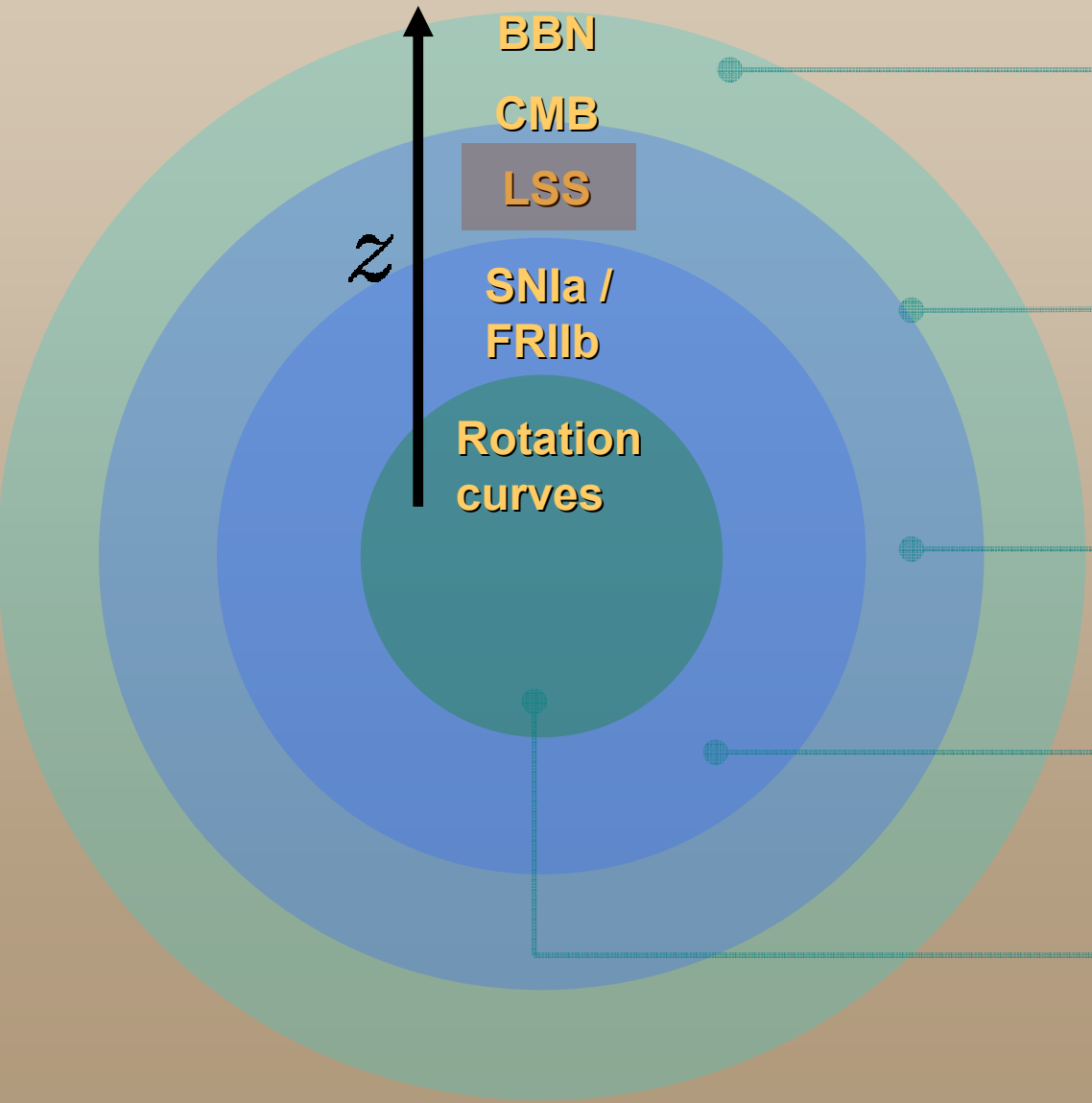
Cosmological tests and approximation methods



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The PN philosophy

$$T_{\alpha\beta} = \left(\begin{array}{cc} \text{Energy density} & \text{Momentum density} \\ \text{Energy flux density} & \text{Momentum flux density} \end{array} \right)$$

$$T_{00} > T_{0a} > T_{ab}$$

$g_{\alpha\beta}$ and u^{α} expandable in powers of c

PN order scheme

Order \ Metric	g_{00}	g_{0a}	g_{ab}
Newtonian	2	0	0
0.5PN	0	0	0
1PN	4	3	2
1.5PN	5	0	0
2PN	6	5	4
2.5PN	7	6	5
3PN	...	...	6
3.5PN	...	...	7

c^{-n}

Numbers correspond to order in inverse powers of the speed of light

CPNA metric ansatz & strategy

Provide metric ansatz by hand

Velocities / EM tensor

Connection / Curvature / FEQs

Impose GC / „Solve" FEQs

Determine form of the metric

EOMs / EMPT

$$
\begin{aligned}
g_{00} &= 1 - \frac{2U}{c^2} + O\left(c^{-4}\right) \\
g_{0a} &= h_{0a} = O\left(c^{-3}\right) \\
g_{ab} &= -a^2\delta_{ab} + O\left(c^{-2}\right)
\end{aligned}
$$

Determine final set FEQs / EOMs

Start over

Numerics

Metric & Field equations (1CPNA)

$$
\begin{aligned}
g_{00} &= 1 + \frac{2}{c^2}\left[\psi - U + \frac{1}{c^2}\left(U^2 - 2\Phi\right)\right] + O\left(c^{-6}\right) \\
g_{0a} &= \frac{1}{c^3}\left(4U_a - \chi_{,a}\right) + O\left(c^{-5}\right) \\
g_{ab} &= a^2\left(-1 - \frac{2U}{c^2}\right)\delta_{ab} + O\left(c^{-4}\right)
\end{aligned}
$$

1CPN metric

$$
\begin{aligned}
\nabla^2 U &= -4\pi G a^2 \rho \\
\nabla^2 \Phi &= -4\pi G a^2 \rho \phi \\
\nabla^2 \psi &= -4\pi G a^2 \rho V \\
\phi &= a^2 v^2 + U + \frac{1}{2}\Pi + \frac{3p}{2\rho} \\
V &= -\frac{3}{4\pi G \rho}\left(\frac{\ddot{a}}{a} + \frac{3}{c^2}\frac{\dot{U}\dot{a}}{a} + \frac{1}{c^2}\ddot{U}\right)
\end{aligned}
$$

„Poisson like“ FEQ hierarchy

$$
\begin{aligned}
\dot{\rho} + \nabla\left(\rho \underline{v}\right) + 3\frac{\dot{a}}{a}\rho &= 0 \\
2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} &= 0
\end{aligned}
$$

Constraints from GC

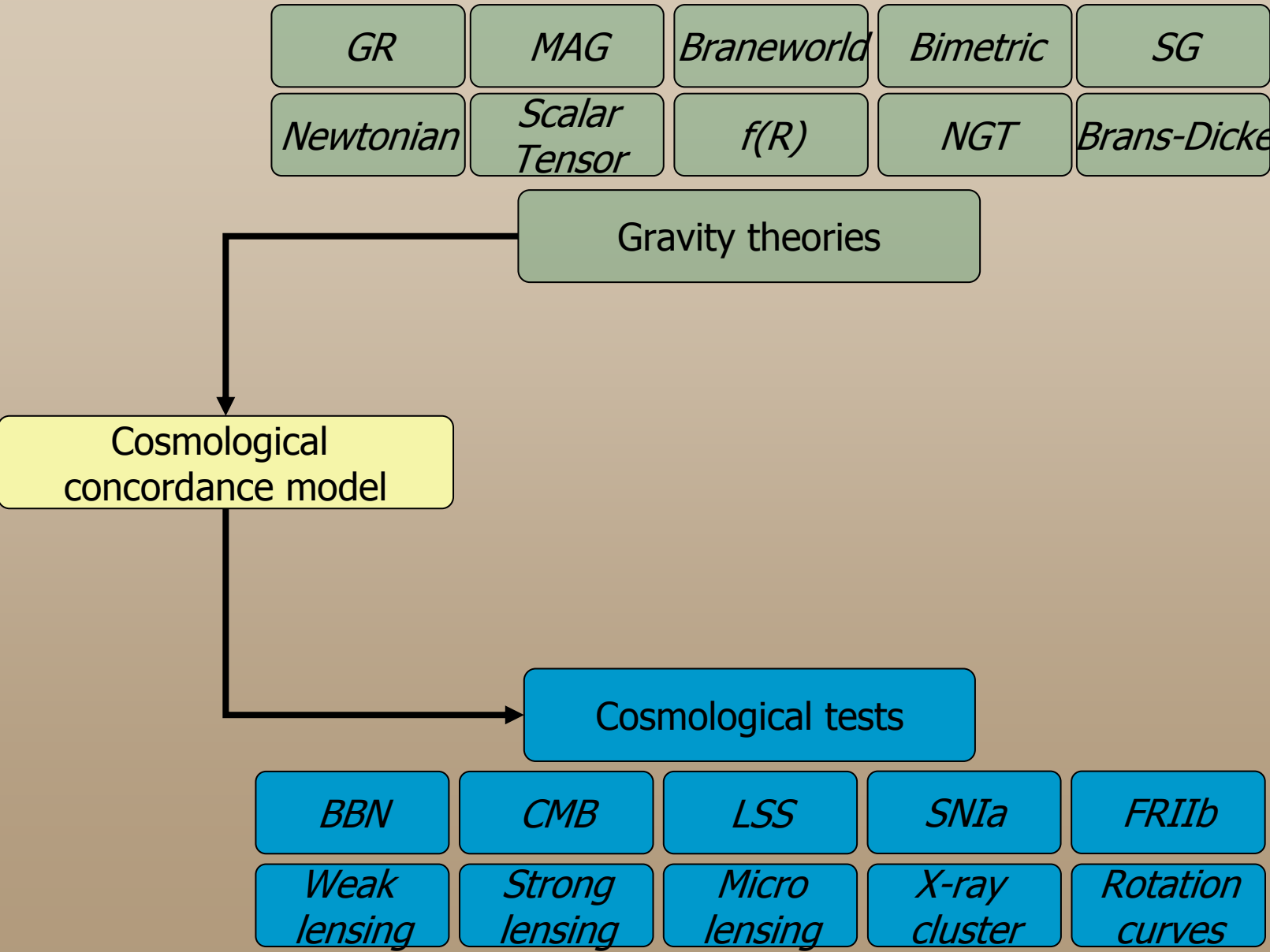
Equations of motion (1CPNA)

$$T^{\alpha\beta}{}_{;\beta} = 0$$

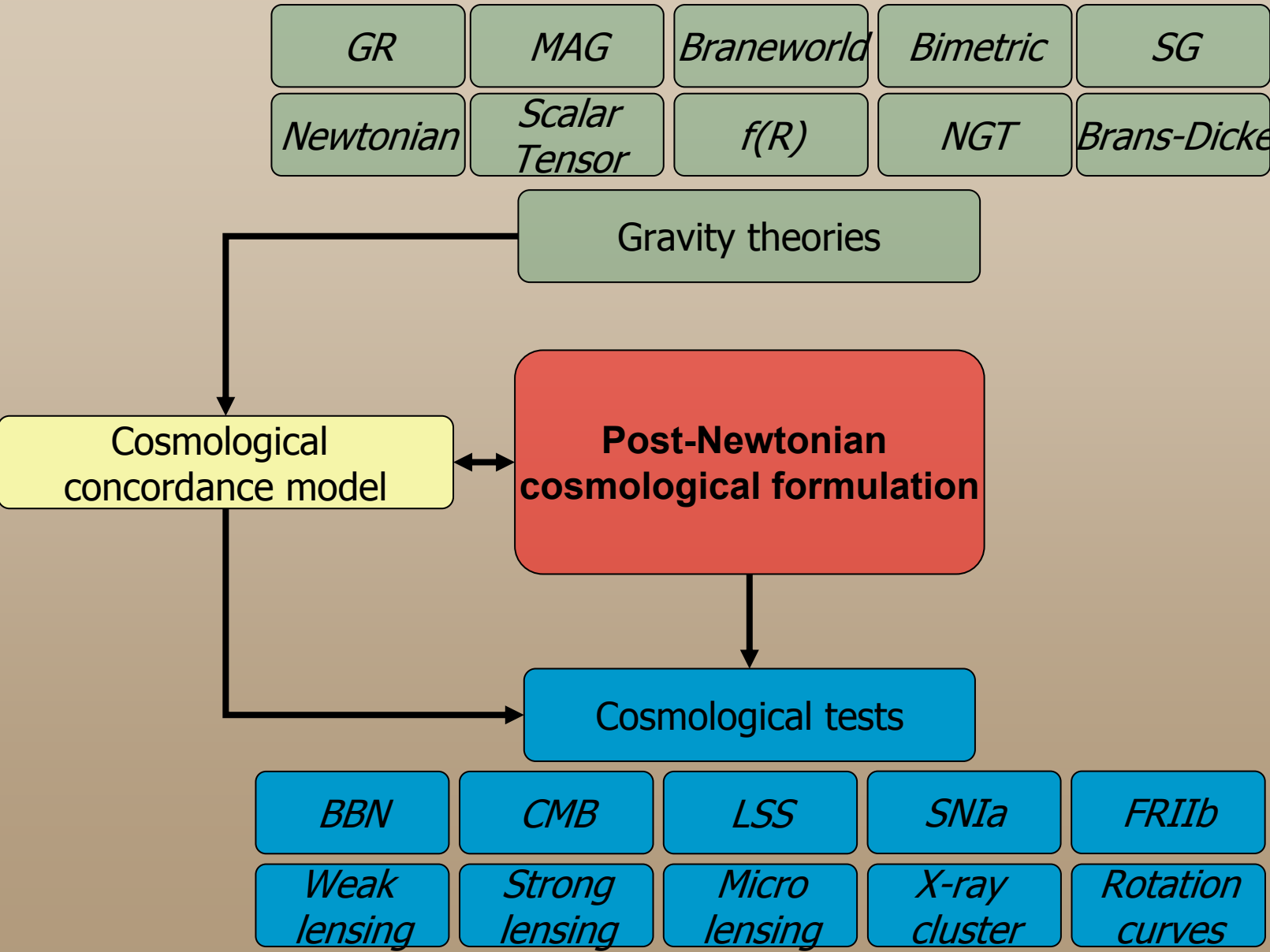
1PN form of the metric (general form)

$$
\begin{aligned}
T^{0\alpha}{}_{;\alpha} &= c\left\{\frac{1}{a^3}\left(a^3\rho\right)_{,t} + \left(\rho v^a\right)_{,a}\right\} \\
&+\frac{1}{c}\left\{\left(\rho\Pi\right)_{,t} + \dot\rho\left(a^2v^2 + 2U\right) + 3\frac{\dot a}{a}\left(p + 2\rho U + \rho\Pi\right) + \rho\left[a^2\left(v^2\right)_{,t} + 3\dot V\right]\right. \\
&\left.+\left[\rho v^a\left(\Pi + a^2v^2\right)\right]_{,a} + \left(pv^a\right)_{,a} + \rho v^a\left(3V - U\right)_{,a} + 2U\left(v^a\rho\right)_{,a}\right\} + O\left(c^{-3}\right) \\
T^{b\alpha}{}_{;\alpha} &= \left\{\left(\rho v^a v^b\right)_{,a} + \left(\rho v^b\right)_{,t} + \frac{1}{a^2}\left(p_{,b} - \rho U_{,b}\right) + 5\frac{\dot a}{a}\rho v^b\right\} \\
&+\frac{1}{c^2}\left\{\left(\rho\Pi v^b\right)_{,t} + \left(\rho\Pi v^b v^a\right)_{,a} + \left(pv^b\right)_{,t} + \left(pv^b v^a\right)_{,a} - V_{,b}\rho v^2\right\} \\
&+\rho a^2\left[\left(v^b v^a v^2\right)_{,a} + \left(v^b v^2\right)_{,t}\right] \\
&+\frac{\rho}{a^2}U_{,b}\left[2\left(V - U\right) - \Pi - \frac{p}{\rho} - a^2v^2\right] \\
&+2U\left[\left(\rho v^b v^a\right)_{,a} + \left(v^b\rho\right)_{,t}\right] \\
&+v^b\left[\rho\left(5\frac{\dot a}{a}\Pi + 7\dot a a v^2 + 10\frac{\dot a}{a}U + 5V_{,a}v^a + U_{,a}v^a + \dot U + 5\dot V\right)\right. \\
&\left.+\dot\rho a^2v^2 + 5\frac{\dot a}{a}p + \rho_{,a}v^a a^2v^2\right] \\
&+\frac{1}{a^2}\left[\frac{1}{2}\sigma_{,b}\rho - 2Vp_{,b} + \rho\left(h_{0a,b} - h_{0b,a}\right)v^a - \rho\dot h_{0b}\right]\Big\} + O\left(c^{-4}\right)
\end{aligned}
$$

Testing gravitational theories



Testing gravitational theories



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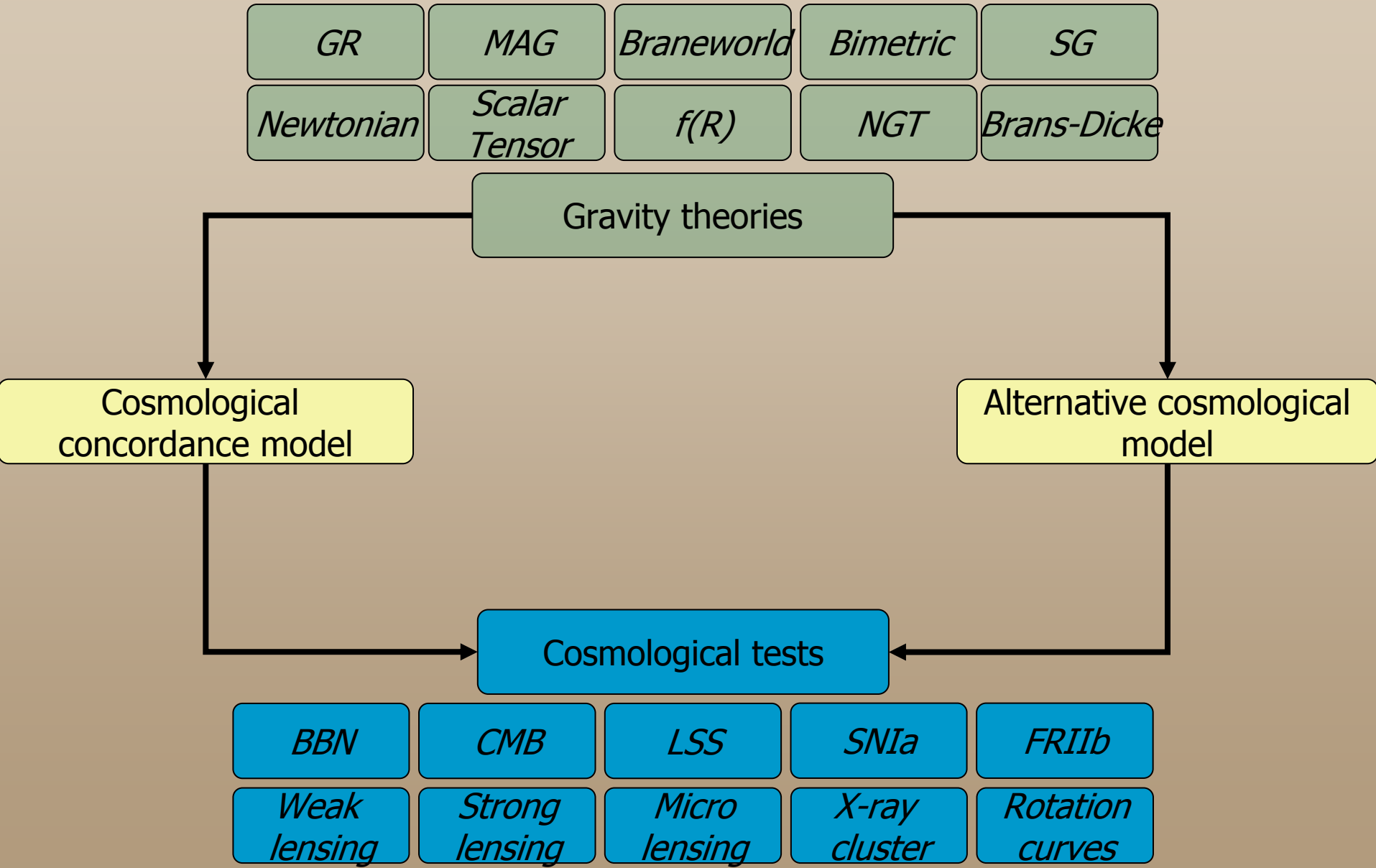
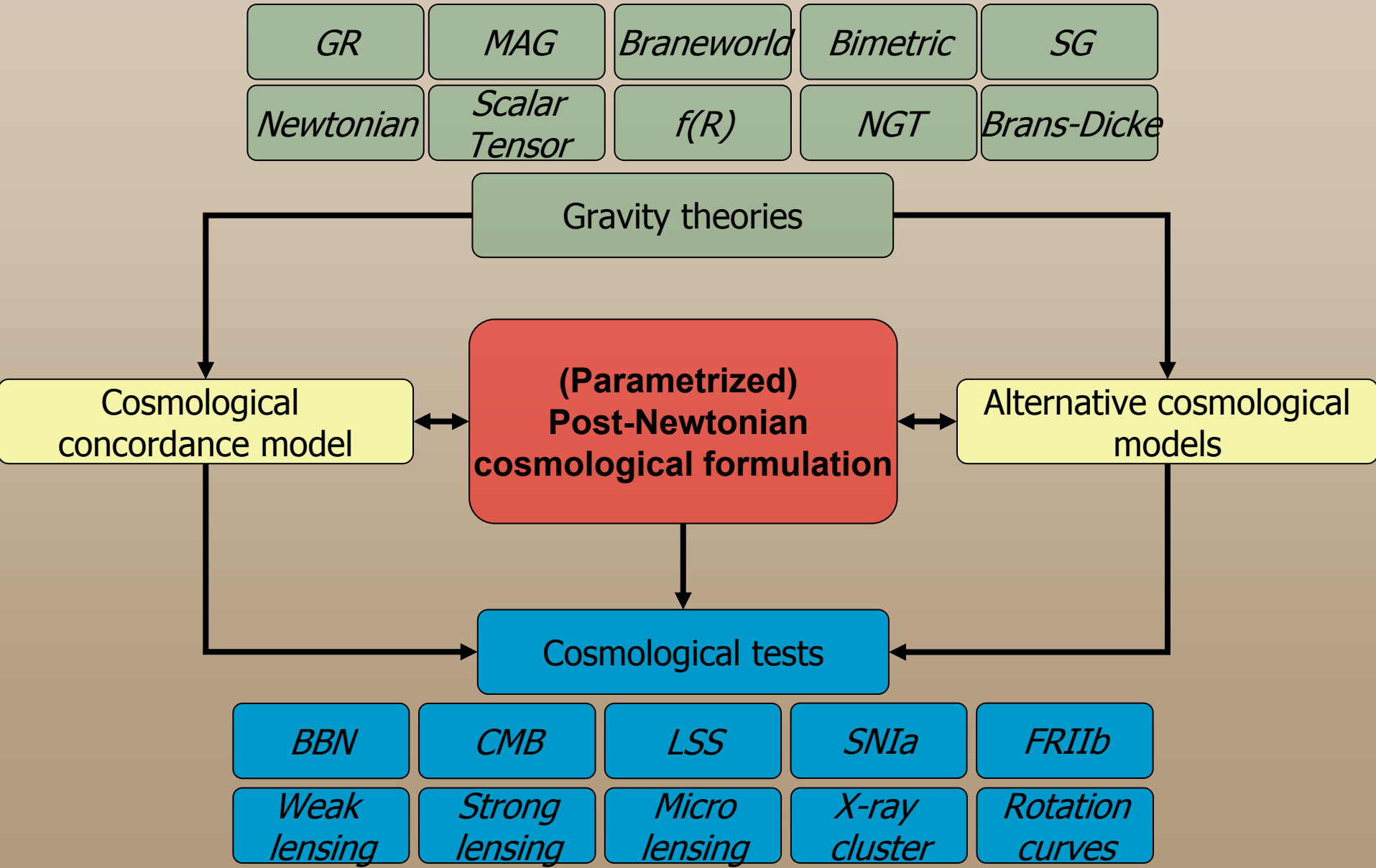
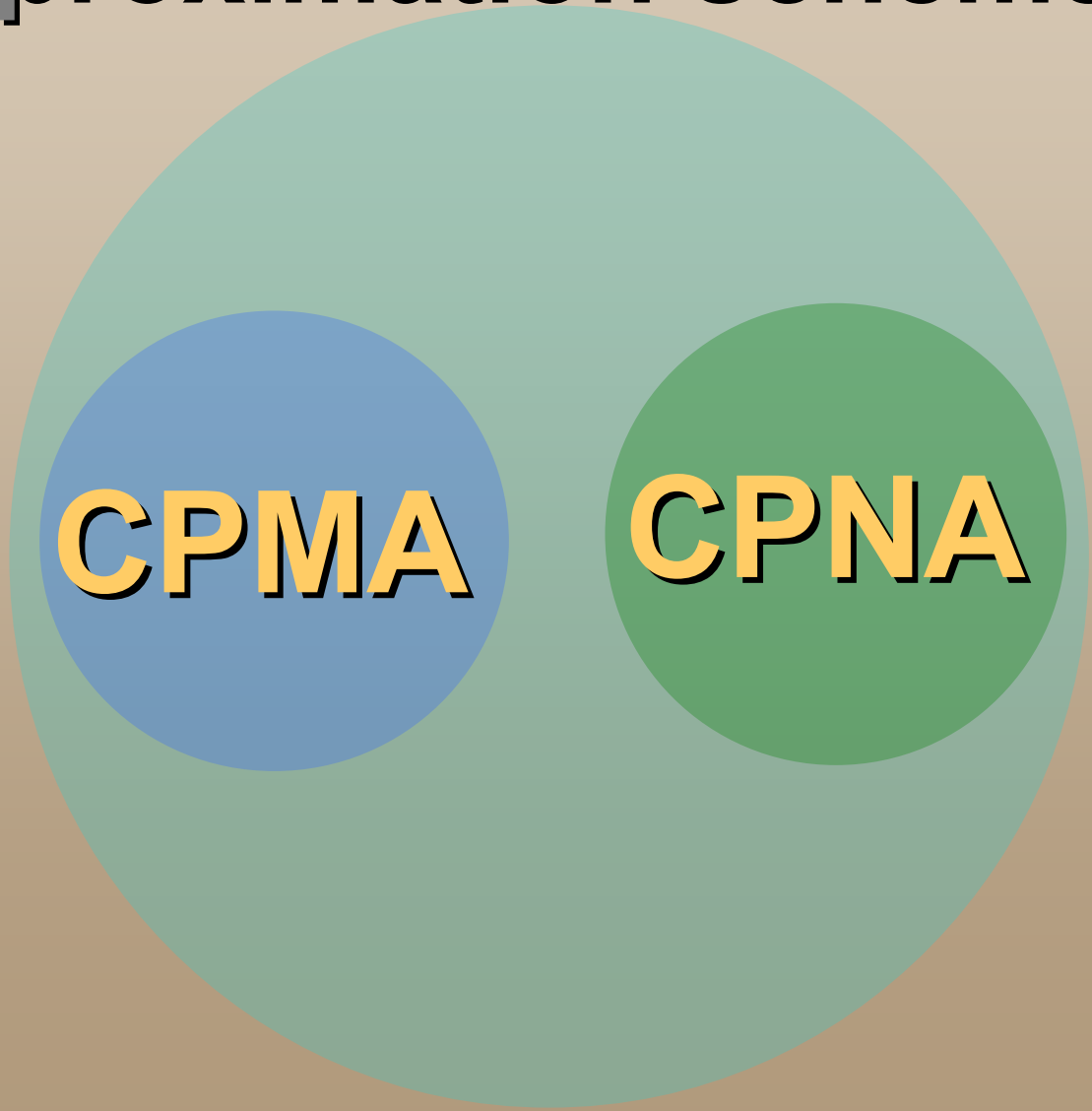
Testing gravitational theories



Testing gravitational theories



Combination of approximation schemes



Summary & Outlook

i. We successfully derived the **FEQs / EOMs / EMPT to 1st CPNA order** (following Chandrasekhar's ordering scheme)

ii. We are currently searching for a form of the FEQs and EOMs which is most suitable for **numerical simulations**

iii. We implemented and tested a **CA package** of routines for Maple (to be released soon!)

iv. We want **higher orders** and a **combination** of different approximation methods

Collaborators

- Jai-chan Hwang

  (Kyungpook National Unversity)

- Hyerim Noh

  (Korea Astronomy and Space Science Institute)

Publications

- Hwang et al. astro-ph/0507085
- Puetzfeld et al. (in preparation)

Last words...