Limits on Lorentz Violation from Gamma-Ray Bursts

A. Sakharov
ETHZ, CERN

COSMO-05, Bonn, Germany

OUTLINE

- Manifestation of quantum gravity in "low" energy phenomena
- Variation structures in GRBs light curves
- Spectral time lags in emission of GRBs and violation of Lorentz invariance
- Conclusions

in collaboration with
J. Ellis, N. Mavromatos, D. Nanopoulos and E.K.G. Sarkisyan

Quantum gravity approaches

The existance of the lower bound at which space-time responses actively to the presence of energy, may lead to violation of Lorentz symmetry. In fact, different approaches to this theory lead to this result.

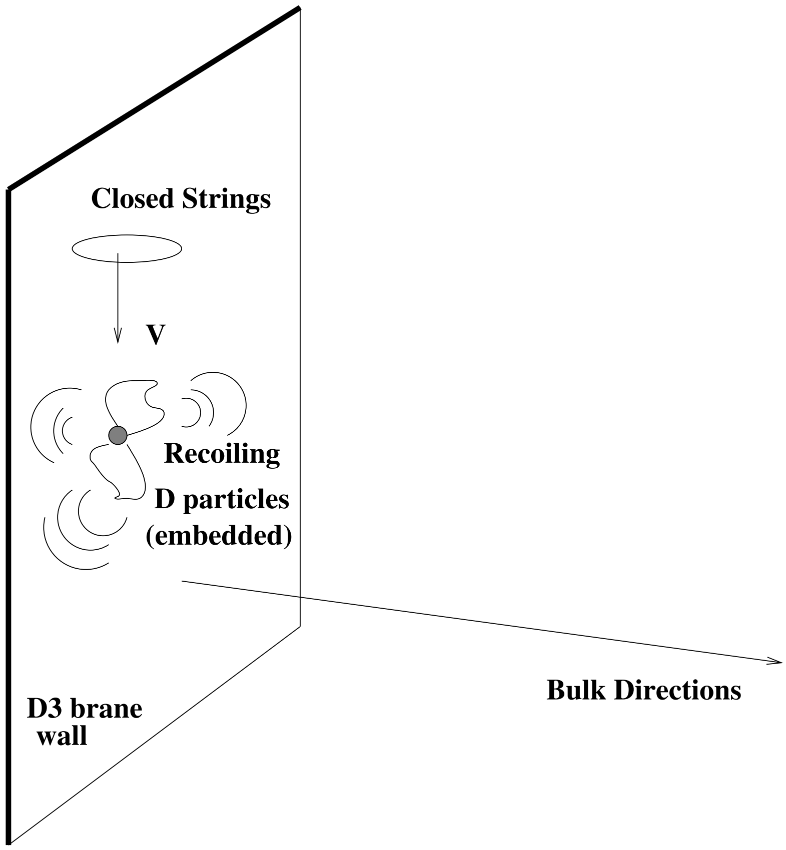
- Liouville strings (J. Ellis, N. Mavromatos, D. Nanopoulos 1997, 1998, 1999)
- Effective field theory approach (R.C. Mayers, M. Pospelov 2003)
- Space-time foam (L.J. Garay 1998)
- Loop quantum gravity (R. Gambini, J.Pullin 1999)
- String theory (A. Matusis, L Suskind, N. Tombas 2000)
- Noncomutative geometry (G. Amelino-Camelia 2001)

In the approximation $E \ll M$, the distortion of the standard dispersion relations may be represented as an expansion in E/M:

$$\boxed{E^2 = m^2 + p^2(1 + \xi_1(p/M) + \xi_2(p/M)^2 + \ldots)}$$

$$\boxed{\xi_1 < 0; \quad v = c(1 - \tfrac{E}{M}); \quad n(E) = 1 + \tfrac{E}{M}}$$

D-brain recoil



$$G_{ij} = \delta_{ij}, \quad G_{00} = -1, \quad G_{0i} \sim \bar{U}_i$$

Metric perturbations $h_{\mu\nu}$ about the flat spacetime

$$h_{0i} \simeq \bar{U}_i \quad \bar{U}/c = O(E/M_D c^2)$$

Dispersion analysis

The background metric

$$G_{00} \equiv -h, \qquad \mathcal{G}_i = -\frac{G_{0i}}{G_{00}}, \quad i = 1, 2, 3$$

Maxwell's equations in a medium with $1/\sqrt{h}$ playing the rôle of the electric and magnetic permeability

$$\nabla \cdot B = 0, \qquad \nabla \times H = \frac{1}{c}\frac{\partial}{\partial t} D = 0$$

$$\nabla \cdot D = 0, \qquad \nabla \times E = -\frac{1}{c}\frac{\partial}{\partial t} B = 0$$

$$D = \frac{E}{\sqrt{h}} + H \times \mathcal{G}; \quad B = \frac{H}{\sqrt{h}} + \mathcal{G} \times E$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial^2 t} B - \nabla^2 B - 2\left(\overline{U}.\nabla\right)\frac{1}{c}\frac{\partial}{\partial t} B = 0$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial^2 t} E - \nabla^2 E - 2\left(\overline{U}.\nabla\right)\frac{1}{c}\frac{\partial}{\partial t} E = 0$$

The one dimensional wave solution

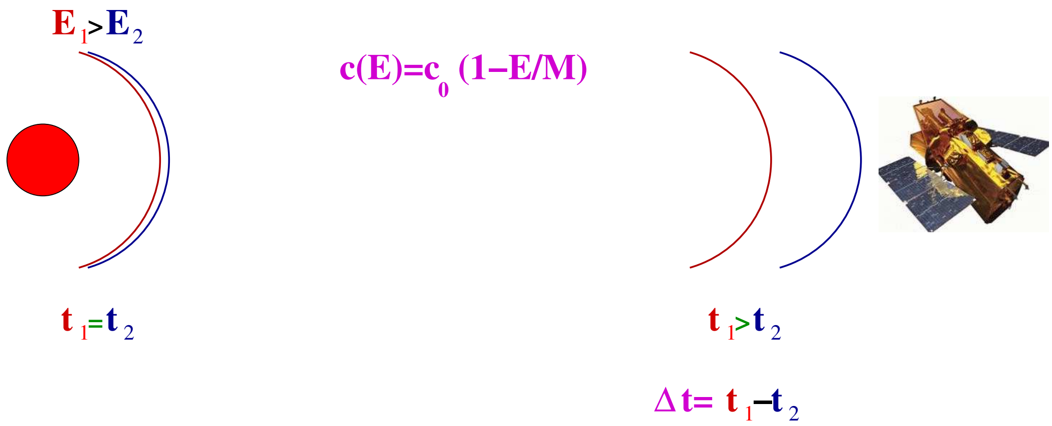
$$E_y(x,t) = E_0 e^{ikx-\omega t}; \quad B_z(x,t) = B_0 e^{ikx-\omega t}$$

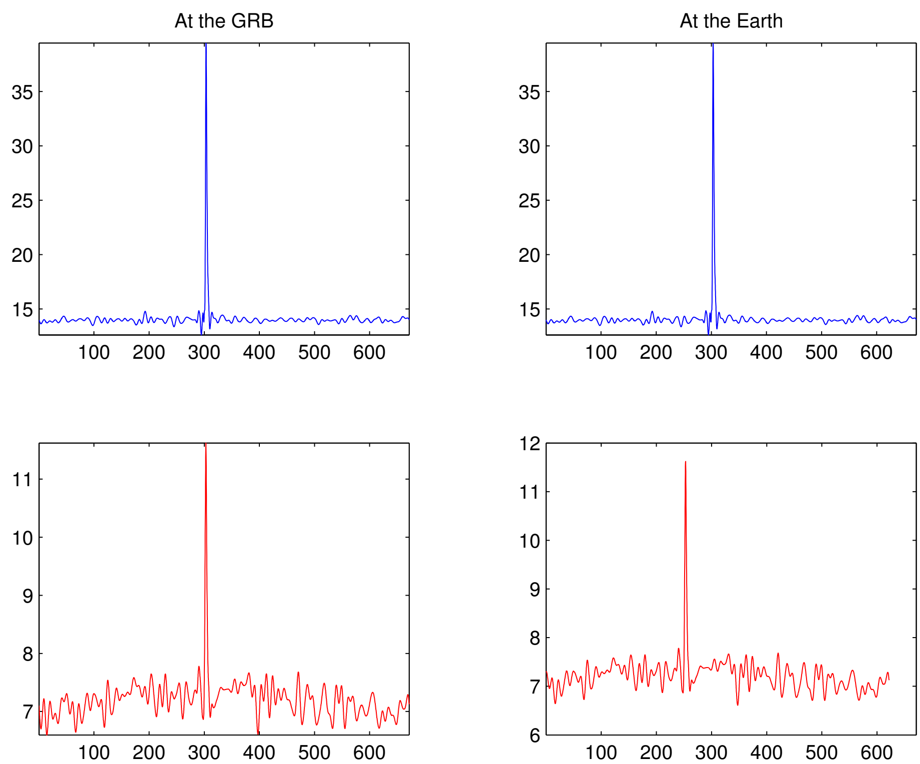
with dispersion relation

$$k^2 - \omega^2 - 2\overline{U}k\omega = 0$$

Time-of-flight studies

The modification of the group velocity would affect the simultaneity of the arrival times of photons with different energies from distant sources (G. Amelino-Camelia, J. Ellis, N. Mavromatos, D. Nanopoulos, S. Sarkar 1998).





Light propagation

Light propagation from remote objects is affected by the expansion of the Universe and depends upon the cosmological model: $\Omega_{total} = \Omega_\Lambda + \Omega_M = 1$; $\Omega_\Lambda \simeq 0.7$.

$$dt = -H_0^{-1} \frac{dz}{(1+z)h(z)}; \quad h(z) = \sqrt{\Omega_\Lambda + \Omega_M (1+z)^3}$$

$$udt = -H_0^{-1} \frac{udz}{(1+z)h(z)}$$

$$\Delta L = H_0^{-1} \int_0^z \frac{\Delta u dz}{(1+z)h(z)}$$

We consider two photons traveling with velocities very close to c**, whose present day energies are** E_1 **and** E_2**. At earlier epochs, their energies would have been blueshifted by a factor** $1+z$**.**

$$\Delta u = \frac{\Delta E (1+z)}{M}$$

$$\Delta t = H_0^{-1} \frac{\Delta E}{M} \int_0^z \frac{dz}{h(z)}$$

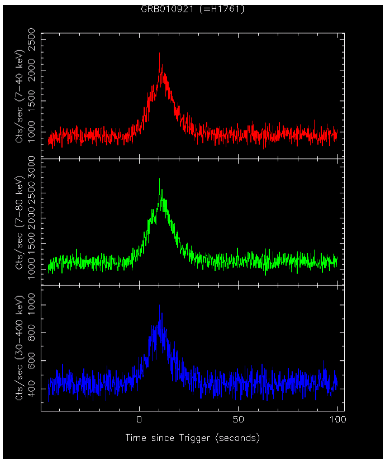
GRB's

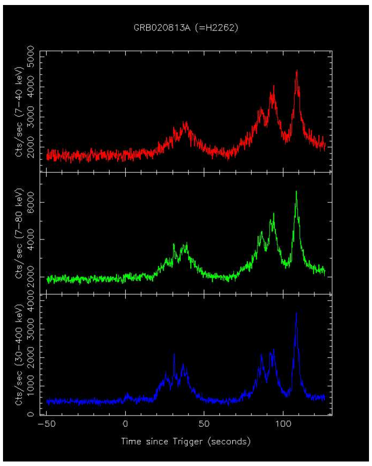
- High energies γ-rays
- Cosmological distances
- Short duration transients
- Known spread in distances

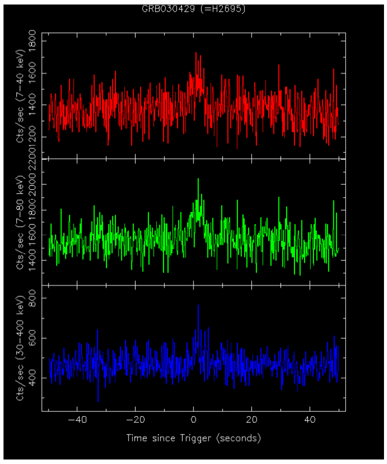
A key issue in such probes is to distinguish the effects of the quantum-gravity medium from any intrinsic delay in the emission of photons of different energies by the source.

GRBs are short, non-thermal bursts of γ-rays.

The distance is cosmological $z \simeq 0.1 - 5$.







We correlate genuine variation points of the light curves recorded by BATSE, HETE and SWIFT in the highest energy band relative to those in the lowest energy band.

32 GRBs

	GRB	z	Distance (Mpc)
BATSE (64ms)	970508	0.835	2884.1
	971214	3.418	6733.33
	980329	3.9	7104.7
	980703	0.966	3222.19
	990123	1.600	4546.52
	990308	1.2	3765.71
	990510	1.619	4579.68
	991216	1.020	3354.18
	990506	1.3060	3989.28
HETE (164ms)	010921	0.45	1723.36
	020124	3.198	6543.49
	020903	0.25	1008.43
	020813	1.25	3872.8
	020819	0.41	1586.86
	021004	2.33	5626.37
	021211	1.01	3330.05
	030226	1.99	5168.68
	030323	3.372	6694.79
	030328	1.52	4403.31
	030329	0.168	691.58
	030429	2.66	6011.65
	040924	0.859	2948.01
	041006	0.716	2553.37
	050408	1.2357	3841.2
SWIFT (64ms)	050319	3.24	6580.92
	050401	2.9	6261.98
	050416	0.653	2368.52
	050505	4.3	7374.34
	050525	0.606	2226.01
	050603	2.821	6182.08
	050724	0.258	1038.6
	050730	3.967	7152.09

Wavelet transform

Standard Fourier analysis loses the information about the time localisation of a given frequency component!

Wavelet transforms (WT) are used to represent signals which require for their specification not only a set of typical frequencies (scales) s, but also knowledge of the coordinate u neighbourhoods where these properties are important.

$$\boxed{Wf(u,s) = \int_a^b f(t)\psi^*_{u,s}(t)dt}$$

Wavelets are a special class of oscillating functions with finite support

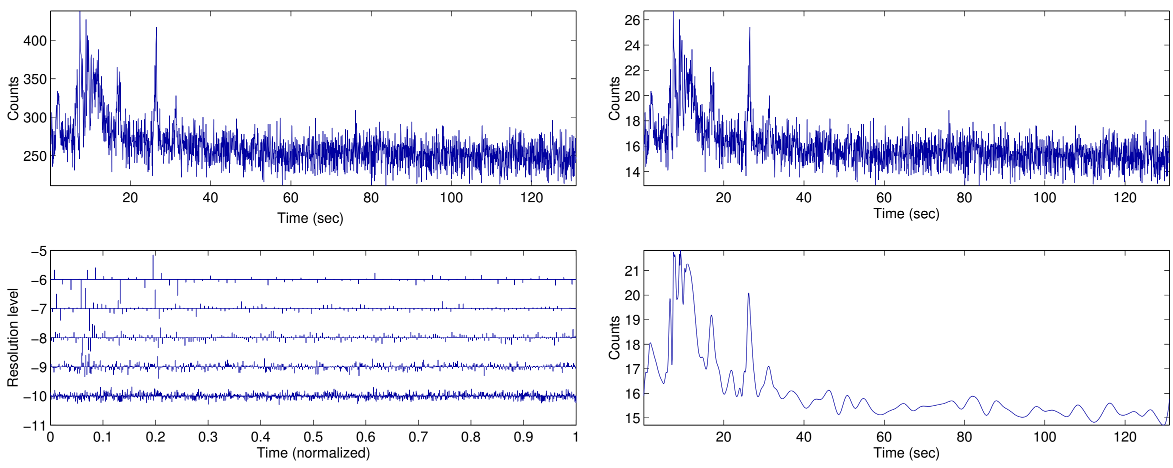
$$\boxed{\psi_{u,s} = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)}$$

The action of s on the mother wavelet ψ is a dilation ($s > 1$) or contraction ($s < 1$): the shape of the function is unchanged, it is simply spread out or squeezed.

- **Discrete wavelet transform (DWT)**
- **Continuous wavelet transform (CWT)**

Nonparametric estimation of GRBs

- **We apply the DWT with an appropriate basis to a GRB's light curve breaking the given intensity profile down into a coarse approximation at a given time scale, that can be extended to successive levels of residual detail on finer and finer scales.**
- **We apply the thresholding procedure, which deletes wavelet coefficients below the threshold value, and diminishes the others by the threshold value.**
- **We reconstruct the intensity profile by the inverse DWT using the thresholded wavelet coefficients.**



The intensity profile of GRB990308 estimated by the wavelet shrinkage procedure at the level $L = 6$.

Detection of singularities with CWT

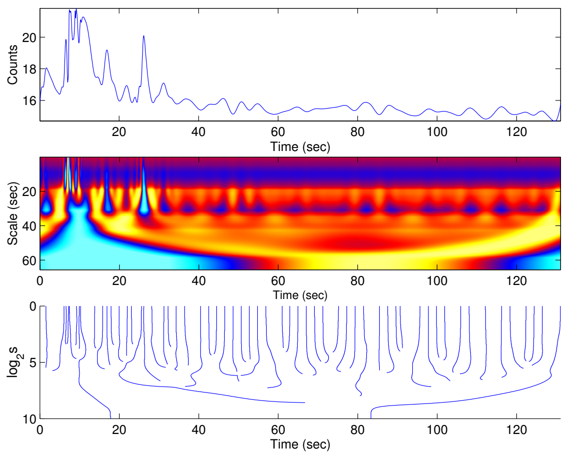
$$\boxed{|f(t) - p_\nu(t)| \leq \mathcal{K}|t - \nu|^\alpha}$$

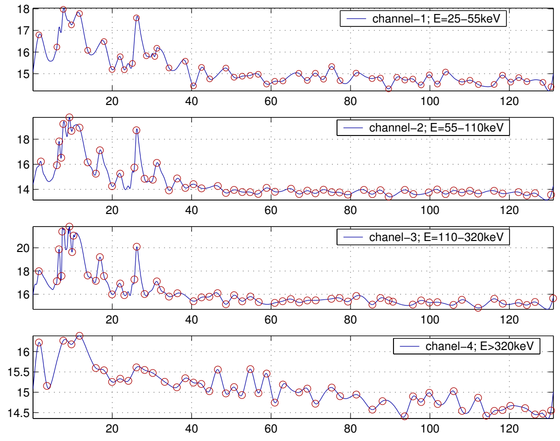
If $0 \leq \alpha < 1$ then $p_\nu(t) = f(\nu)$ and the Lipschitz condition becomes

$$\boxed{|f(t) - f(\nu)| \leq \mathcal{K}|t - \nu|^\alpha}$$

The function f is not differentiable at point ν and α characterizes the singularity type.

The CWT amplitude decays with the scale
$|Wf(u,s)| \propto s^{\alpha+1/2}$.



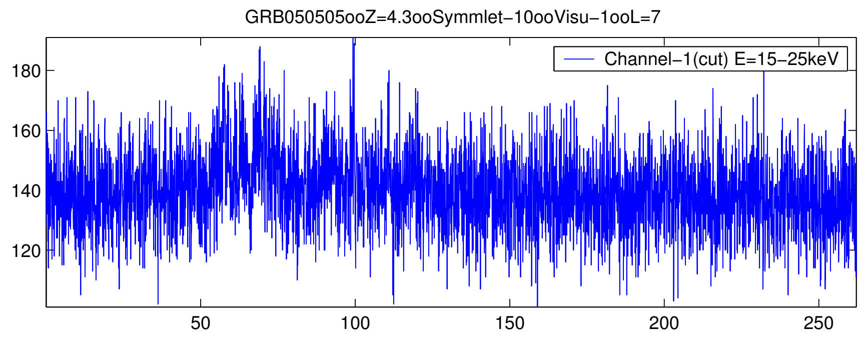


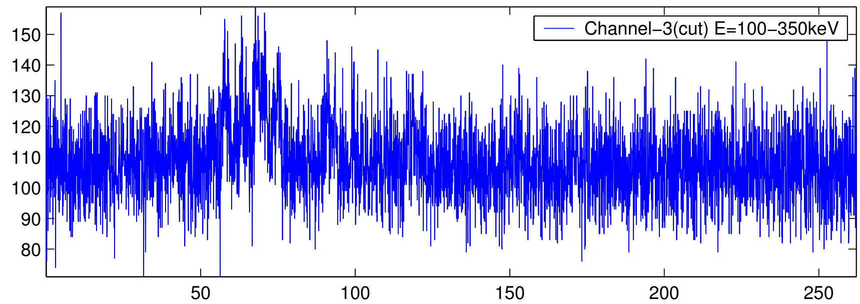
$$\boxed{\log_2|Wf(u,s)| \approx \left(\alpha + \frac{1}{2}\right)\log_2 s + const}$$

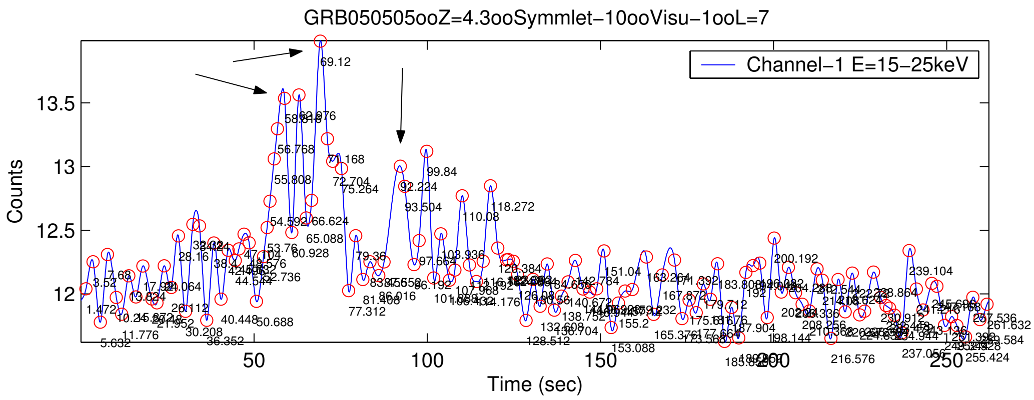
Genuine variation points

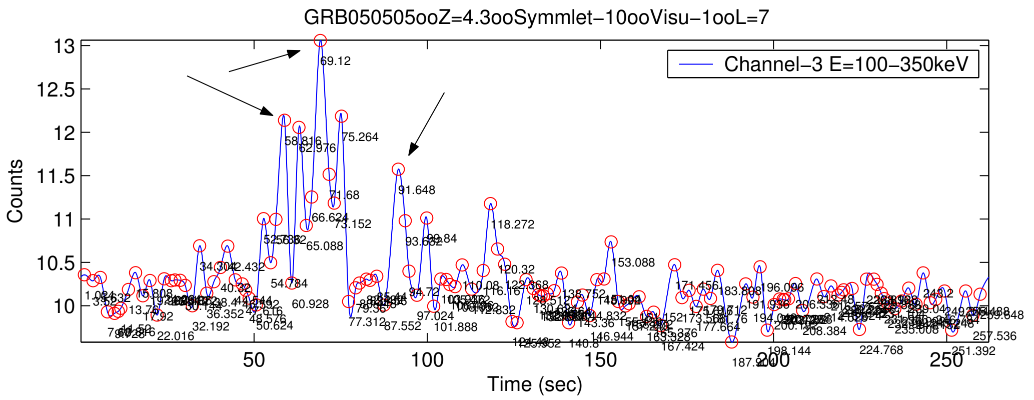
$$\boxed{\alpha + 1/2 \leq 1.5}$$

Lipschitz conterparts









about 1000 simulations with Poissonian noise

Fitting equation

Lorentz violating contribution can be accompanied by an appriory unknown intrinsic time lag

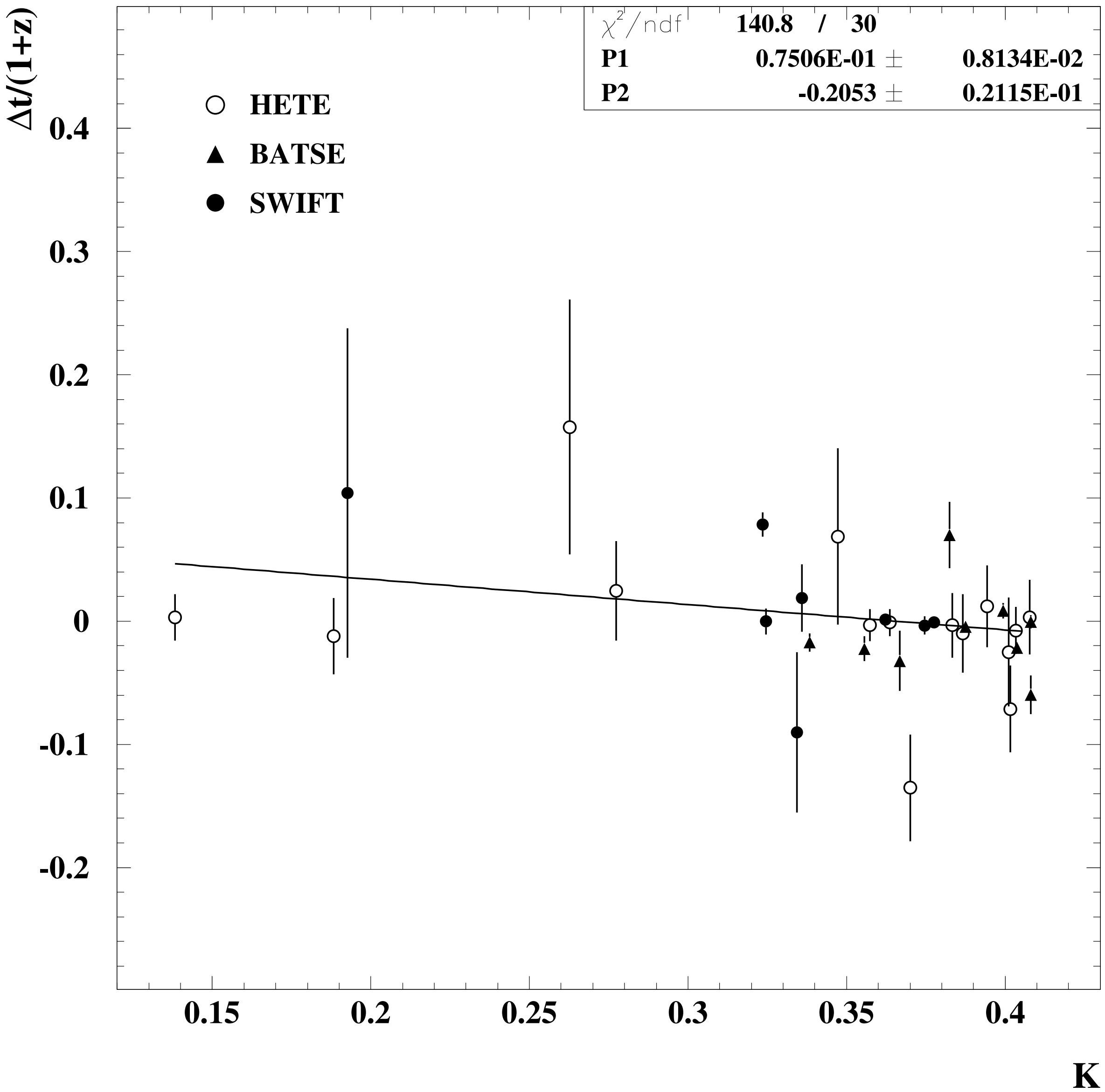
$$\boxed{\Delta t_{\rm obs} = \Delta t_{LV} + b_{\rm sf}(1+z)}$$

Linear fit

$$\boxed{\frac{\Delta t_{\rm obs}}{1+z} = a_{\rm LV} K + b_{\rm rf}}$$

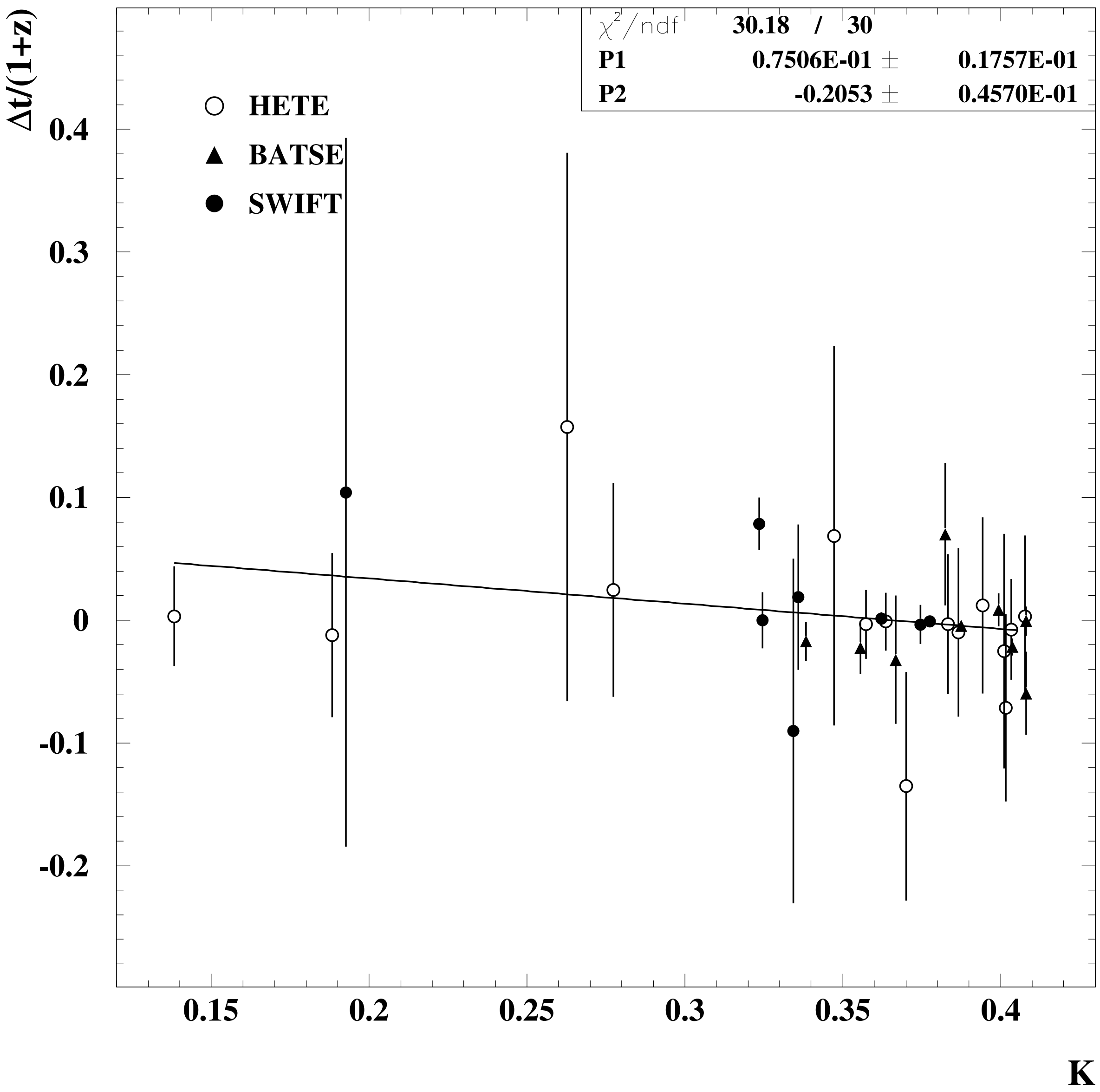
$$\boxed{a_{\rm LV} = H_0^{-1}\frac{\Delta E}{M} \qquad K = \frac{1}{1+z}\int_0^z \frac{dz}{h(z)}}$$

Linear fit



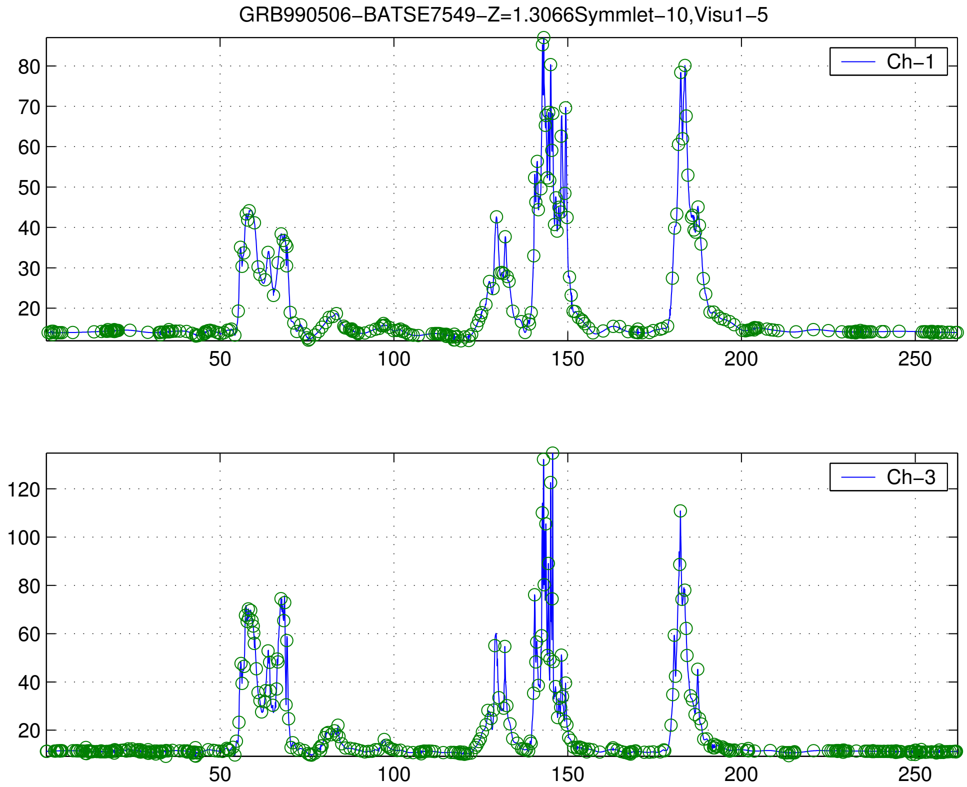
$$\frac{\Delta t}{1+z} = -0.2053(\pm 0.0211)K + 0.0750(\pm 0.0081)$$

Linear fit $[\chi^2/\mathrm{ndf}]^{1/2}$



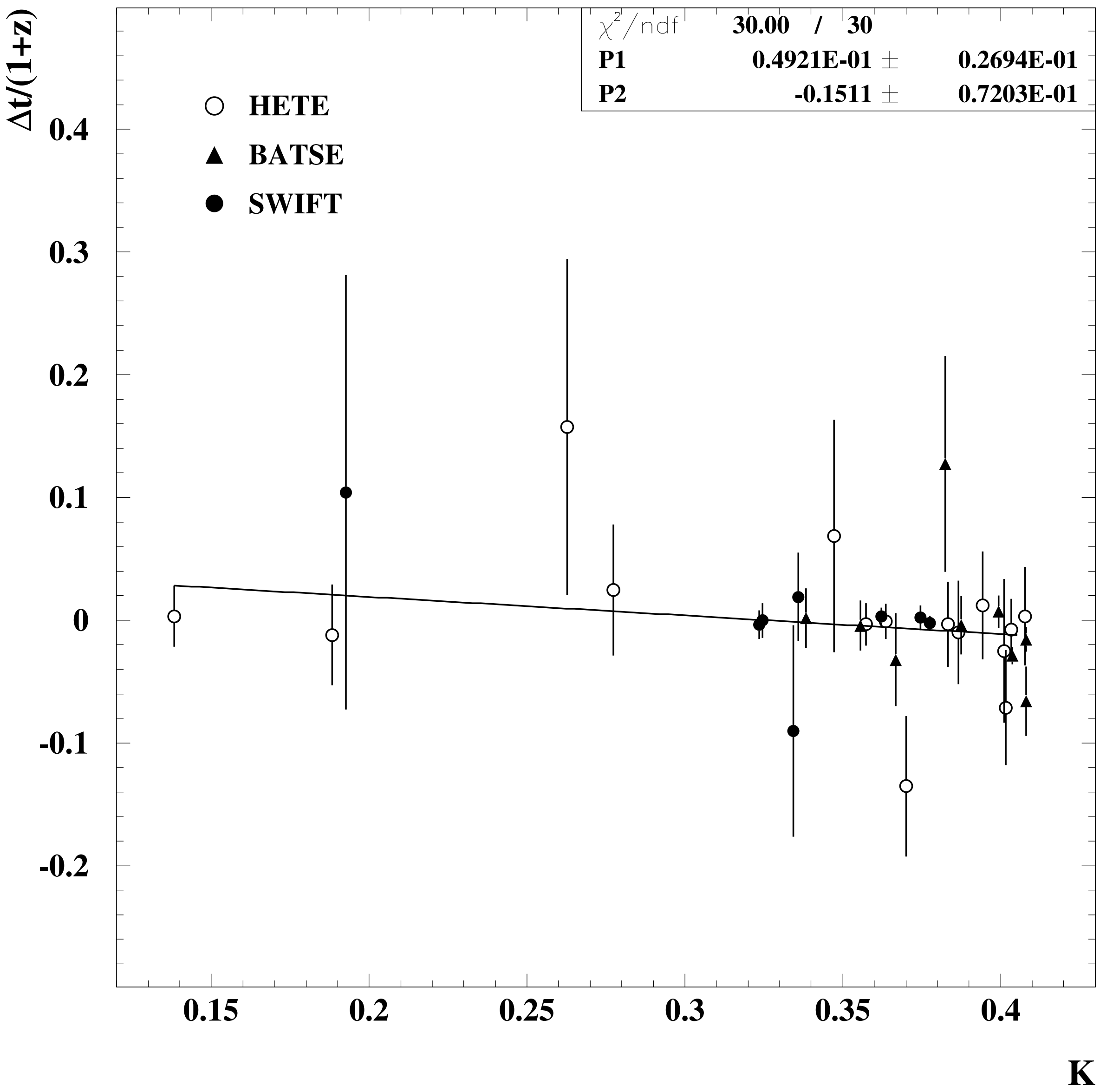
$$\frac{\Delta t}{1+z} = -0.2053(\pm 0.0457)K + 0.0750(\pm 0.0176)$$

Di-weighting



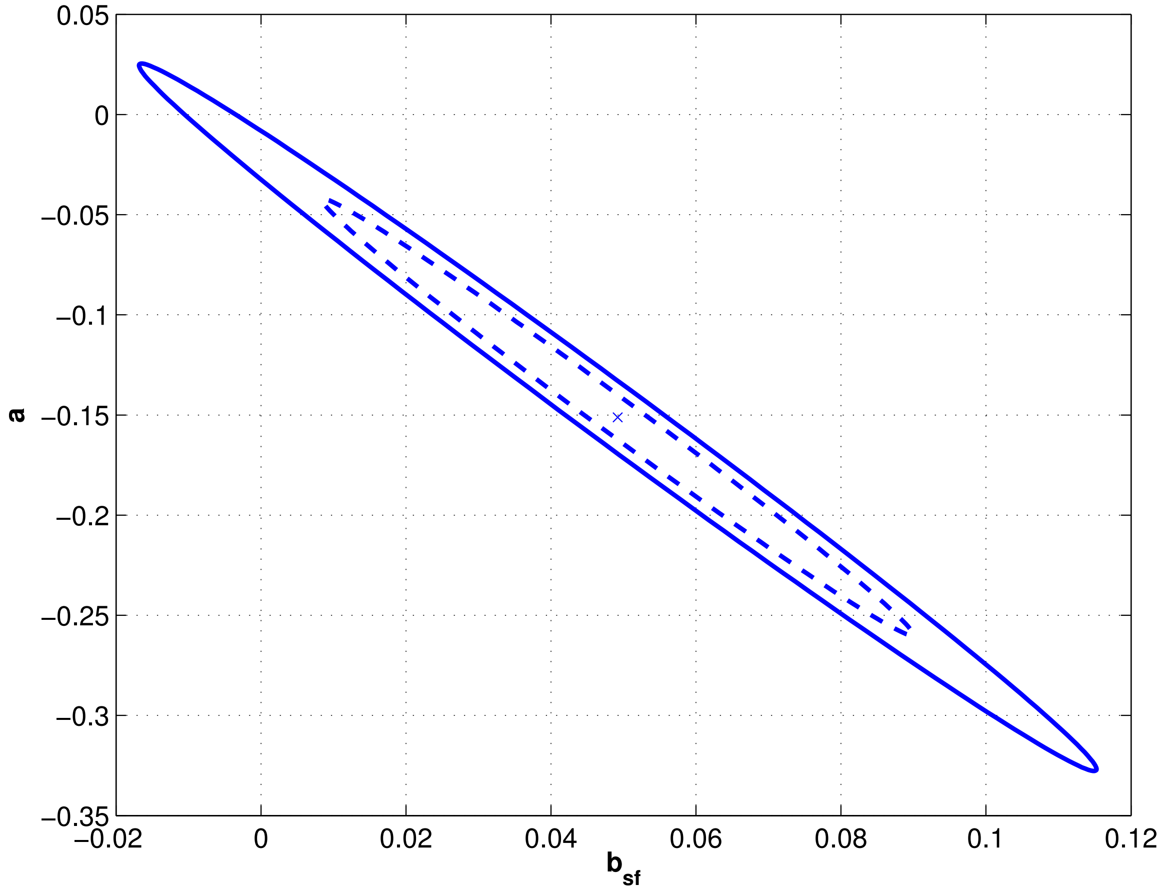
-Selected only those variation points at which the amplitudes saturate 70% of maximal count rate for a given light curve

Di-weighting fit $[\chi^2/\mathrm{ndf}]^{1/2}$



$$\frac{\Delta t}{1+z} = -0.1511(\pm 0.0720)K + 0.0492(\pm 0.0269)$$

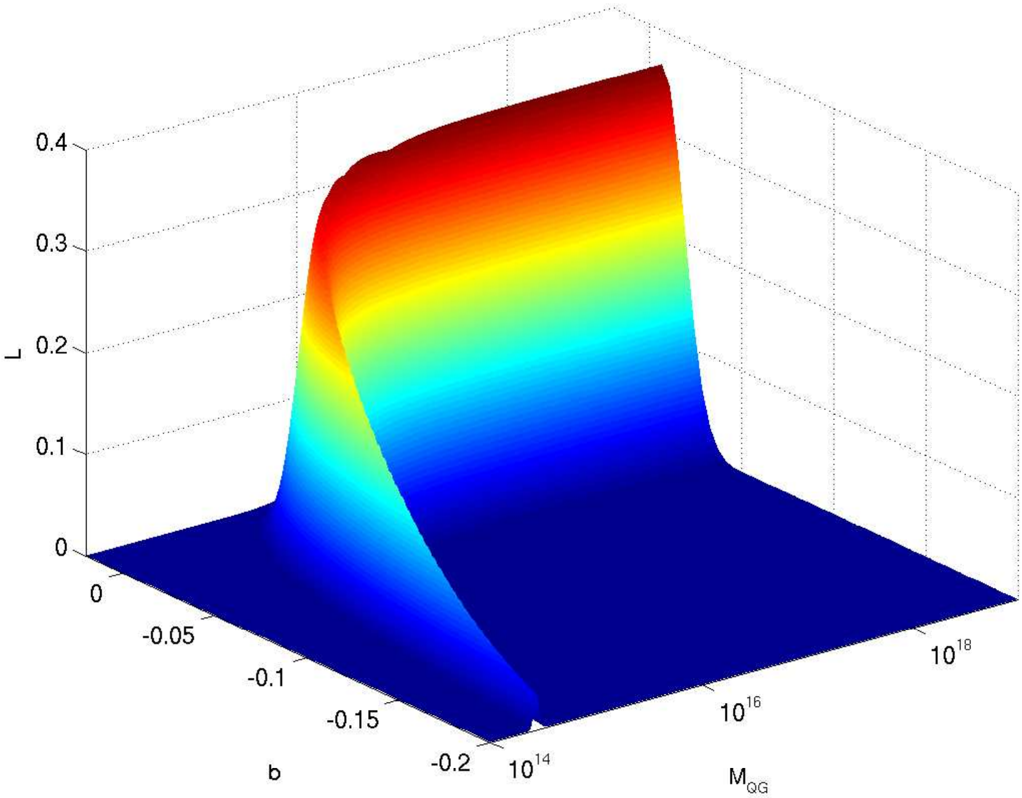
Slope-intercept



Compilation of the Lorentz violation limit

$$L \propto \exp(-\chi^2(M)/2)$$

$$\chi^2(M, b_{sf}) = \sum_{\mathcal{D}} \left[\frac{\frac{\Delta t_i}{1+z_i} - a(M)K_i - b_{\rm sf}}{\frac{\sigma_i}{1+z_i}} \right]^2$$



$$\frac{\int_{\mathcal{M}}^{\infty} L_{\rm marg}(\xi)d\xi}{\int_{0}^{\infty} L_{\rm marg}(\xi)d\xi} = 0.95; \quad \mathcal{M} = 10^{19}\text{GeV}$$

$$M \geq 4.1 \cdot 10^{16}\ \text{GeV}$$

Conclusions

Using the arrival times of variation features extracted from 32 GRBs measured by BATSE, HETE and SWIFT instruments we find a systematic tendency at 2σ level for more energetic photons to arrive earlier than less energetic ones.

We establish a statistically robust lower limit $M > 4.1 \times 10^{16}$ GeV on the scale of violation of Lorentz invariance.

The tendency becomes more significant (5σ level) if the appriory unknown spectral evolution of GRBs is not taken into account.