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Cosmological Gravitomagnetism and Mach's Principle

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- FRW, $k = 0, \pm 1$ plus cosmolog. vector perturb. ($\equiv$ vorticity perturbat.: $\text{div}\,\vec{v} = 0$, $\text{curl}\,\vec{v} \neq 0$), asymptotic FRW

- question:
  will rotating matter in the universe exactly drag the spin axes of gyroscopes here relative to the directions (geodesics) to galaxies in asymptotic FRW?

  - forces: Gravitomagnetism

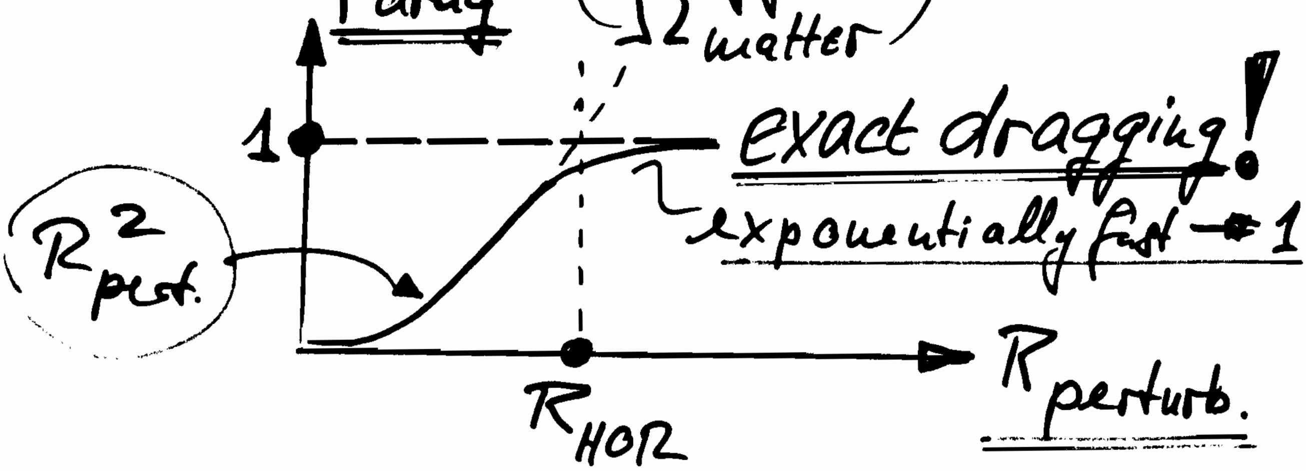
gravitomagnetism on FRW background:
$k=0$

Our results:

- 1) the influence of rotating matter beyond $R_{\dot{H}}$ (particle horizon) is $\boxed{\text{exponentially suppressed.}}$

- 2) for the special case of a homogeneous rotation of cosmic matter ($\vec{\Omega}_{\text{matter}}$) inside $R_{\text{perturb.}}$:

$$f_{\text{drag}} \equiv \left(\frac{\Omega_{\text{gyro}}}{\Omega_{\text{matter}}} \right)$$



$$R_{\dot{H}} \equiv \frac{1}{\sqrt{-\dot{H}}}$$

3) for the most general vector perturbation

spin axes of gyroscopes here
axis directions of local inertial frames there

exactly follow

weighted average rotating matter out there!

$$\boxed{\begin{aligned}
&\vec{\Omega}_{gyro} \equiv -\tfrac{1}{2}\vec{B}_g(\vec{r}_{gyro}) \\
&\vec{B}_g(P) = -4G_N \int d^3r_Q \, [\vec{n}_{PQ} \times \vec{J}_\varepsilon(Q)] \, Y_\mu(r_{PQ}) \\
&\text{Yukawa force} = Y_\mu(r) = \frac{-d}{dr}\frac{e^{-\mu r}}{r}
\end{aligned}}$$

$\mu^2 \equiv -4\dot{H}$, $\frac{2}{\mu} \equiv$ $\dot{H}$-dot radius

$\vec{J}_\varepsilon$ = energy current $= (\rho + p)\vec{v}$

Gravito-magneto-dynamics: like Ampère's law except exponential cutoff

$(\vec{n} \times \vec{J}_\varepsilon) = \frac{1}{r}$(density of kinetic ang. momentum $\vec{L}$

everything measured relative to geodesics from gyroscope at P to asy. quas

Mach's Hypothesis: Valid for any eq. of state, any pert. FRW

Mach's Principle (Mach 1)

Ernst Mach (1880's, Prag)

Mach's formulation ↕ many formulation

axes of (local) non-rotating frames
i.e. spin axes of gyroscopes

↓ | what physical cause governs time-evolution?

<u>precisely follow</u> (are <u>dragged exactly</u>)
"<u>some average</u>" of the motion of matter in the Universe

NB: Other authors later proposed <u>many alternative formulations</u>, <u>inequivalent</u>, of "Mach's Principle": <u>wrong</u> (within cosmological Gen. Relat.)

NB: <u>Mach had no mechanism</u>
<u>GR</u>: <u>Gravitomagnetism</u>

NB: Mach: <u>"What average"?</u> // our answer: <u>below</u>

NB: <u>exact dragging</u> required by Mach's Principle
<u>not a tiny dragging effect</u> as expected for GP B.

NB: Mach's Principle <u>cannot apply</u> to <u>GR for solar system</u>

cosmological perturb. theory for vector sector

Jim Bardeen (1980)

all divergences zero

- theorems:

① slicing of space-time: unique Σ_t
lapse un-perturbed: $\delta g_{00} = 0$

② intrinsic geometry of each Σ_t
unperturbed: Euclidean 3-space E^3

- our choice of coordinates in E^3:
Cartesian

$$\boxed{ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx^i dx^j + 2\beta_i\, dx^i dt}$$

- asy. FRW ($\beta_i \to 0$ asy), gauge completely fixed
- $\beta^i(\vec{x},t) = -\Psi_{\text{Bardeen}}(t)\, Q^{(1)i}(\vec{x})$

- "star-fixed coord" (in BH physics) $\Rightarrow$
our coord. fixed to quasars in asy. FRW

- coordinate artefacts? gauge artefacts?
No: everything measured relative to geodesics on Σ_t from gyro to asy quasars

Gravitomagnetism:

- É. Cartan → work with LONB's ↔ FIDO's ← Kip Thorne

local ortho-normal Bases — fiducial observers

$$\bar{e}_{\hat{0}}\Big|_P = \bar{u}(\text{FIDO})\Big|_P$$

$$\bar{e}_{\hat{i}}\Big|_P = \bar{e}_{\hat{i}}(\text{FIDO})\Big|_P$$

hat on index → LONB component

- our choice of FIDO's

1) 3-velocity $(d\vec{x}/dt)$ of FIDO's, relative to $\bar{n}_\Sigma \Rightarrow$ shift vector $\vec{\beta}$

→ i.e. $\boxed{(dx^i/dt)_{FIDO} = 0}$ (FIDO at rest in our coord) (choice of congruence)

2) $\boxed{\bar{e}_{\hat{i}}(\text{FIDO})}$ → fixed to geodesics to galaxies in asympt. FRW. geodesics on Σ_t

Gravitomagnetism I:

- $\boxed{\text{general operational def. of } (\vec{E}_g, \vec{B}_g)}$

independent of perturbation theory

fund. measurement by FIDO's: compulsory to use LONTI's

$$\begin{cases} \dfrac{d}{dt} p_{\hat{i}} \equiv m E^{(g)}_{\hat{i}} \\ \dfrac{d}{dt} S_{\hat{i}} = \big(\underbrace{\vec{\Omega}_{gyro}}_{\text{precess}} \wedge \vec{S}\big)_{\hat{i}} \end{cases}$$

for free-falling, quasistatic test particle.

$$\Omega^{gyro}_{\hat{i}} \equiv -\frac{1}{2} B^{g}_{\hat{i}}$$

$\boxed{\text{for gyro at rest relative to FIDO}}$

(*) [NB: for free-falling obs.: $\vec{E}_g = 0$
for non-spinning nonrotat. obs. (relat. to gyro): $\vec{B}_g = 0$
it all depends on choice of FIDO's.]

- Comparing geodesic eq. + Fermi transport eq. with op. def. $(\vec{E}_g, \vec{B}_g)$ above $\Longrightarrow$ equiv. definitions

$$\big(\underbrace{\omega_{\hat{i}\hat{0}}}_{\text{L-boost}}\big)_{\hat{0}} \equiv -E^{(g)}_{\hat{i}} \qquad \big(\underbrace{\omega_{\hat{i}\hat{j}}}_{\text{rotation}}\big)_{\hat{0}} \equiv -\frac{1}{2} \varepsilon_{\hat{i}\hat{j}\hat{k}} B^{(g)}_{\hat{k}}$$

- Compute remaining $(\omega_{\hat{a}\hat{b}})_{\hat{c}}$ via 1st Cartan eq. for linear vorticity perturb. on Minkowski:

$$(\omega_{\hat{i}\hat{0}})_{\hat{j}} = -\frac{1}{2} \varepsilon_{\hat{i}\hat{j}\hat{k}} B^{(g)}_{\hat{k}} \qquad (\omega_{\hat{i}\hat{j}})_{\hat{k}} = 0$$

Gravitomagnetism II

- **Equations of motion for free-falling particles** in Minkowski space plus vector perturbations:

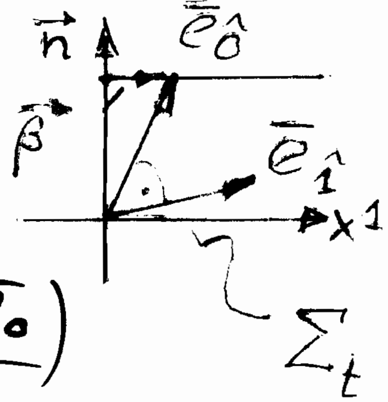
geodesic eq.:

$$\frac{d}{dt}(p_{\hat{i}}) = \mathcal{E}\left[\vec{E}_g + (\vec{v} \wedge \vec{B}_g)\right]_{\hat{i}}$$

- no term bilinear in $\vec{v}$ for arbitrary $v \leq c$ (also for photons)!
- $\mathcal{E}$ = energy of particle measured by FIDO
- compare to Lorentz law for el. magn.

- when computing $(\omega_{\hat{a}\hat{b}})_{\hat{c}}$ from LONB

$$\begin{cases} \vec{e}_{\hat{0}} = \vec{e}_0 \\ \vec{e}_{\hat{i}} = \left(\dfrac{\vec{e}_i + \beta_i \vec{e}_0}{a}\right) \end{cases}$$



it turns out / follows: $\vec{\beta} = \vec{A}_g$

$$\vec{B}_g = \mathrm{curl}\,\vec{A}_g, \quad \vec{E}_g = -\partial_t \vec{A}_g$$ (vector part of Mink.)

$$\mathrm{curl}\,\vec{E}_g + \partial_t \vec{B}_g = 0$$

Einstein's $G_{\hat{0}\hat{i}}$ equation

for FIDO's axes fixed to asymptotic galaxies

① on Minkowski background:

$$\left(\vec{\nabla} \times \vec{B}_g\right) = -(16\pi G_N)\,\vec{J}_\varepsilon$$

- $\vec{J}_\varepsilon$ = measure energy current = moment. density
  div $\vec{J} \equiv 0$ in vorticity sector
- analogous to Ampère's law,
  but Maxwell term ($\partial_t E$) absent.
- $G_{\hat{0}\hat{i}}$ → eq. at fixed time → momentum constraint

② on FRW background, $k = (0, \pm 1)$

additional term: $-4\dot{H}\vec{\beta} \equiv \mu^2 \vec{\beta}$ $(\mu^2 > 0)$

$$\left(-\Delta + \mu^2\right)\vec{A}_g = -(16\pi G_N)\,\vec{J}_\varepsilon$$

solution for $\vec{A}_g$: ~ Yukawa potential $\left(\frac{e^{-\mu x}}{r}\right)$

$\Downarrow$

exponential cutoff

(details next page)

Final result for
Gravitomagnetism on FRW background, $K=0,-1$
open universe

$$\vec{\Omega}_{gyro} = -\frac{1}{2}\vec{B}_g(\vec{r}_{gyro})$$

(open univ.) $$\vec{B}_g(P) = -4G_N \int d(vol)\left[\vec{n}_{PQ} \times \vec{J}_\varepsilon(Q)\right] Y_\mu(\chi_{PQ})$$

valid for every choice of FIDO!

- χ = geodesic distance from gyroscope (P) to source (Q)
- $R(\chi) = \{\chi,\ \sin\chi,\ \sinh\chi\}$ for $k = (0, +1, -1)$

– Yukawa potential $= \frac{1}{R}e^{-\mu\chi}$

– Yukawa force $\equiv Y_\mu = \left(\frac{-d}{d\chi}\right)$(Yukawa potent.)

$K=+1$ FRW background:

method of mirror images (sources) relative to antipodal points.

- FRW, $K=0,-1$ background:

go arbitrarily close to Minkowski space or de Sitter space
$\Rightarrow$ exact dragging ✓ Mach's principle ✓ END