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Stability of Dilaton in String Gas Cosmology

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Introduction

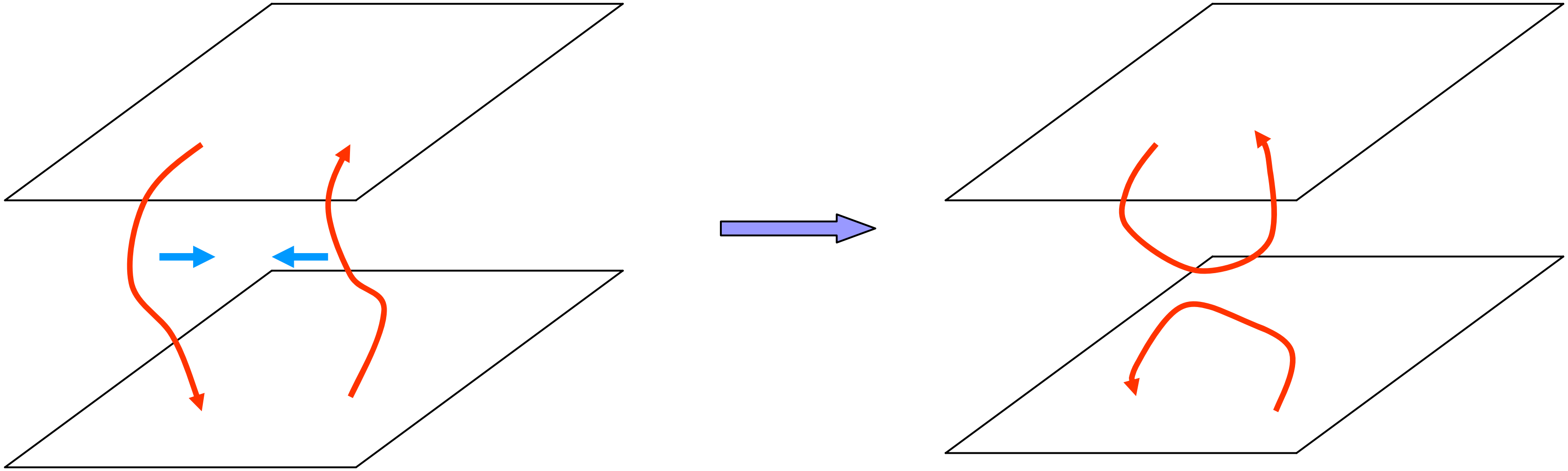
- The superstring theory predicts 10-d.
- The most important issue is the origin of 4-d and the stability of moduli.
- The flux compactification is well known. However, it is difficult to explain 4-d and stabilize the volume moduli in this approach.
- The string gas cosmology may explain 4-d and can stabilize the volume moduli.

String Gas Cosmology

Brandenberger & Vafa (1989)

- **The 10-d universe is toroidally compactified.**
- **The universe is filled with a closed string gas.**
- **4-d spacetime becomes large due to annihilation of windings.**
- **Other 6-d are expected to remain small and stable.**
- **The dilaton ϕ should be stable ($g_s = e^{\phi}$ string coupling).**

Pair annihilation of windings can occur only in less than 3 spatial dimensions.

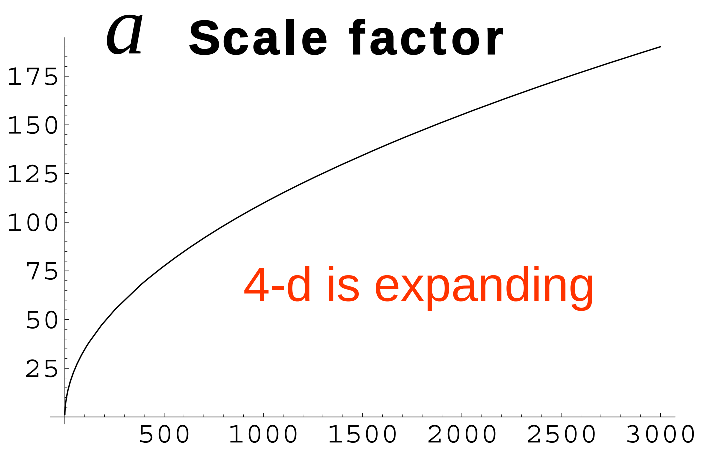


Stability of Dilaton: numerical results

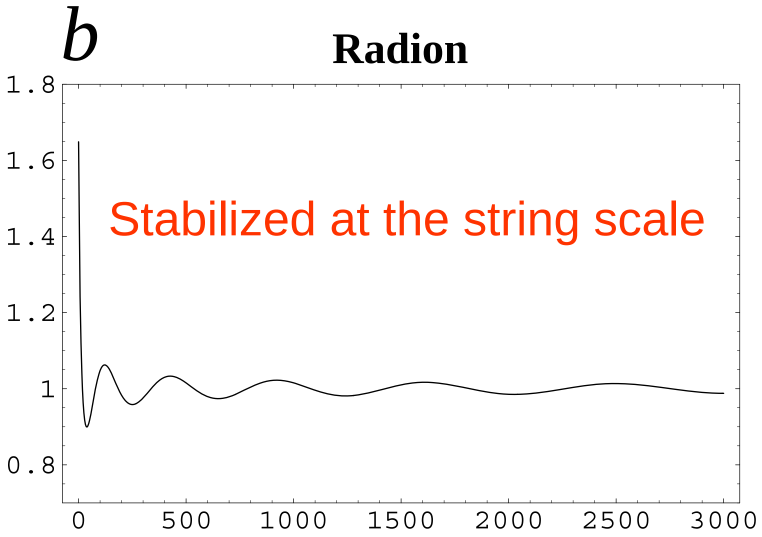
To test the picture in the simplest case, we performed numerical calculation.

$$ds^2 = \underbrace{-dt^2 + a^2(t)\delta_{ij}dx^i dx^j}_{\text{4-d universe}}$$

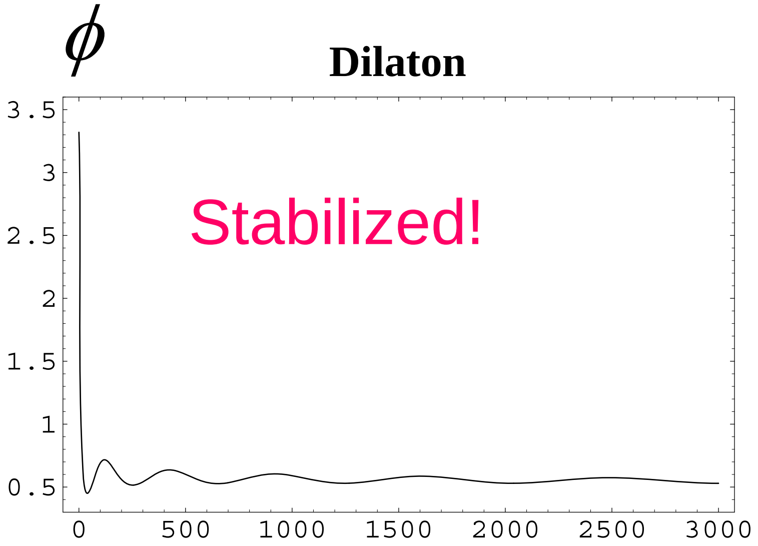
$$\underbrace{+b^2(t)\delta_{ab}dy^a dy^b}_{\text{6-d internal space}}$$



4-d is expanding



Stabilized at the string scale



Stabilized!

The goal of this talk

- **To understand why the dilaton is stabilized in our numerical results.**

  Cf. Watson & Brandenberger 2003

- **To construct a 4-d effective action in a T- dual invariant manner.**

  Cf. Battefeld & Watson 2004

- **To show the stability of all moduli in a simple compactification model.**

  Cf. Brandenberger, Cheung, and Watson 2005

Plan of this talk

- Review of the string spectrum
- T-dual invariant 4-d effective action
- Stable compactification
- Conclusion

Review of the string spectrum

String spectrum

String mass spectrum in compactified spacetime

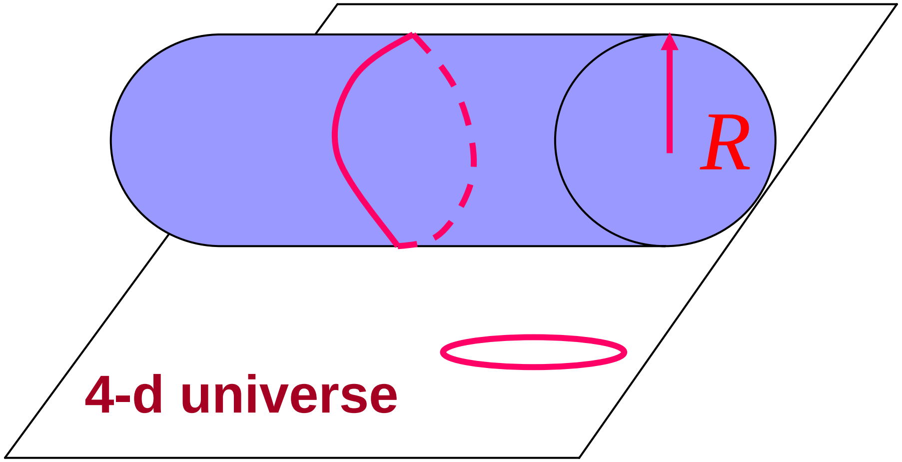
momentum **winding** **oscillator**

$$M^2 = \frac{n^2}{R^2} + m^2 R^2 + 2\left(N + \bar{N} - 2\right)$$

$$N - \bar{N} = n m$$

$$N = \sum_{n=1}^{\infty} n \, a_n^{\mu\dagger} a_{\mu n}$$

$$\bar{N} = \sum_{n=1}^{\infty} n \, \bar{a}_n^{\mu\dagger} \bar{a}_{\mu n}$$



◆ massless modes

$$n = m = 0 \, , \, N = \bar{N} = 1$$

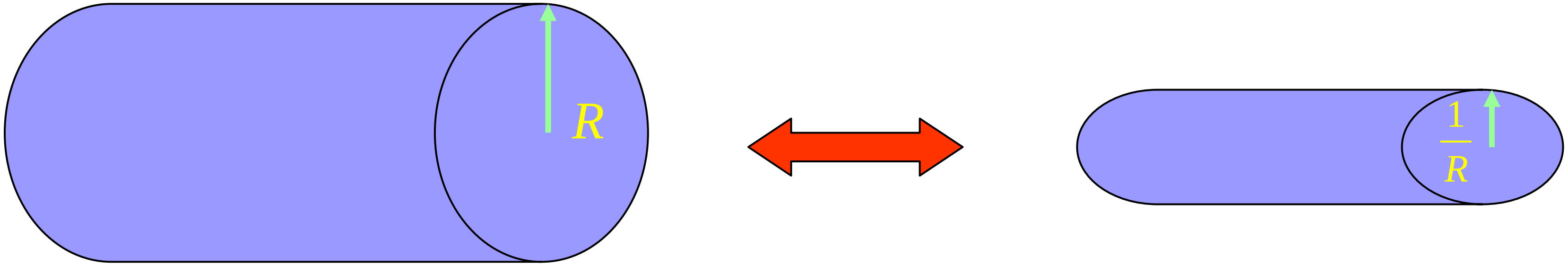
$$a_1^{\mu\dagger} \, \bar{a}_1^{\nu\dagger} \, | 0; k^\mu > \quad \supset$$ **graviton, 2 form , dilaton**

μ, ν are 10-d spacetime indices.

Target space duality

T-duality

$$R \leftrightarrow \frac{1}{R} \qquad n \leftrightarrow m$$



$$\boxed{R = \ell_s = 1}$$ **is the self-dual radius.**

string scale

- massless modes at self dual point

Patil & Brandenberger 2005

$$n = m = \pm 1\,,\ N = 1\,,\ \bar{N} = 0$$

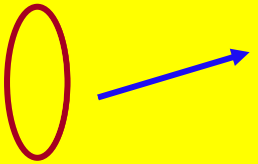
$$n = -m = \pm 1\,,\ N = 0\,,\ \bar{N} = 1$$

T-dual invariant

4-d effective action

Low energy effective action

$G_{\mu\nu}$ **graviton** $B_{\mu\nu}$ **2 form** ϕ **dilaton**



Weyl invariance

$$S = \frac{1}{2\kappa_{10}^{2}}\int d^{10}x\sqrt{-G}\,e^{-2\phi}\left[\overset{(10)}{R}+4\left(\partial\phi\right)^{2}-\frac{1}{12}H^{2}\right] \qquad H = dB$$

- **Dimensional reduction**

$$ds^{2} = \underset{\text{4-d universe}}{\underline{g_{\mu\nu}(x)dx^{\mu}dx^{\nu}}} + \underset{\text{6-d toroidal space}}{\underline{\gamma_{ab}(x)dy^{a}dy^{b}}} \qquad y^{a} \sim y^{a}+1$$

☆ $\sqrt{\gamma}e^{-2\phi} \equiv e^{-2\bar{\phi}} \Leftrightarrow \bar{\phi} = \phi - \frac{1}{2}\log\sqrt{\gamma}$ **shifted dilaton**

☆ $H_{\mu ab} = \partial_{\mu}B_{ab}(x)$

T-dual invariant 4-d effective action

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-2\bar{\phi}}\Big[R+4\left(\partial\bar{\phi}\right)^2$$

$$\underbrace{-\frac{1}{4}\gamma^{ac}\gamma^{bd}\partial^{\mu}\gamma_{ab}\partial_{\mu}\gamma_{cd}-\frac{1}{4}\gamma^{ab}\gamma^{cd}\partial^{\mu}B_{ac}\partial_{\mu}B_{bd}}_{\text{T-dual invariant}}\Big]$$

T-dual transformation

$$\tilde{\bar{\phi}}=\bar{\phi}$$

$$\tilde{\Gamma}=\left(\Gamma-B\Gamma^{-1}B\right)^{-1}$$

$$\tilde{B}=-\Gamma^{-1}B\left(\Gamma-B\Gamma^{-1}B\right)^{-1}$$

Matrix notation

$$\left(\Gamma\right)_{ab}=\gamma_{ab}$$

$$\left(B\right)_{ab}=B_{ab}$$

Action for a string gas

In the conventional cosmology, it is assumed the relativistic particles occupy the universe. As the everything comes from a string, it is natural to imagine the universe filled with a string gas.

Here, the adiabaticity is assumed, namely, the matter action is represented by the action of a string gas with constant background fields γ_{ab}, B_{ab} replaced by functions of spacetime coordinates $\gamma_{ab}(x), B_{ab}(x)$.

$$S_{gas} = -\mu_4 \int d^4x \sqrt{-g_{00}}\ E$$

Comoving number density of a string gas

$$E = \sqrt{g^{ij} p_i p_j + \underbrace{M^2(\gamma_{ab}, B_{ab})}_{\text{T-dual invariant}}}$$

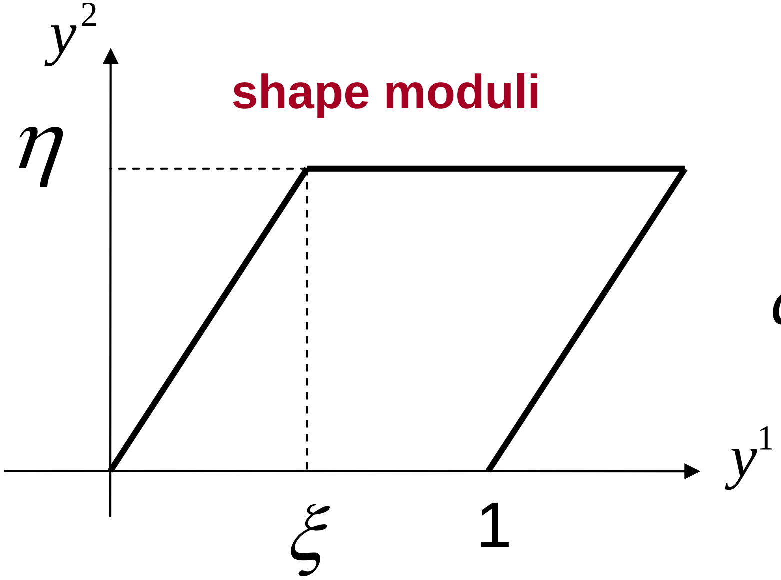
4-d momentum

Stable Compactification

A model of Compactfication

$$M_4 \otimes T_2 \otimes T_2 \otimes T_2$$

We can analyze each torus separately.



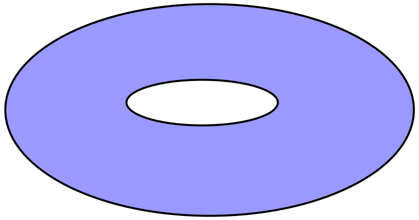
shape moduli

volume moduli

$$ds^2 = \frac{b^2}{\eta}\left[\left(dy^1 + \xi dy^2\right)^2 + \eta^2\left(dy^2\right)^2\right]$$

$$\Gamma = \frac{b^2}{\eta}\begin{pmatrix} 1 & \xi \\ \xi & \xi^2 + \eta^2 \end{pmatrix}$$

Identify the opposite sides



$$B = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix}$$

flux moduli

4-d effective action: Einstein frame

$$S_E = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[R - 2\overset{\text{Shifted dilaton}}{\left(\partial\bar{\phi}\right)^2} - 2\overset{\text{volume}}{\left(\partial\log b\right)^2} - \overset{\text{flux}}{\frac{1}{2b^4}\left(\partial\beta\right)^2}$$

$$- \overset{\text{shape}}{\frac{1}{2\eta^2}\left\{\left(\partial\eta\right)^2 + \left(\partial\xi\right)^2\right\}}\Big]$$

$$-\mu_4\int d^4x\sqrt{-g_{00}}\sqrt{g^{ij}p_ip_j + e^{2\bar{\phi}}\underbrace{M^2(b,\eta,\xi,\beta)}_{\text{Mass of a string}}}$$

↑

Effective potential

T-duality and Self-dual point

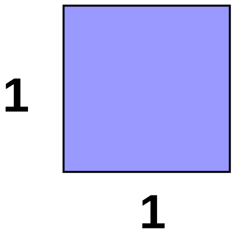
T-dual transformation

$$\tilde{\eta} = \frac{\eta}{\eta^2 + \xi^2} \qquad \tilde{\xi} = -\frac{\xi}{\eta^2 + \xi^2}$$

$$\tilde{b}^2 = \frac{b^2}{b^4 + \beta^2} \qquad \tilde{\beta} = -\frac{\beta}{b^4 + \beta^2}$$

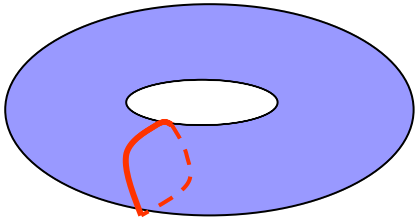
Self-dual point $\eta = 1, \xi = 0, b = 1, \beta = 0$

$$B_{ab} = 0$$



Moduli Stabilization I

The first kind of string gas
massless at the self dual point

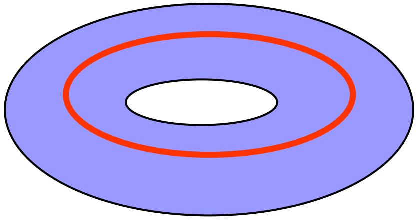


Mass formula for this wrapped string gas

$$M_1^2 = \frac{1}{\eta b^2}(\xi - \beta)^2 + \frac{\eta}{b^2} + \frac{b^2}{\eta} - 2$$

→ **Flat direction** $\xi = \beta \qquad \eta = b^2$

Moduli Stabilization II



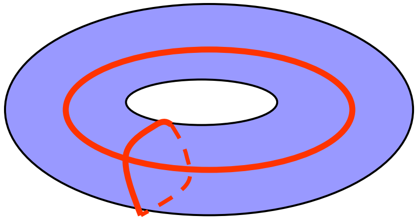
The second kind of string gas
massless at the self dual point

Mass formula for this wrapped string gas

$$M_2^2 = \frac{1}{\eta b^2}(1+\beta\xi)^2 + \frac{\eta\beta^2}{b^2} + \frac{b^2\xi^2}{\eta} + \eta b^2 - 2$$

⇨ **Flat direction** $\beta^2 = \frac{b^4\xi^2}{\eta^2}$ $1+\beta\xi = -\frac{\eta^2\beta}{\xi}$

Stable compactification



Let us consider both contributions together. As the would-be flat directions are orthogonal to each other, the flat direction disappears at the end of the day.

Volume, shape, and flux moduli get stabilized!

Moduli stabilization induces the dilaton stabilization

- **Effective potential**

$$-\mu_4\int d^4x\sqrt{-g_{00}}\sqrt{g^{ij}p_ip_j+e^{2\overline{\phi}}\underline{M^2(b,\eta,\xi,\beta)}}$$

The dilaton potential disappears!

- **Equation for the dilaton**

X : moduli

$$\underline{\frac{d^2}{dt^2}\left(e^{-2\phi}\right)+3H\frac{d}{dt}\left(e^{-2\phi}\right)}+\underline{6\dot{X}\frac{d}{dt}\left(e^{-2\phi}\right)}=0$$

Hubble damping

Modulation due to moduli oscillation

We have thus understood our numerical result and shown that the dilaton is marginally stable.

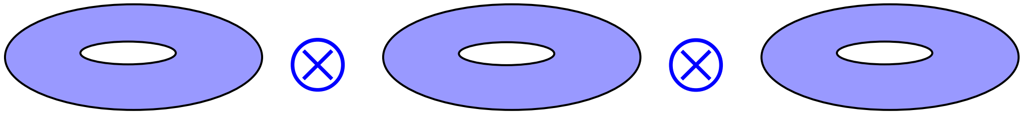
Conclusion

Further challenges in String Gas cosmology

- It is necessary to connect this string gas phase to the standard cosmological phase.
- In the matter dominated phase, the dilaton will start to run again. We need to invent a mechanism to stabilize the dilaton in this phase. We guess the loop corrections may stabilize the dilaton.
- We need to explain the origin of structure of the universe in the context of string gas scenario.

Summary

- We have clarified why the dilaton shows the damped oscillation in numerical results.
- We have constructed the T-dual invariant 4-d effective action.
- We have shown the stability of

$M_4 \otimes$ 

compactification.