Non-Gaussian perturbations with two scalar fields

Antti Väihkönen

Helsinki Institute of Physics
and
University of Helsinki,
Finland

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Motivation

Talk based on two papers:

(I) K. Enqvist, AV: hep-ph/0405103 (JCAP)

"Non-Gaussian perturbations in hybrid inflation"

(II) AV: astro-ph/0506304

"Comment on non-Gaussianity in hybrid inflation"

First consistent 2*nd* order perturbation computation of a two field model presented in (I)

Later Lyth and Rodríguez re-estimated [1] the result and computed it with δN formalism [2]

$\longrightarrow$ disagreeing results

Aim of my work in (II) was to tackle the discrepancy

[1] D. Lyth, Y. Rodriguez: astro-ph/0502578 (PRD)
[2] D. Lyth, Y. Rodriguez: astro-ph/0504045

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Outline of the Talk

- Explain what is Non-Gaussianity
- *2nd* order CPT [3] computation for hybrid inflation
- Key points of δN approach and the result
- Compare and discuss the results

[3] Cosmological perturbation theory

Gaussianity vs. Non-Gaussianity

Gaussian

- free field theory
- no interactions
- linear EOM
- Fourier modes independent
- 2-point correlator enough

Non-Gaussian

- interactions
- non-linear EOM
- Fourier modes coupled
- *n*-point correlators needed
- parameterizing difficult

- Observations agree with Gaussianity – do not demand it
- Models predict additional Non-Gaussianity
  - Different models – different amount and type
- Observations only now starting to be sensitive enough
- Requires going to 2*nd* order

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Metric and matter fields

$$\begin{cases} g_{00} = -(1 + 2\phi_1 + \phi_2) \\ g_{ij} = a^2(1 - 2\psi_1 - \psi_2) \end{cases} \Rightarrow \begin{array}{c} G_{\mu\nu} \\ \text{to } 2nd \text{ order} \end{array}$$

$$\begin{cases} \varphi = \varphi_0 + \varphi_1 + \frac{1}{2}\varphi_2 \\ \sigma = \sigma_0 + \sigma_1 + \frac{1}{2}\sigma_2 \end{cases} \Rightarrow \begin{array}{c} T_{\mu\nu} \\ \text{to } 2nd \text{ order} \end{array}$$

$\Longrightarrow$ Einstein equations to *2nd* order

Curvature perturbation $\mathcal{R}$

- With one field in 1*st* order $\quad \mathcal{R} = \psi + H\frac{\delta\varphi}{\dot{\varphi}}$
- Extended for two fields
- Expanded to 2*nd* order $\quad \mathcal{R} = \mathcal{R}_1 + \frac{1}{2}\mathcal{R}_2$

Demand invariance in change of slicing

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Applying the Formalism to Hybrid Inflation

Potential $V = V_0 - \frac{1}{2}m_0^2\sigma^2 + \frac{1}{4}\lambda\sigma^4 + \frac{1}{2}m^2\varphi^2 + \frac{1}{2}g^2\sigma^2\varphi^2$

where $V_0 = m_0^2/4\lambda$

Key features

- Transverse field at the bottom of the valley, $\sigma_0 = 0$
- No terms linear in σ in the potential

σ contribution decouples

- No σ in 1*st* order Einstein equations (Klein–Gordon only)
- No $\sigma\varphi$-mixing terms in 2*nd* order (σ additional)

- Formalism applies to other models also (e.g. Linde–Mukhanov)

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Estimating the expression for $\mathcal{R}_{2,\sigma}$

- Scales outside horizon
- Effective masses small, $m \lesssim H$
- $\mathcal{R}_1$ roughly constant
- $H,\ m_\sigma, m,\ \eta, \eta_\sigma$ roughly constant
- time evolution of $\epsilon,\ \varphi_0, \varphi_1, \sigma_1$ important

Make an order of magnitude estimate $|\Delta^{-1}(\partial_i \mathcal{R}_1 \partial^i \mathcal{R}_1)| \sim |\mathcal{R}_1|^2$

The result

$$\mathcal{R}_{2,\sigma} \sim \mathcal{O}(\epsilon, \eta, \eta_\sigma)\, e^{2\Delta N(\eta - \eta_\sigma)}\, |\mathcal{R}_1|^2 \,.$$

$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \varphi}\right)^2, \quad \eta \equiv \frac{M_P^2}{V}\frac{\partial^2 V}{\partial \varphi^2}, \quad \eta_\sigma \equiv \frac{M_P^2}{V}\frac{\partial^2 V}{\partial \sigma^2}$

δN Approach

- Each point in space treated separately

  surrounded by FRW universe; local expansion parameter N
- Inhomogeneous behaviour of the universe obtained by patching the points together

Curvature perturbation ζ

Definition $$g_{ij} = a^2(t)\, e^{2\zeta(t,\boldsymbol{x})}\, \gamma_{ij}(t,\boldsymbol{x})$$

Obtained from $$\zeta = \frac{\partial N}{\partial \phi_i}\, \delta\phi^i + \frac{1}{2}\, \frac{\partial^2 N}{\partial \phi_i \partial \phi_j}\, \delta\phi^i \delta\phi^j$$

The Result

$$\zeta_{2,\sigma} \sim \mathcal{O}(\eta_\sigma)\, e^{2\Delta N(\eta-\eta_\sigma)}\, |\zeta_1|^2 \,.$$

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Comparing the results

CPT

$$\mathcal{R}_{2,\sigma} \sim \mathcal{O}(\epsilon, \eta, \eta_\sigma)\, e^{2\Delta N(\eta-\eta_\sigma)}\, |\mathcal{R}_1|^2\,.$$

δN

$$\zeta_{2,\sigma} \sim \mathcal{O}(\eta_\sigma)\, e^{2\Delta N(\eta-\eta_\sigma)}\, |\zeta_1|^2\,.$$

- Results are of the same form and of the same order
- The exponential factor does not provide enhancement
  $\longrightarrow$ Non-Gaussianity unobservable

Drop non-local terms $\Delta^{-1}(\partial_i g \partial^i g)$, $\Delta^{-1}(g\Delta g)$ in CPT case
$\longrightarrow \mathcal{O}(\eta_\sigma)$ coefficient only.
Justification?

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Summary and Outlook for the Future

Summary

- The results of two different methods (CPT, δN) shown to agree
- Provides confidence on the applicability of the methods

Open questions

- Non-local terms $\Delta^{-1}(\partial_i g \partial^i g)$ and $\Delta^{-1}(g \Delta g)$
- Mode-mode couplings in second order: preheating studies[a] implicate they might be important

[a] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamäki, AV: astro-ph/0411394 (PRL), hep-ph/0501076 (JCAP), hep-th/0502185 (JHEP to appear)

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