Non-Gaussianity from Multiple-Field Inflation

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Formalism: astro-ph/0405185, /0504508

Single-field case: astro-ph/0410486

Multiple-field case: astro-ph/0506704

1. **Non-linear formalism** *[astro-ph/0504508, /0405185]*

Fully non-linear equation of motion:

- for gauge-invariant, smoothed $\zeta_i^A \equiv \frac{\dot{\phi}^A}{\dot{\phi}} \partial_i \ln a - \frac{\kappa}{\sqrt{2\tilde{\epsilon}}} \partial_i \phi^A$
- for multiple fields ϕ^A and non-trivial field metric G_{AB}
- in long-wavelength approximation
- with stochastic source terms $\mathcal{S}_i^A$ and $\mathcal{J}_i^A$ describing short-wavelength effects

$$\begin{cases} \mathcal{D}_t \zeta_i^A - \theta_i^A = \mathcal{S}_i^A \\ \mathcal{D}_t \theta_i^A + 3\theta_i^A + \left(\frac{\mathcal{D}^A \partial_B V}{H^2} + 3(\tilde{\epsilon} + \tilde{\eta}^{\parallel}) - 6\tilde{\epsilon} \frac{\dot{\phi}^A \dot{\phi}_B}{\dot{\phi}^2} \right) \zeta_i^B = \mathcal{J}_i^A \end{cases}$$

$\mathcal{D}$ = covariant derivative; V = potential

[leading-order slow roll; gauge choice $t = \ln(aH)$]

$$\begin{cases} \mathcal{D}_t \zeta_i^A - \theta_i^A = \mathcal{S}_i^A \\ \mathcal{D}_t \theta_i^A + 3\theta_i^A + \left(\frac{\mathcal{D}^A \partial_B V}{H^2} + 3(\tilde{\epsilon} + \tilde{\eta}^{\parallel}) - 6\tilde{\epsilon}\, \frac{\Pi^A \Pi_B}{\Pi^2} \right) \zeta_i^B = \mathcal{J}_i^A \end{cases}$$

All coefficients are inhomogeneous and depend on $(t, \mathbf{x})$ through $H(t, \mathbf{x})$, $\phi^A(t, \mathbf{x})$ and $\Pi^A(t, \mathbf{x}) \equiv \dot{\phi}^A / N$:

$$\tilde{\epsilon} \equiv \frac{\kappa^2}{2} \frac{\Pi^2}{H^2}, \qquad \tilde{\eta}^{\parallel} \equiv -3 - \frac{\Pi^A \partial_A V}{H \Pi^2}. \qquad \kappa^2 \equiv 8\pi / M_{\rm pl}^2$$

$\Rightarrow$ **Constraint** relations close non-linear system:

$$\partial_i (\ln H) = \tilde{\epsilon}\, \frac{\Pi_A}{\Pi}\, \zeta_i^A, \qquad \partial_i \phi^A = -\frac{\sqrt{2\tilde{\epsilon}}}{\kappa}\, \zeta_i^A,$$

$$\mathcal{D}_i \Pi^A = -\frac{\sqrt{2\tilde{\epsilon}}}{\kappa} H \left[\theta_i^A + \left((\tilde{\epsilon} + \tilde{\eta}^{\parallel}) \delta_B^A - \tilde{\epsilon}\, \frac{\Pi^A \Pi_B}{\Pi^2} \right) \zeta_i^B \right].$$

$\mathcal{S}_i^A, \mathcal{J}_i^A$ = stochastic sources **emulating effect of quantum fluctuations** entering long-wavelength system

$$\begin{cases} \mathcal{D}_t \zeta_i^A - \theta_i^A = \mathcal{S}_i^A & \lim_{t\to-\infty} \zeta_i^A = 0, \quad \lim_{t\to-\infty} \theta_i^A = 0 \\ \mathcal{D}_t \theta_i^A + 3\theta_i^A + \left(\frac{\mathcal{D}^A \partial_B V}{H^2} + 3(\tilde{\epsilon} + \tilde{\eta}^{\parallel}) - 6\tilde{\epsilon}\, \frac{\Pi^A \Pi_B}{\Pi^2} \right) \zeta_i^B = \mathcal{J}_i^A \end{cases}$$

$$(\mathcal{S}_i^A, \mathcal{J}_i^A) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, \dot{\mathcal{W}}(k) \left(\zeta_{\mathrm{lin}\ B}^A, \theta_{\mathrm{lin}\ B}^A\right)(k, \mathbf{x}) \alpha^B(\mathbf{k}) \mathrm{i} k_i\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \text{c.c.}$$

$\zeta_{\mathrm{lin}\ B}^A = \frac{-\kappa}{a\sqrt{2\tilde{\epsilon}}} Q_{\mathrm{lin}\ B}^A$ full linear solution *[Gr.Nibbelink & BvT hep-ph/0107272]*

$\mathcal{W} = \exp\left(-k^2 R^2/2\right)$ Gaussian window, smoothing length R

$R \equiv c/(aH) = c\,\mathrm{e}^{-t}$, with c number sufficiently > 1, $c \approx$ 3–5

$\alpha^A(\mathbf{k})$ = Gaussian complex random numbers, satisfying:

$$\langle \alpha^A(\mathbf{k}) \alpha_B^*(\mathbf{k}') \rangle = \delta^3(\mathbf{k} - \mathbf{k}') \delta_B^A, \qquad \langle \alpha^A(\mathbf{k}) \alpha_B(\mathbf{k}') \rangle = 0.$$

Approximations for analytic treatment

N.B.: The following approximations are only necessary for analytic calculations, not for numerical treatment!

Slow-roll expansion

Consider only leading-order effects in slow-roll parameters.

Perturbation expansion

- **1st order** (linear approximation): all coefficients take homogeneous background values.
- **2nd order**: Expand coefficients as ($C = \tilde{\epsilon}, \tilde{\eta}^{\parallel}$, etc.):

$$C(t,\mathbf{x}) = C^{(0)}(t) + C^{(1)}(t,\mathbf{x}) = C^{(0)} + \partial^{-2}\partial^{i}(\partial_i C)^{(1)}$$

and use constraints to find $(\partial_i C)^{(1)} = (\ldots)_A^{(0)} \theta_i^{(1)\,A} + (\ldots)_A^{(0)} \zeta_i^{(1)\,A}$

2. **Single-field inflation** *[astro-ph/0410486]*

$$\begin{cases} \dot{\zeta}_i - \theta_i = \mathcal{S}_i & \lim_{t\to-\infty} \zeta_i = 0, \quad \lim_{t\to-\infty} \theta_i = 0 \\ \dot{\theta}_i + 3\theta_i + \left(\frac{\partial^2 V/\partial\phi^2}{H^2} - 3(\tilde{\epsilon} - \tilde{\eta}) \right) \zeta_i = \mathcal{J}_i \end{cases}$$

2. Single-field inflation *[astro-ph/0410486]*

First order:

$$\dot{\zeta}_i^{(1)} = \mathcal{S}_i^{(1)}, \qquad \lim_{t\to-\infty} \zeta_i^{(1)} = 0$$

$$\zeta^{(1)}(R,\mathbf{x}) = -\frac{\kappa}{2\sqrt{2}} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \frac{1}{k^{3/2}} \frac{H_{\mathcal{H}}}{\sqrt{\tilde{\epsilon}_{\mathcal{H}}}} e^{-k^2R^2/2} \alpha(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{c.c.}$$

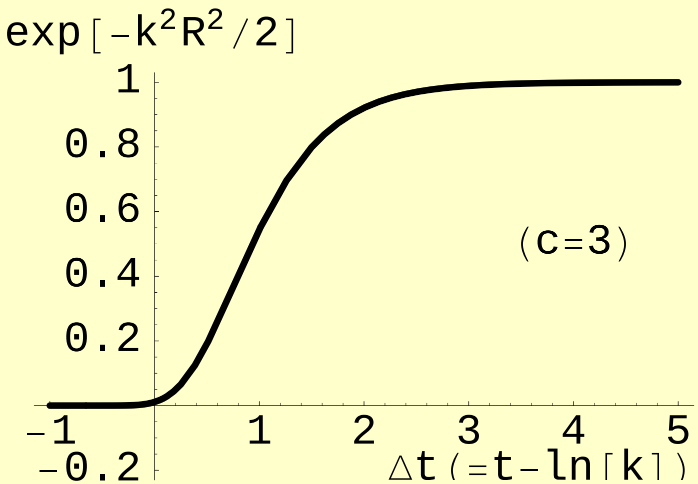
with

$$\zeta \equiv \partial^{-2}\partial^i \zeta_i$$

$$R = c/(aH) = c\,e^{-t}$$

Subscript $\mathcal{H}$: evaluate when $k = aH$

$$e^{-k^2R^2/2} \xrightarrow{R\to 0} 1$$



Second order:

$$\dot{\zeta}_i^{(2)} = (2\tilde{\epsilon}^{(0)} + \tilde{\eta}^{(0)})\,\zeta^{(1)}\mathcal{S}_i^{(1)}$$

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Three-point correlator (bispectrum): (→ **non-Gaussianity**)

$$\langle\zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\zeta(\mathbf{x}_3)\rangle^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = (2\pi)^3\delta^3(\sum \mathbf{k}_s)\left[f(\mathbf{k}_1,\mathbf{k}_2)+f(\mathbf{k}_1,\mathbf{k}_3)+f(\mathbf{k}_2,\mathbf{k}_3)\right]$$

$$f(\mathbf{k},\mathbf{k}') \equiv \frac{\kappa^4}{16}\frac{1}{k^3(k')^3}\frac{H^2_{\mathcal{H}}}{\tilde{\epsilon}_{\mathcal{H}}}\frac{H^2_{\mathcal{H}'}}{\tilde{\epsilon}_{\mathcal{H}'}}\left[(2\tilde{\epsilon}_{\mathcal{H}'}+\tilde{\eta}_{\mathcal{H}'})\frac{k^2}{k^2+(k')^2}\frac{k^2+\mathbf{k}\cdot\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|^2}+(\mathbf{k}\leftrightarrow\mathbf{k}')\right]$$

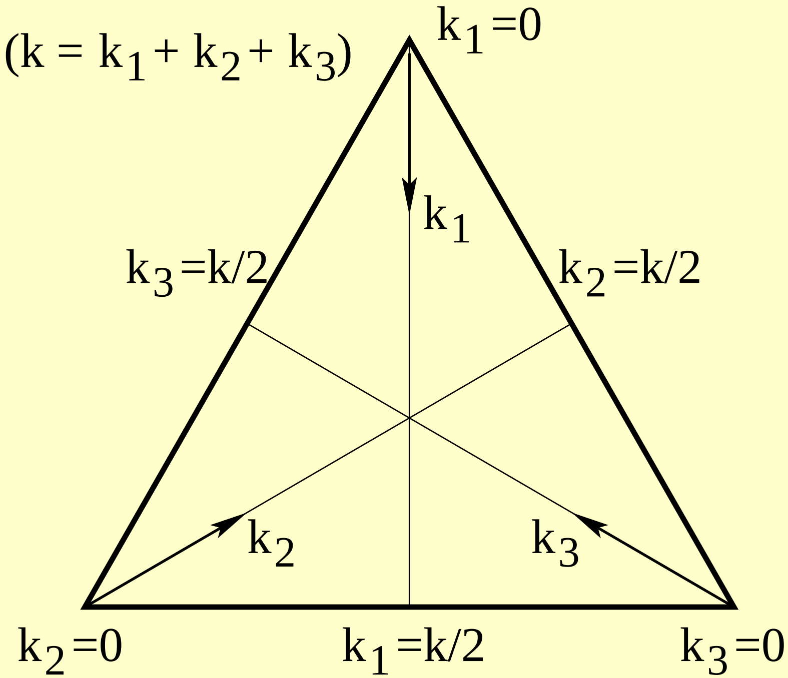
Limit $k_3 \ll k_1, k_2$ (and hence $\mathbf{k}_1 = -\mathbf{k}_2 \equiv \mathbf{k}$):

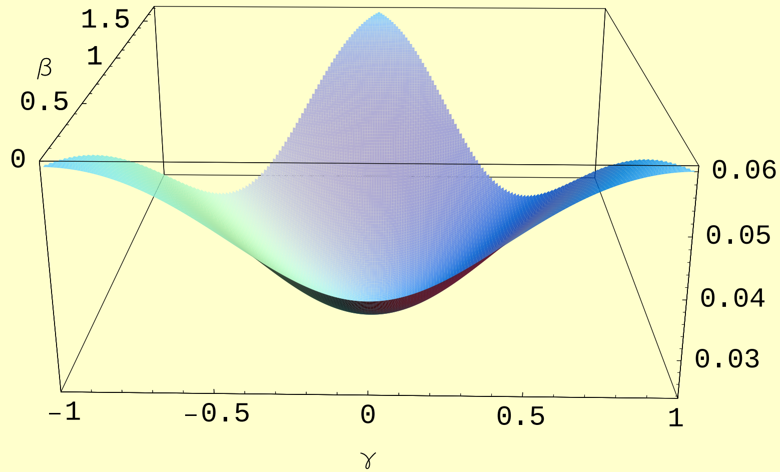
$$\langle\zeta\zeta\zeta\rangle^{(2)} = 2\,\frac{\kappa^4}{16}\frac{1}{k^3k_3^3}\frac{H^2_{\mathcal{H}}}{\tilde{\epsilon}_{\mathcal{H}}}\frac{H^2_{\mathcal{H}_3}}{\tilde{\epsilon}_{\mathcal{H}_3}}(2\tilde{\epsilon}_{\mathcal{H}_3}+\tilde{\eta}_{\mathcal{H}_3})$$

$$\Rightarrow \quad \boxed{f_{\mathrm{NL}} = \frac{\langle\zeta\zeta\zeta\rangle^{(2)}}{(\langle\zeta\zeta\rangle^{(1)})^2} = -\tilde{n} \quad \rightarrow \text{unobservable.}}$$

(with $\tilde{n} = n-1$ scalar spectral index)

$$\text{"}f_{\rm NL}\text{"} = \langle\zeta\zeta\zeta\rangle/(k^3\langle\zeta\zeta\rangle)^2 \times k_1^3k_2^3k_3^3/[(k_1^2+k_2^2+k_3^2)/2]^{3/2}, \text{ for } 2\tilde{\epsilon}+\tilde{\eta} = 0.03:$$





Other computations in literature:

- tree-level action *[Maldacena astro-ph/0210603]*;
- 2nd order Einstein *[Acquaviva et al. astro-ph/0209156]*.

Ours: much **simpler**; **multiple fields** incorporated from start;
convenient for **numerical implementation**.

3. Multiple-field inflation *[astro-ph/0506704]*

Basis: $\Pi^A \rightsquigarrow e_1^A$, $\mathcal{D}_t\Pi^A \rightsquigarrow e_2^A$, etc. by orthogonalization:

$$e_1^A \equiv \frac{\Pi^A}{\Pi}, \qquad e_2^A \equiv \frac{(\delta^A_B - e_1^A e_{1\,B})\mathcal{D}_t\Pi^B}{|(\delta^A_B - e_1^A e_{1\,B})\mathcal{D}_t\Pi^B|}, \qquad \text{etc.}$$

Define: $\boxed{\zeta^m_i \equiv \mathbf{e}_{m\,A}\zeta^A_i}$ etc.

Slow-roll parameter $\tilde{\eta}^A$ is now vector:

$$\tilde{\eta}^A = \frac{\mathcal{D}_t\Pi^A}{NH\Pi} = -\frac{3H\Pi^A + G^{AB}\partial_B V}{H\Pi},$$

$\tilde{\eta}^{\parallel} = e_1^A\tilde{\eta}_A$ (eff. single-field), $\tilde{\eta}^{\perp} = e_2^A\tilde{\eta}_A$ (truly multiple-field).

[Groot Nibbelink & BvT hep-ph/0011325]

Multiple-field equations (leading-order slow roll):

$$\begin{cases} \mathcal{D}_t \zeta_i^A - \theta_i^A = \mathcal{S}_i^A & \lim_{t\to-\infty} \zeta_i^A = 0, \qquad \lim_{t\to-\infty} \theta_i^A = 0 \\ \mathcal{D}_t \theta_i^A + 3\theta_i^A + \left(\frac{\mathcal{D}^A \partial_B V}{H^2} + 3(\tilde{\epsilon} + \tilde{\eta}^{\parallel}) - 6\tilde{\epsilon} \frac{\Pi^A \Pi_B}{\Pi^2} \right) \zeta_i^B = \mathcal{J}_i^A \end{cases}$$

Multiple-field equations (leading-order slow roll):

$$\begin{cases} \dot{\zeta}_i^m - \theta_i^m = \mathcal{S}_i^m & \lim_{t\to-\infty} \zeta_i^m = 0, \qquad \lim_{t\to-\infty} \theta_i^m = 0 \\ \dot{\theta}_i^m + 3\theta_i^m + \left(\frac{V_{mn}}{H^2} + 3(\tilde{\epsilon} + \tilde{\eta}^{\parallel})\delta_{mn} - 6\tilde{\epsilon}\delta_{m1}\delta_{n1} + 3Z_{mn} \right) \zeta_i^n = \mathcal{J}_i^m & \end{cases}$$

with $V_{mn} \equiv e_m^A e_n^B \mathcal{D}_A \partial_B V$ and $Z_{mn} \equiv e_{m\,A} \mathcal{D}_t e_n^A / (NH)$.

Combine into single vector equation:

$$\dot{v}_i^a + A_{ab} v_i^b = b_i^a, \qquad \lim_{t\to-\infty} v_i^a = 0,$$

$$v_i \equiv (\zeta_i^1, \theta_i^1, \zeta_i^2, \theta_i^2, \ldots)^T, \qquad b_i \equiv (\mathcal{S}_i^1, \mathcal{J}_i^1, \mathcal{S}_i^2, \mathcal{J}_i^2, \ldots)^T.$$

Two-field case: $A = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 3 & -6\tilde{\eta}^{\perp} & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 3\chi & 3 \end{pmatrix}, \qquad \chi \equiv \frac{V_{22}}{3H^2} + \tilde{\epsilon} + \tilde{\eta}^{\parallel}.$

Perturbative expansion:

$$\dot{v}_i^{(1)a} + A_{ab}^{(0)} v_i^{(1)b} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \dot{\mathcal{W}}(k) X_{am}^{(1)}(k) \alpha^m(\mathbf{k}) \mathrm{i} k_i \, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \text{c.c.}$$

$$\dot{v}_i^{(2)a} + A_{ab}^{(0)} v_i^{(2)b} = v^{(1)c} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} \dot{\mathcal{W}}(k) \bar{X}_{amc}^{(1)}(k) \alpha^m(\mathbf{k}) \mathrm{i} k_i \, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \text{c.c.} \quad - v^{(1)c} \bar{A}_{abc}^{(0)} v_i^{(1)b}$$

with e.g. $v^{(1)c} \bar{A}_{abc}^{(0)} = \partial^{-2} \partial^i (\partial_i A_{ab})^{(1)} \equiv A_{ab}^{(1)}$.

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$$\dot{v}_i^{(2)a} + A_{ab}^{(0)} v_i^{(2)b} = v^{(1)c} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\, \dot{\mathcal{W}}(k) \bar{X}_{amc}^{(1)}(k) \alpha^m(\mathbf{k}) \mathrm{i} k_i\, \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \text{c.c.} \; - v^{(1)c} \bar{A}_{abc}^{(0)} v_i^{(1)b}$$

with e.g. $v^{(1)c} \bar{A}_{abc}^{(0)} = \partial^{-2} \partial^i (\partial_i A_{ab})^{(1)} \equiv A_{ab}^{(1)}$.

⇒ Solve with **Green's function** (same for each order!):

$$v_i^{(1,2)a}(t,\mathbf{x}) = \int_{-\infty}^{t} \mathrm{d}t' G_{ab}(t,t') [\text{r.h.s.}]_b^{(1,2)}(t',\mathbf{x})$$

$$\frac{\mathrm{d}}{\mathrm{d}t} G_{ab}(t,t') + A_{ac}^{(0)}(t) G_{cb}(t,t') = 0, \qquad \lim_{t \to t'} G_{ab}(t,t') = \delta_{ab}.$$

Define $v^{(1)}_{am}(k,t) \equiv \int_{-\infty}^{t} \mathrm{d}t'\, G_{ab}(t,t')\dot{\mathcal{W}}(k,t')X^{(1)}_{bm}(k,t') \quad \Rightarrow$

Power spectrum:

$$\langle\zeta^1\zeta^1\rangle^{(1)}(k) \quad = \quad 2\, v^{(1)}_{1m}(k,t)v^{(1)}_{1m}(k,t)$$

Bispectrum:

$$\langle\zeta^1\zeta^1\zeta^1\rangle^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = (2\pi)^3\delta^3(\sum\mathbf{k}_s)\left[f(\mathbf{k}_1,\mathbf{k}_2)+f(\mathbf{k}_1,\mathbf{k}_3)+f(\mathbf{k}_2,\mathbf{k}_3)\right]$$

$$f(\mathbf{k},\mathbf{k}') \quad \equiv \quad 4\,\frac{k^2+\mathbf{k}\cdot\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|^2}\, v^{(1)}_{1m}(k,t)v^{(1)}_{1n}(k',t)\int_{-\infty}^{t}\mathrm{d}t'\, G_{1a}(t,t')v^{(1)}_{cn}(k',t')$$
$$\times\left[\bar{X}^{(1)}_{amc}(k,t')\dot{\mathcal{W}}(k,t') - \bar{A}^{(0)}_{abc}(t')v^{(1)}_{bm}(k,t')\right] + \mathbf{k}\leftrightarrow\mathbf{k}'.$$

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$$v_{1m}^{(1)}(k,t)v_{1n}^{(1)}(k',t)\int_{-\infty}^{t}\mathrm{d}t'\,G_{1a}(t,t')v_{cn}^{(1)}(k',t')\left[\bar{X}_{amc}^{(1)}(k,t')\dot{\mathcal{W}}(k,t')-\bar{A}_{abc}^{(0)}(t')v_{bm}^{(1)}(k,t')\right]$$

$\bar{X}$ term:

- Comes from perturbing stochastic sources
- Not exact in limit 3 equal momenta because linear solution in source?
- However, small anyway (confirmed by *[Seery & Lidsey astro-ph/0506056]*)

$\bar{A}$ term:

- Comes from perturbing matrix A in equation of motion
- Absent in single-field case
- Integrated effect due to continued influence of isocurvature modes on adiabatic mode on super-horizon scales
- Treated fully non-linearly in our formalism

Large non-Gaussianity possible from $\bar{A}$ term!

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4. **Two-field slow-roll example** *[astro-ph/0506704]*

Constant slow-roll parameters $\Rightarrow$ analytic calculation possible.

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Constant slow-roll parameters $\Rightarrow$ analytic calculation possible.

Result:

$$
\begin{aligned}
f(\mathbf{k},\mathbf{k}') \equiv \frac{\kappa^4}{16}\frac{1}{k^3k'^3}\frac{H^4}{\tilde\epsilon^2}\frac{k^2+\mathbf{k}\cdot\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|^2}\Bigg\{&(2\tilde\epsilon+\tilde\eta^{\parallel})J_1 \\
+4\left(\frac{\tilde\eta^{\perp}}{\chi}\right)^2 g(\chi,\Delta t)\Big[&(2\tilde\epsilon+\tilde\eta^{\parallel})\,(2J_1-J_3-J_4)+\frac{\chi}{2}\,(J_3-J_2)+\psi_1\,(I_1-I_4) \\
&+\chi\left(-\tilde\epsilon+2\tilde\eta^{\parallel}-\frac{\tilde\xi^{\perp}}{\tilde\eta^{\perp}}\right)I_1\Big] \\
+16\left(\frac{\tilde\eta^{\perp}}{\chi}\right)^4 g^2(\chi,\Delta t)\Big[&(2\tilde\epsilon+\tilde\eta^{\parallel})\,(J_1-J_3-J_4+J_6)+\frac{\chi}{2}\,(J_3-J_2-J_6+J_5) \\
&+\psi_1\,(I_1-I_4-I_2+I_3)+\chi\left(-\tilde\epsilon+2\tilde\eta^{\parallel}-\frac{\tilde\xi^{\perp}}{\tilde\eta^{\perp}}\right)(I_1-I_2) \\
&+\frac{\chi}{2\tilde\eta^{\perp}}\,\psi_2\,(I_2-I_3)+\frac{\chi^2}{2(\tilde\eta^{\perp})^2}\,\omega\,I_2\Big]\Bigg\}+\mathbf{k}\leftrightarrow\mathbf{k}'
\end{aligned}
$$

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4. **Two-field slow-roll example** *[astro-ph/0506704]*

Constant slow-roll parameters $\Rightarrow$ analytic calculation possible.

Result of the form:

$$\text{"}f_{\rm NL}\text{"} = \frac{\text{bispectrum}}{(\text{power spectrum})^2} = \mathcal{O}\left(\tilde{\epsilon}, \tilde{\eta}^{\parallel}\right) J + \mathcal{O}\left((\tilde{\eta}^{\perp})^2\right) I$$

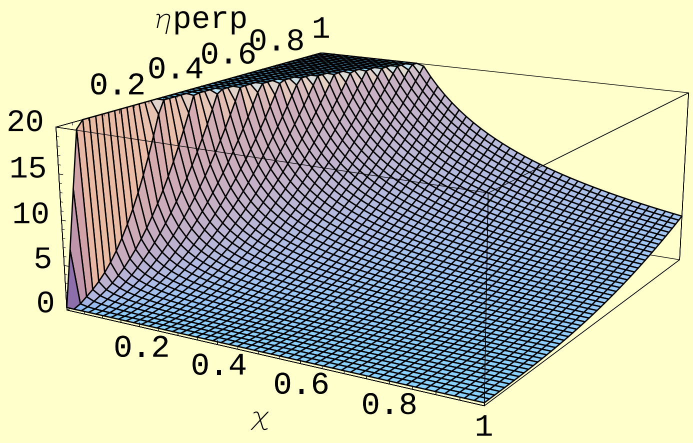
First term comes from $\bar{X}$, integral $J \leq 1$

$$\Rightarrow f_{\rm NL} = \mathcal{O}\left(\tilde{\epsilon}, \tilde{\eta}^{\parallel}\right) \sim 10^{-2}, \text{ like single-field case.}$$

Second term comes from $\bar{A}$, integral $I \sim \min(\Delta t, \chi^{-1})$

$$\Rightarrow f_{\rm NL} = \mathcal{O}\left((\tilde{\eta}^{\perp})^2/\chi\right) \sim 1!$$

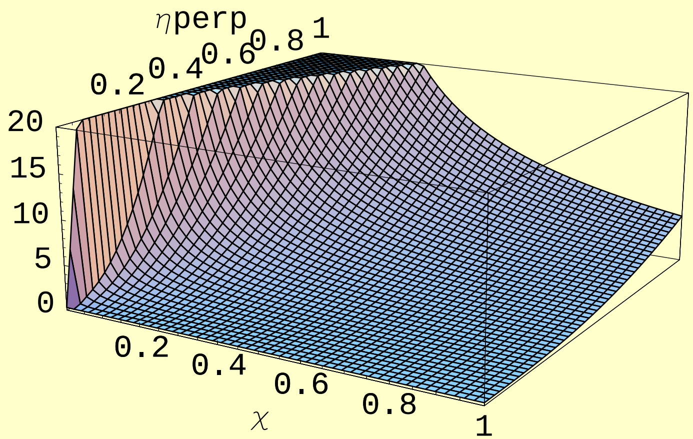
(because $\tilde{\eta}^{\perp} \sim 10^{-1}$ easily possible in multiple-field models)



Analytic result for $f_{\rm NL}(\tilde{\eta}^{\perp}, \chi)$

in limit $k_3 \ll k_1, k_2$ (vertex triangle)

for $\Delta t = 50$, $\tilde{\epsilon}, \tilde{\eta}^{\parallel} = 0.05$, 2nd-order $= 0.003$.



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in limit $k_3 \ll k_1, k_2$ (vertex triangle)

for $\Delta t = 50$, $\tilde{\epsilon}, \tilde{\eta}^{\parallel} = 0.05$, 2nd-order $= 0.003$.

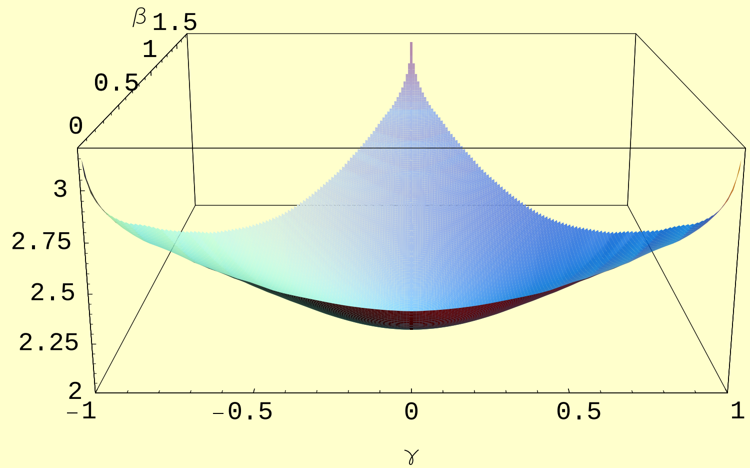
Full momentum dependence "$f_{\rm NL}$",

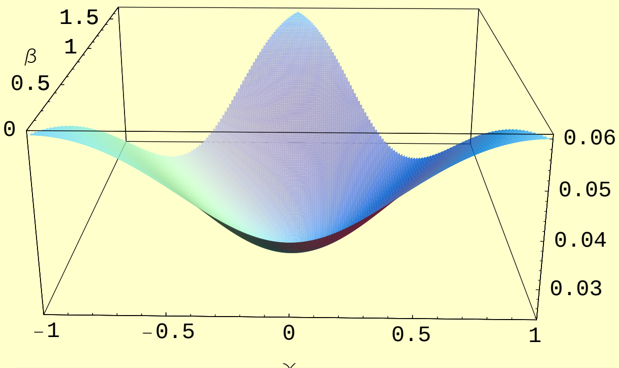
same normalisation as single-field,

for $\tilde{\eta}^{\perp} = 0.2$, $\chi = 0.01$.

Value at vertex $= 15.6$.

Ratio $\bar{A}$ to $\bar{X}$ at centre $= 200$.





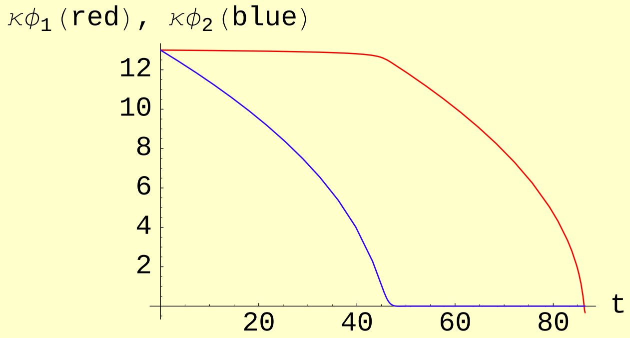
$\Rightarrow$ **Multiple-field non-Gaussianity**

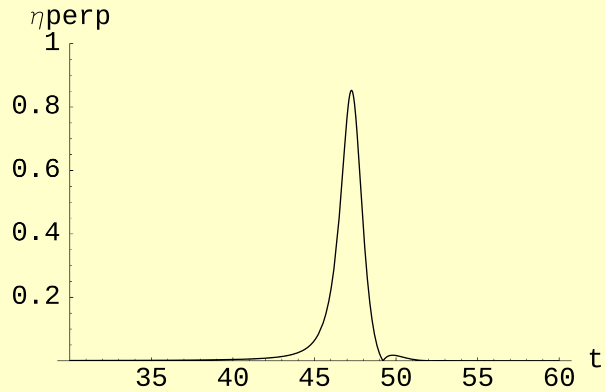
about **two orders** of magnitude

larger than single-field case!

Semi-analytic calculation quadratic potential

Explicit, simple model: $V = \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2$





$m_1 = 10^{-5}\kappa^{-1}$

$m_2/m_1 = 9$

initial cond.:

$\phi_1 = \phi_2 = 13\kappa^{-1}$

Results semi-analytic Green's function calculation:

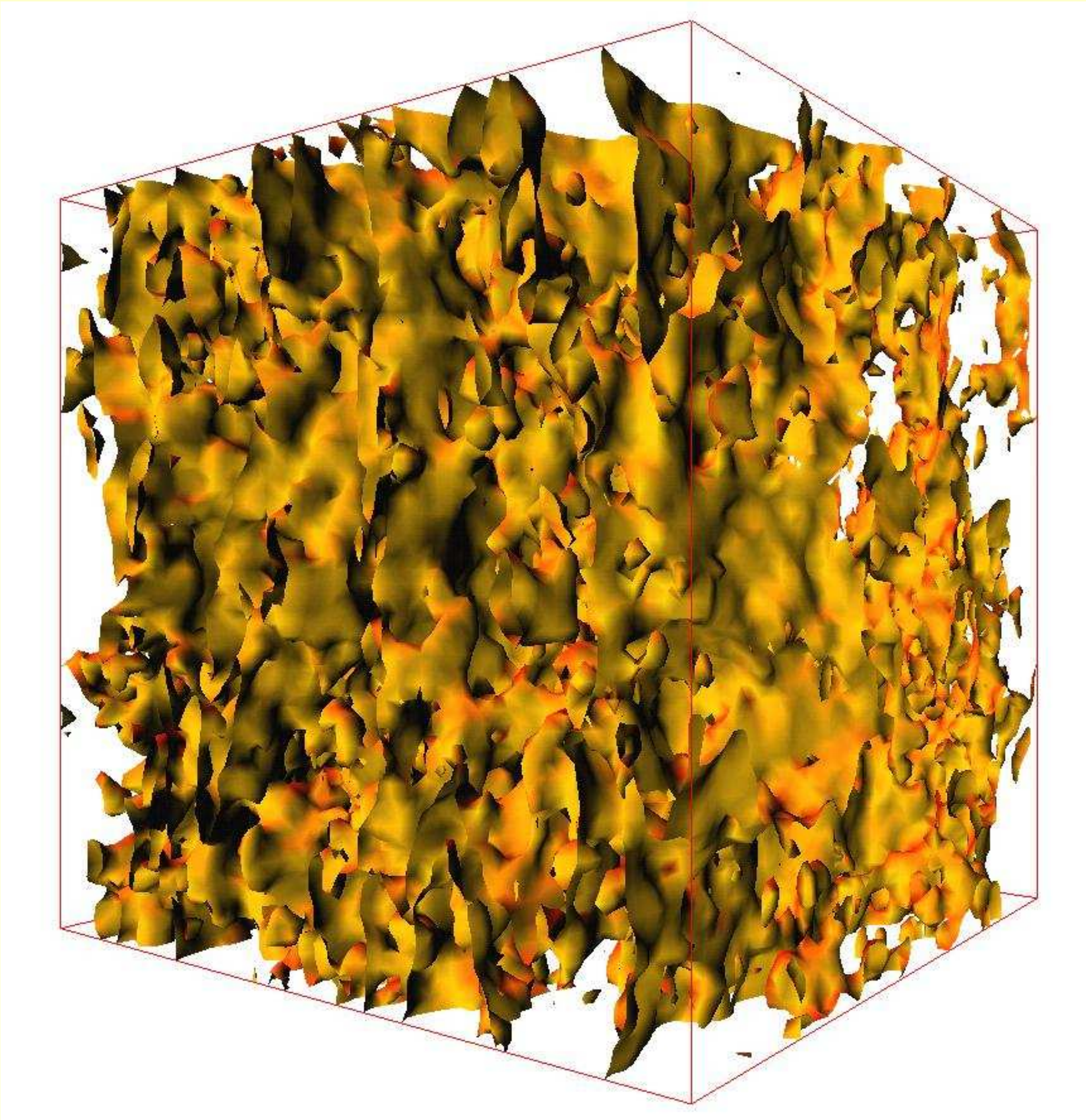
	$f_{\rm NL}$	$\bar{A}/\bar{X}$ contribution
$k_1 = k_2 = k_3$	1.0	74
$k_3 = e^{-9}k_1 = e^{-9}k_2$	1.9	26

(k_1 exits horizon 58 e-folds before end inflation)

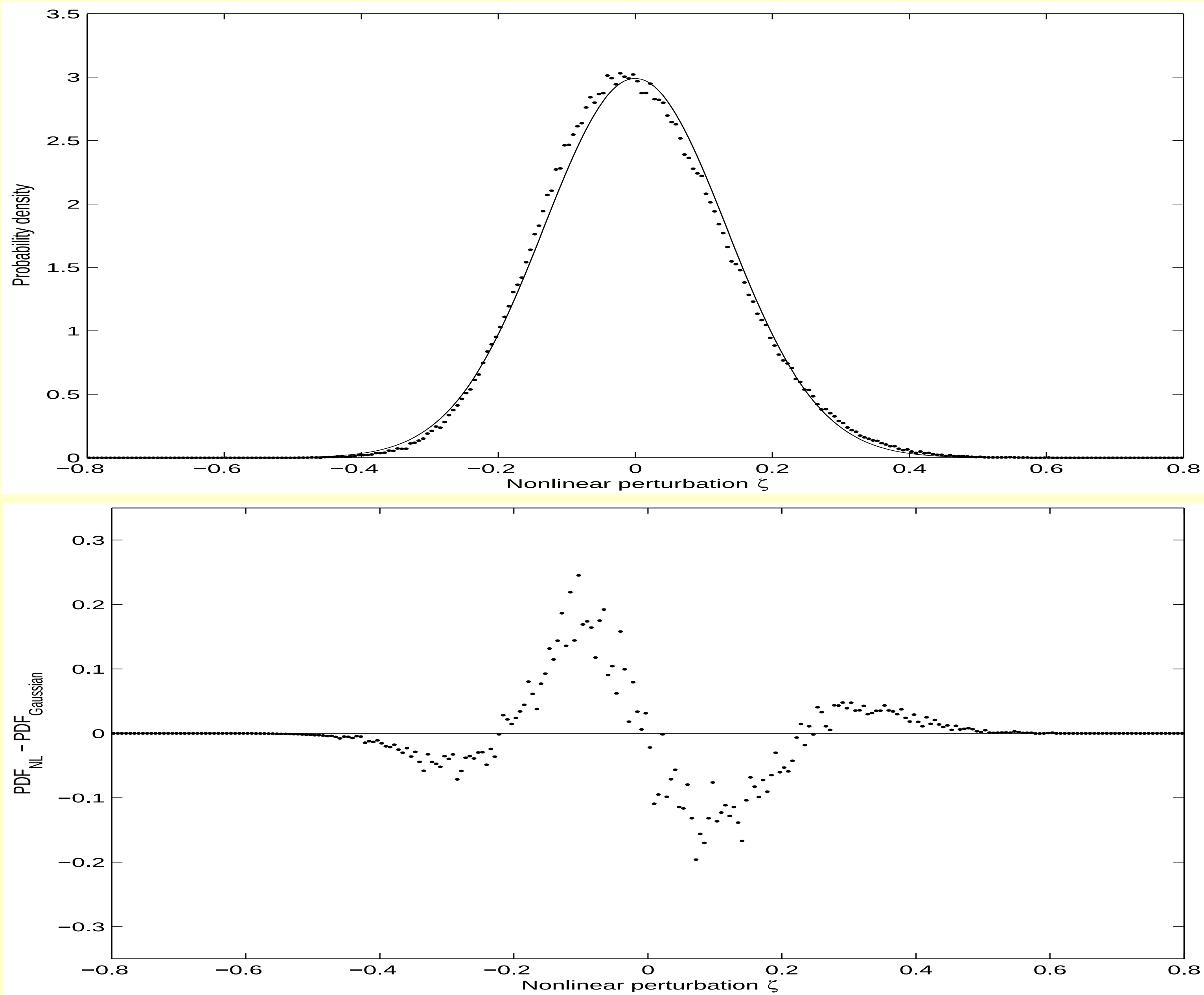
Agreement approximation $(2/3)^{3/2}(9/4)(\tilde{\eta}^{\perp})^2\Delta t$ with averages.

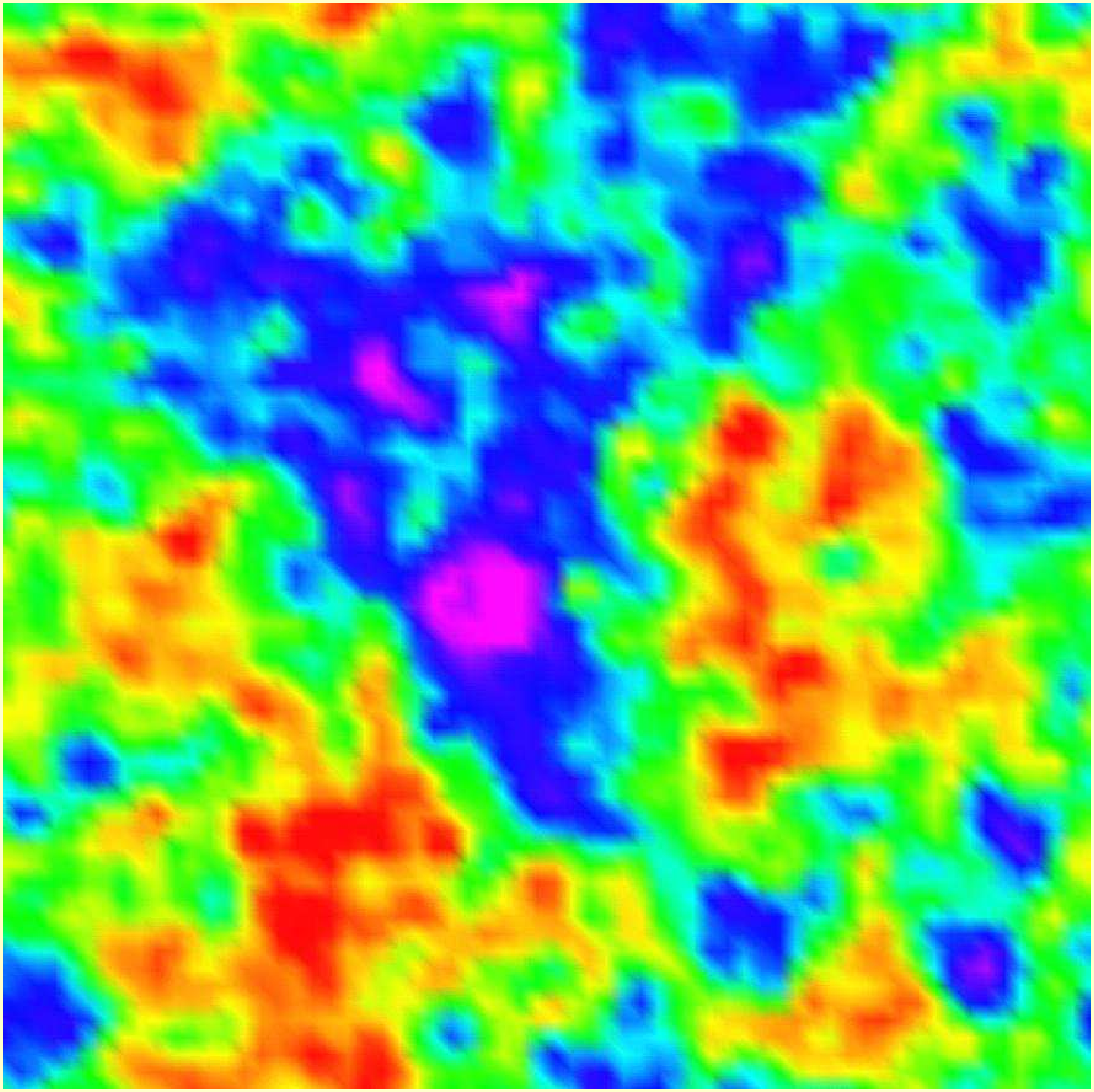
N.B.: Spectral index $n = 0.93$ not compromised.

5. Numerical results (single field, preliminary)



Realization of $\zeta(\mathbf{x})$.


Probability density
Nonlinear perturbation ζ
PDF_NL - PDF_Gaussian
Nonlinear perturbation ζ



6. Conclusions

- Developed **non-linear** formalism **multiple-field** inflat.
- Well-suited to **numerical** implementation.
- Allows explicit **(semi-)analytic** 2nd-order pert. calc.:
  - General expression **bispectrum** in terms of background and horizon-crossing linear pert.;
  - Full momentum dependence.
- Worked out two-field slow-roll case $\Rightarrow$ **Non-Gaussianity** in **multiple-field** inflation can be **much larger** than in single-field case $\rightarrow$ observable?

  (Depending on $\tilde{\eta}^{\perp}$, χ; due to integrated super-horizon influence isocurv. mode.)
- Confirmed $f_{\mathrm{NL}} \sim 1$ in simple quadratic model.