Viable inhomogeneous universe without dark energy from primordial inflation

David L. Wiltshire

University of Canterbury, New Zealand

E-prints DLW: gr-qc/0503099, v5.1

www2.phys.canterbury.ac.nz/∼dlw24/gr-qc-0503099.html

+ astro-ph/0509nnn (in preparation)

B.M.N. Carter, B.M. Leith, S.C.C. Ng, A.B. Nielsen and DLW, astro-ph/0504192

Concordance model has problems

TO NAME A FEW...

- Observed emptiness of voids..."a crisis for cosmology" Peebles, ApJ 557 (2001) 495
- Complex galaxies at $z = 2$, quasars at $z = 6$, epoch of reionization at $z \sim 20^{+10}_{-9}$ all just a bit "too early".
- With WMAP primordial nucleosynthesis bounds were pushed beyond those previously accepted, and the spectral index changed in order to fit the ratio of heights of the first two Doppler peaks
- Various CMBR anisotropy anomalies are seen at large angles (low spherical harmonic multipoles) in WMAP data. E.g., Land and Magueijo, astro-ph/0502237; Copi, Huterer, Schwarz and Starkman, astro-ph/0508047.

Universe today is lumpy

- Universe was smooth at last scattering, but no longer. . .
- Average galaxies are located on bubble walls surrounding voids.
- Two recent surveys estimate that 40% of the volume of the universe is contained in voids of diameter $(29.8 \pm 3.5)h^{-1}$ Mpc, and $(29.2 \pm 2.7)h^{-1}$ Mpc (Hoyle and Vogeley, ApJ **566** (2002) 641; ApJ **607** (2004) 751).
- There may be evidence of a few voids ten times larger (Tomita, MNRAS **326** (2001) 287; astro-ph/0505157).
- Is the fact that “dark energy” has a value which becomes important at just the epoch when things go non-linear more than a coincidence?

The fitting problem

- In an inhomogeneous universe, how do we average over lumps to make a smooth geometry [Ellis and Stoeger CQG **4** (1987) 1697]?
- How relevant are the idealized homogeneous isotropic geometries that we use, upon which all our cosmological parameter estimates are based?
- How many levels of structure are relevant?
- How do we deal with choices of gauge, such as time parameters?
- In inhomogeneous Lemaître–Tolman–Bondi models we typically get inhomogeneous time characterizations. How do we choose an average cosmic time?

The fitting problem

- Much recent interest in the “gravitational back reaction problem”, starting from the homogeneous state at last scattering [see Ellis and Buchert, gr-qc/0506106 and references therein]
- Focus is on cosmological perturbation theory.
- Kolb, Matarrese, Notari and Riotto hep-th/0503117 claim *Primordial inflation explains why the universe is accelerating today* with *no dark energy* and a Fermilab press release, stories carried by Reuters
- A stampede of order a dozen papers are rushed out to say Kolb, Matarrese, Notari and Riotto are wrong, because without dark energy one cannot get cosmic acceleration [e.g., S. Räsänen, astro-ph/0504005].

The fitting problem

- At the same time I realised there is another solution to the fitting problem, with no dark energy, no cosmic acceleration, and better still, quantitative predictions; and I rushed (too quickly) to get the results out: gr-qc/0503099, astro-ph/0504192.
- Assume like Kolb, Matarrese, Notari and Riotto that (1): we live in an underdense bubble, $\mathcal{S}$, in a spatially flat universe with bulk metric

$$\mathrm{d}s^2_{\mathrm{bulk}} = -\mathrm{d}\tau^2 + \bar{a}^2(\tau)[\mathrm{d}\bar{r}^2 + \bar{r}^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)],$$

where $\bar{a}(\tau) = \bar{a}_i(\tau/\tau_i)^{2/[3(1+w)]}$, $w > -1$.
(2) There is no dark energy, just radiation, ordinary baryonic matter, maybe CDM, and (3) all inhomogeneity grew from primordial inflation perturbations.

The average geometry

- We see an average geometry with a single isotropic scale factor $a(t)$. With a local Copernican principle, at largest scale the fitting problem must involve an average FRW geometry

$$\mathrm{d}\widetilde{s}^2 = -\mathrm{d}t^2 + \widetilde{a}^2(t)\left[\frac{\mathrm{d}r^2}{1+r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right],$$

- BUT the time parameter, t, refers to the the clock of a comoving observer, namely the observer at an average spatial position on a spatial hypersurface.
- Such average observers are in voids.
- Average stars in average galaxies live in non-expanding space, on bubble walls which broke with the Hubble flow over 10 billion years ago.

The average geometry

- No reason to assume that the solution to the fitting problem will match the time parameter of our static local geometry to the expanding space time of comoving observers in voids, where there are almost no galaxies.
- There can be an additional effective gravitational time dilation where mass is concentrated, in bubble walls. These are thick and space still expands within them.
- Geodesics of “comoving observers” are superfluous.
- Only null geodesics (photons) travel cosmological distances and see the average geometry.
- The usual analogy of coins on a balloon is inaccurate.
- Neither we nor the galaxies we observe are at average positions on a spatial hypersurface.

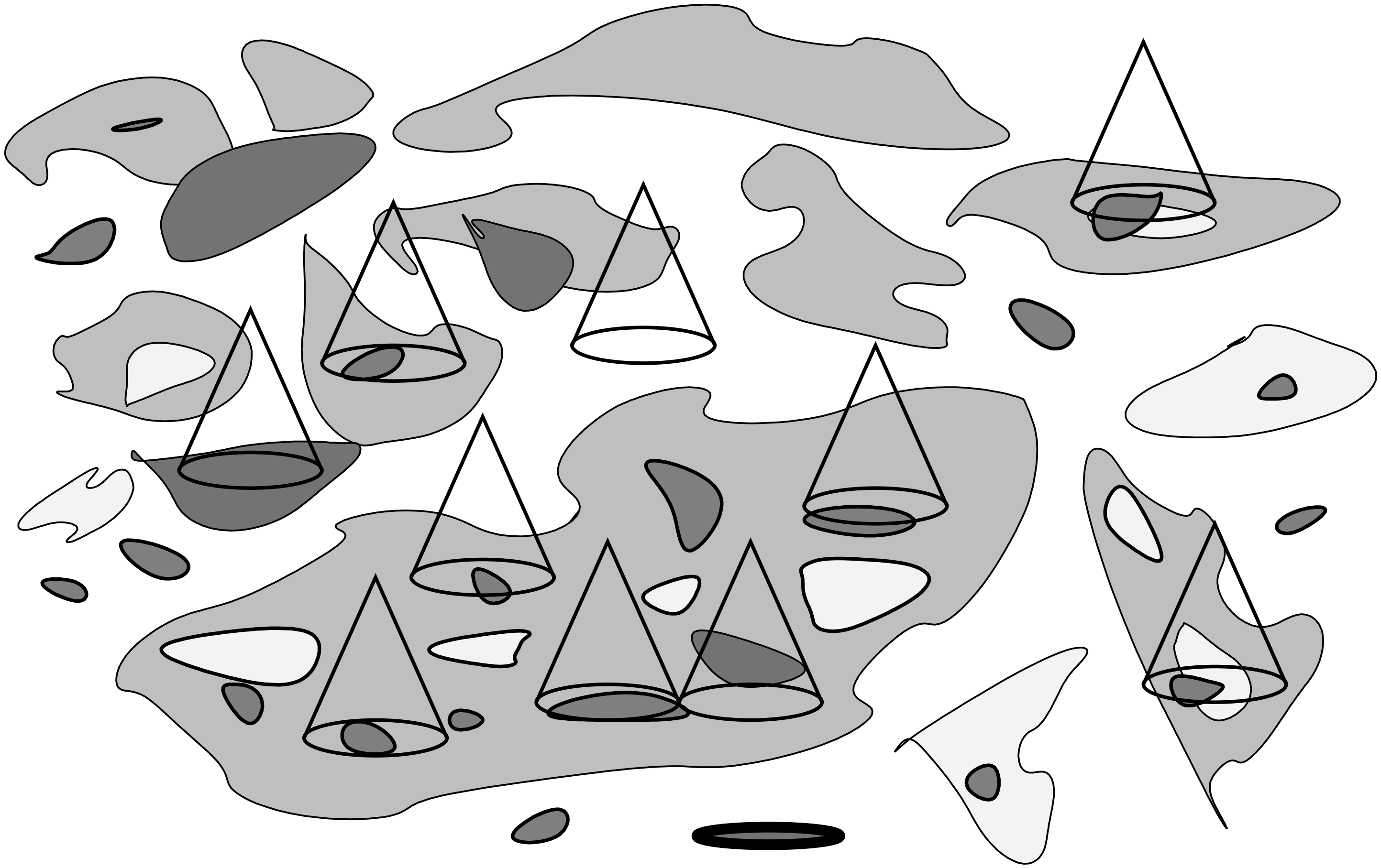
Gedankenexperiment

- With inflation the inhomogeneity is not arbitrary but has a particular structure: it arose from a particular spectrum with scale invariance.
- Scale invariance - at every scale below a cut-off, $\mathcal{B}$, assumed larger than $\mathcal{S}$, there are equal proper volumes of density perturbations with mean density distributed about bulk average.
- On tiny scales relative to $\mathcal{S}$ we have a huge sample; the mean density is therefore close to the bulk.
- In an underdense bubble there is a particle horizon volume selection effect, from “cosmic variance”.
- Suppose $\mathcal{S}$ contains our entire present horizon volume: it is a sample of one, not equal to the bulk average. By assumption it is underdense.

Particle horizon volume selection

- On scales very close to $\mathcal{S}$ very few perturbations can fit into $\mathcal{S}$. The mean of those few perturbations will differ from the bulk average by a $\sqrt{N}$ statistic. Via Sachs–Wolfe this is the well-known cosmic variance.
- Following evolution of an initial small parcel of fluid overdensity embedded in larger regions of under– and over–density with mean of the bulk gives average galaxies presently in bubble walls.
- Following evolution of an initial small parcel of fluid with the initial underdensity of $\mathcal{S}$ gives voids.
- Void regions only enter past light cones of average galaxies well after the matter within them has broken from the Hubble flow.

Evolving perturbations



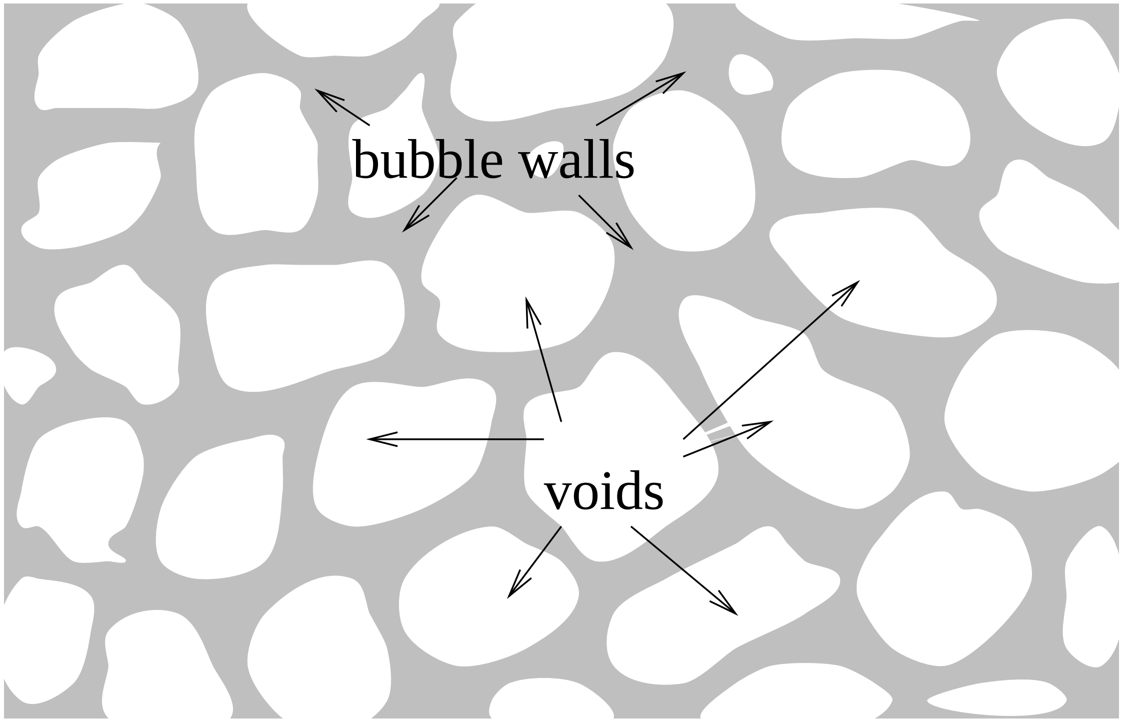
A second homogeneous cosmic time

- In inhomogeneous models such as LTB models, local times are inhomogeneous.
- Inflation gives us a second homogeneous cosmic time: that of the bulk hypersurfaces of true matter homogeneity, which were the surfaces of homogeneity within our past light cone at last scattering.
- Structure forms bottom up: smaller things first, stars, star clusters and proto-galaxy dust clouds, as larger perturbations cross the horizon.
- Clock rates in bound systems are frozen in since the epoch they broke away from the Hubble flow.
- Inflationary perturbations are consistent with the past geometry in our past light cone being that of the bulk when the first bound systems broke with the flow.

Temporal Cosmological Principle

- The statistical and spatial distribution of primordial density perturbations is such that when the first bound systems break away from the Hubble flow and aggregate into larger average galaxies the cosmic time parameter, τ, of the bulk universe comes to be the relevant asymptotic time parameter for bound gravitational systems in average galaxies which measure an isotropic CMBR within $\mathcal{S}$.
- The above specification defines the inertial frames bound systems. In effect, we have a new variant of Mach's principle. *It is the average density of* **all** *matter in the universe, even those parts of the universe which were within the past light cone prior to inflation but which are presently beyond it, defines inertial frames.*

Two sets of isotropic observers



- Differential stretching of space in voids / bubble walls but line of sight to last scattering give same average in both locations
- Both locations are isotropic observers, but mean temperature differs at late times

The average geometry

- The average open FRW geometry is relevant, but must be related to local clocks by the lapse function $\gamma(\tau) \equiv \frac{\mathrm{d}t}{\mathrm{d}\tau}$. The relevant average geometry becomes

$$\mathrm{d}\widetilde{s}^2 = \gamma^2(\tau)\mathrm{d}s^2\,,$$

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(\tau)\left[\frac{\mathrm{d}r^2}{1+r^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right],$$

  and $a \equiv \gamma^{-1}\widetilde{a}$.
- Conformally related geometry is relevant for all measurements defined by null geodesics, including proper distances and volumes, once we insist on a “synchronous gauge” adapted to local clocks.

Various possible Hubble parameters

- the bulk Hubble parameter $\bar{H} = \frac{2}{3(1+w)\tau}$ only measured in $\mathcal{S}$ at early times;
- the comoving (void observer) average parameter

$$\widetilde{H}(t) \equiv \frac{1}{\widetilde{a}} \frac{\mathrm{d}\widetilde{a}}{\mathrm{d}t} \quad ;$$

- the physical Hubble parameter we measure presently

$$H(\tau) \equiv \frac{1}{a} \frac{\mathrm{d}a}{\mathrm{d}\tau} \quad ;$$

- an effective (never measured) parameter

$$H_{\mathrm{eff}}(\tau) \equiv \frac{1}{\widetilde{a}} \frac{\mathrm{d}\widetilde{a}}{\mathrm{d}\tau} \quad ,$$

Matter dominated era

- Spherical expansion model. Background of $\mathcal{S}$ in comoving frame known

$$\widetilde{a} = \frac{a_i \widetilde{\Omega}_i}{2(1-\widetilde{\Omega}_i)}(\cosh\eta - 1)\,,$$
$$H_i t = \frac{\widetilde{\Omega}_i}{2(1-\widetilde{\Omega}_i)^{3/2}}(\sinh\eta - \eta)\,,$$

where $\widetilde{\Omega}_i$ is the initial density contrast of the bubble, $\mathcal{S}$, and a_i a constant to be set.

- Following the standard approach, we assume the initial density parameter is set sufficiently early that it is very close to unity: $\widetilde{\Omega}_i = 1 - \delta_i$, $0 < \delta_i \ll 1$.

Spherical expansion model

- Follow standard spherical collapse model

$$\widetilde{\Omega} = \frac{\widetilde{\rho}}{\bar{\rho}} = \widetilde{\Omega}_i \left(\frac{a_i}{\widetilde{a}}\right)^3 \left(\frac{\bar{a}}{\bar{a}_i}\right)^{2/n} = \frac{18 {H_i}^2 (1-\widetilde{\Omega}_i)^3 \tau^2}{\widetilde{\Omega}_i^2 (\cosh\eta - 1)^3} \, .$$

- Actual bulk critical density. Set $\bar{a} = \bar{a}_i \left(\frac{1}{n}\bar{H}_i\tau\right)^n$, where $n = 2/[3(1+w)]$ and $\bar{H}_i = 3nH_i/2$.

$$\bar{\rho} = \bar{\rho}_i \left(\frac{\bar{a}_i}{\bar{a}}\right)^{2/n} = \frac{3n^2}{8\pi G \tau^2} \, ,$$

- Local (effective) bulk critical density

$$\rho_{\rm cr} = \frac{1}{6\pi G \tau^2} \, .$$

Lapse function

- The lapse function is given by

$$\gamma(\eta) = \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{H_{\mathrm{eff}}}{\widetilde{H}} = \frac{3(\cosh\eta + 1)}{2(\cosh\eta + 2)} \,.$$

- At the present epoch

$$\gamma_0 = \frac{3}{2 + \widetilde{\Omega}_0} \,.$$

- Important "constant": effectively interpolates between present epoch clocks rates when matching the smallest bound systems to those moving freely with the average expansion on the largest cosmological scales in voids. E.g., void CMBR temperature is $\gamma_0 T_0 \simeq 4$K for $T_0 \simeq 2.7$K.

Effective Hubble parameter

- $$H_i\tau = \frac{\widetilde{\Omega}_i(\cosh\eta - 1)^2}{3(1-\widetilde{\Omega}_i)^{3/2}\sinh\eta}\,.$$

  $$\begin{aligned} H_{\rm eff}(\eta) &= \frac{3H_i(1-\widetilde{\Omega}_i)^{3/2}\sinh^3\eta}{\widetilde{\Omega}_i(\cosh\eta-1)^2(\sinh^2\eta+\cosh\eta-1)} \\ &= \frac{H_{\rm eff\,0}(1-\widetilde{\Omega}_0)^{3/2}(\widetilde{\Omega}_0+2)(\cosh\eta+1)^{3/2}}{\widetilde{\Omega}_0(\cosh\eta-1)^{3/2}(\cosh\eta+2)} \end{aligned}$$

  where $\widetilde{\Omega}_0 = 2/(1+\cosh\eta_0)$.

- Observe that at early times, $\eta\sim 0$, $H_{\rm eff}\sim\widetilde{H}$ as expected but at late times, $H_{\rm eff}\sim\frac{3}{2}\widetilde{H}$.

Physically measured parameters

- Physically measured parameters must be referred to conformally related geometry

$$\Omega(\tau) = \gamma^3(\tau)\widetilde{\Omega}(\tau),$$

so that at the present epoch we measure a matter density fraction

$$\Omega_M = \frac{27\widetilde{\Omega}_0}{(2+\widetilde{\Omega}_0)^3}\,.$$

- The solution in the physical range $0 < \widetilde{\Omega}_0 < 1$:

$$\widetilde{\Omega}_0 = \frac{6}{\sqrt{\Omega_M}}\sin\left[\frac{\pi}{6} - \frac{1}{3}\cos^{-1}\sqrt{\Omega_M}\right] - 2\,.$$

Physically measured Hubble parameter

- $H = H_{\rm eff} - \dot\gamma/\gamma$, (overdot $\equiv \tau$ derivative),

$$H(\eta) = \frac{H_0 (2+\widetilde\Omega_0)^2 (1-\widetilde\Omega_0)^{3/2} \sinh\eta\,(\cosh^2\eta + 2\cosh\eta + 3)}{\widetilde\Omega_0 (2+\widetilde\Omega_0^2)(\cosh\eta - 1)^2 (\cosh\eta + 2)^2}\,.$$

- H_0 is the physically measured Hubble constant
- Ratio of measured to void Hubble paramter

$$\frac{H}{\widetilde H} = \frac{3(\cosh^2\eta + 2\cosh\eta + 3)}{2(\cosh\eta + 2)^2}\,,$$

so again $H \simeq \widetilde H$ as $\eta \to 0$, and $H \simeq \frac{3}{2}\widetilde H$ at late times as $\eta \to \infty$. At the present epoch $H_0 = \dfrac{3(2+\widetilde\Omega_0^2)}{(2+\widetilde\Omega_0)^2}\widetilde H_0\,.$

Deceleration parameter

- $q(\tau) = -H^{-2}\ddot{a}/a = -1 - \dot{H}/H^2$, is given by

$$q(\eta) = \frac{7\cosh^2\eta + 10\cosh\eta + 1}{(\cosh^2\eta + 2\cosh\eta + 3)^2}\,.$$

- It is equal to the bulk deceleration parameter $\bar{q} = \frac{1}{2}$ at early times $\eta = 0$, but at late times as $\eta \to \infty$, $q \to 0$, similarly to a Milne universe.
- At the present epoch

$$q_0 = \frac{(14 - 4\widetilde{\Omega}_0 - \widetilde{\Omega}_0^2)\widetilde{\Omega}_0^2}{2(2 + \widetilde{\Omega}_0^2)^2}\,.$$

Expansion age

- $$\tau(\eta) = \frac{\widetilde{\Omega}_0(2+\widetilde{\Omega}_0^2)(\cosh\eta - 1)^{3/2}}{H_0(1-\widetilde{\Omega}_0)^{3/2}(\widetilde{\Omega}_0+2)^2(\cosh\eta+1)^{1/2}} .$$
- Age of the universe of is
  $$\tau_0 = \frac{2(2+\widetilde{\Omega}_0^2)}{(2+\widetilde{\Omega}_0)^2 H_0} .$$
- Using best fit values $\tau_0 = 14.4^{+0.6}_{-0.5}$ Gyr for $\Omega_M = 0.25 \pm 0.5$, or alternatively $\tau_0 = 15.1^{+0.5}_{-0.4}$ Gyr for a universe with only baryonic matter, $\Omega_M = 0.105 \pm 0.025$.

Expansion age

- Importantly, the expansion age is significantly larger at large z: with $H_0 = 62.7^{+1.1}_{-1.7}$ km sec^{-1} Mpc^{-1}, for $\Omega_M = 0.25 \pm 0.5$ we find $\tau = 4.13^{+0.32}_{-0.26}$ Gyr at $z = 2$; $\tau = 1.40^{+0.16}_{-0.13}$ Gyr at $z = 6$; $\tau = 0.29^{+0.05}_{-0.04}$ Gyr at $z = 20$.
- For $\Omega_M = 0.105 \pm 0.025$ then $\tau = 4.73^{+0.24}_{-0.19}$ Gyr at $z = 2$, $\tau = 1.81^{+0.14}_{-0.12}$ Gyr at $z = 6$, and $\tau = 0.44^{+0.06}_{-0.04}$ Gyr at $z = 20$.
- For ΛCDM with WMAP parameters, $H_0 = 71^{+4}_{-3}$ km sec^{-1} Mpc^{-1}, $\Omega_M = 0.27 \pm 0.4$ by comparison $\tau = 3.35^{+0.30}_{-0.44}$ Gyr at $z = 2$; $\tau = 0.95^{+0.15}_{-0.11}$ Gyr at $z = 6$; $\tau = 0.18^{+0.03}_{-0.02}$ Gyr at $z = 20$.
- So with only baryonic matter, early expansion age at least doubled.

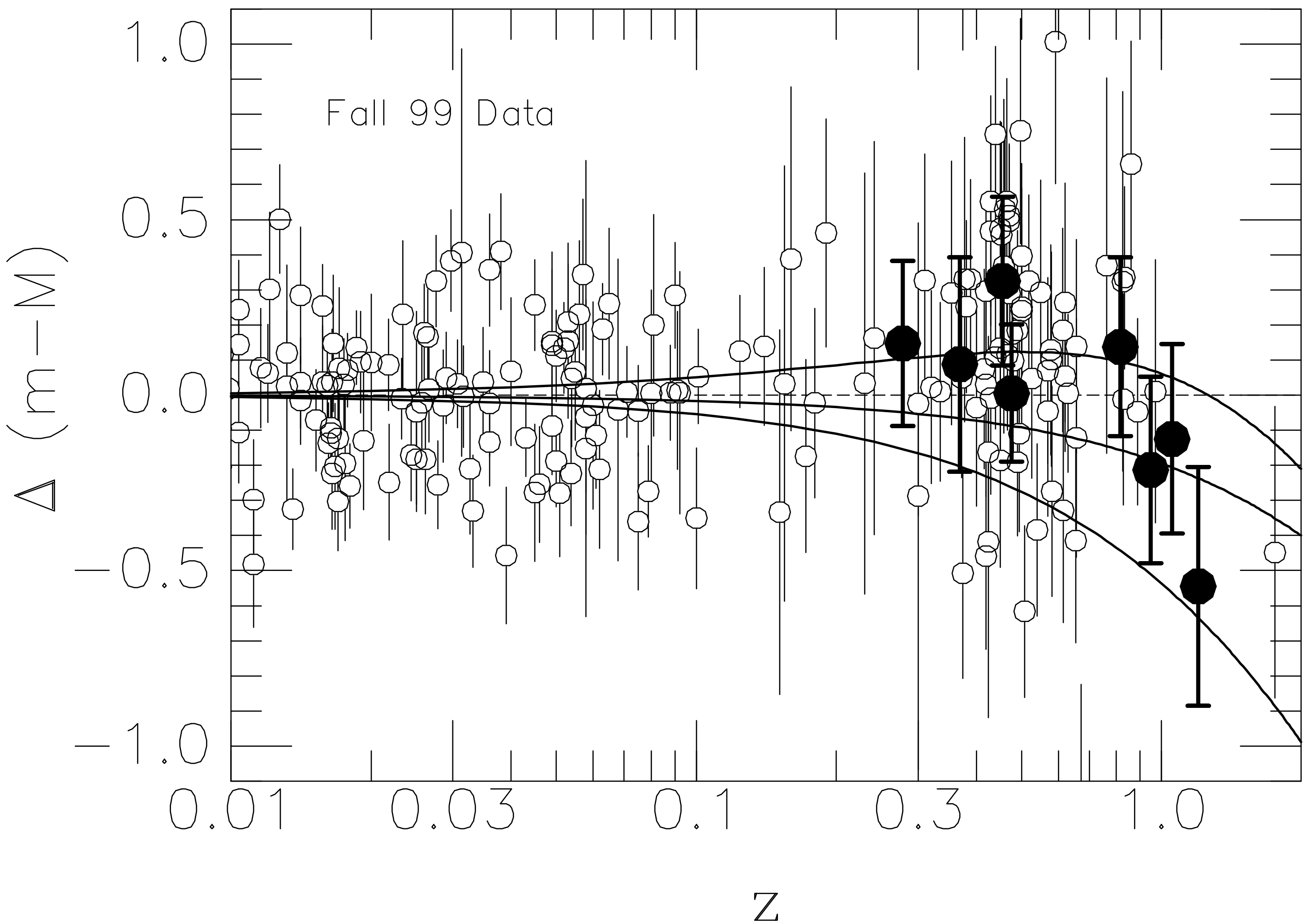
Cosmological redshift

- $$1+z=\frac{a_0}{a}=\frac{\widetilde{a}_0\gamma}{\widetilde{a}\gamma_0}=\frac{(1-\widetilde{\Omega}_0)(2+\widetilde{\Omega}_0)(\cosh\eta+1)}{\widetilde{\Omega}_0(\cosh\eta+2)(\cosh\eta-1)}\,.$$

- Solution involves a quadratic equation in $\cosh\eta$, the physical branch being given by

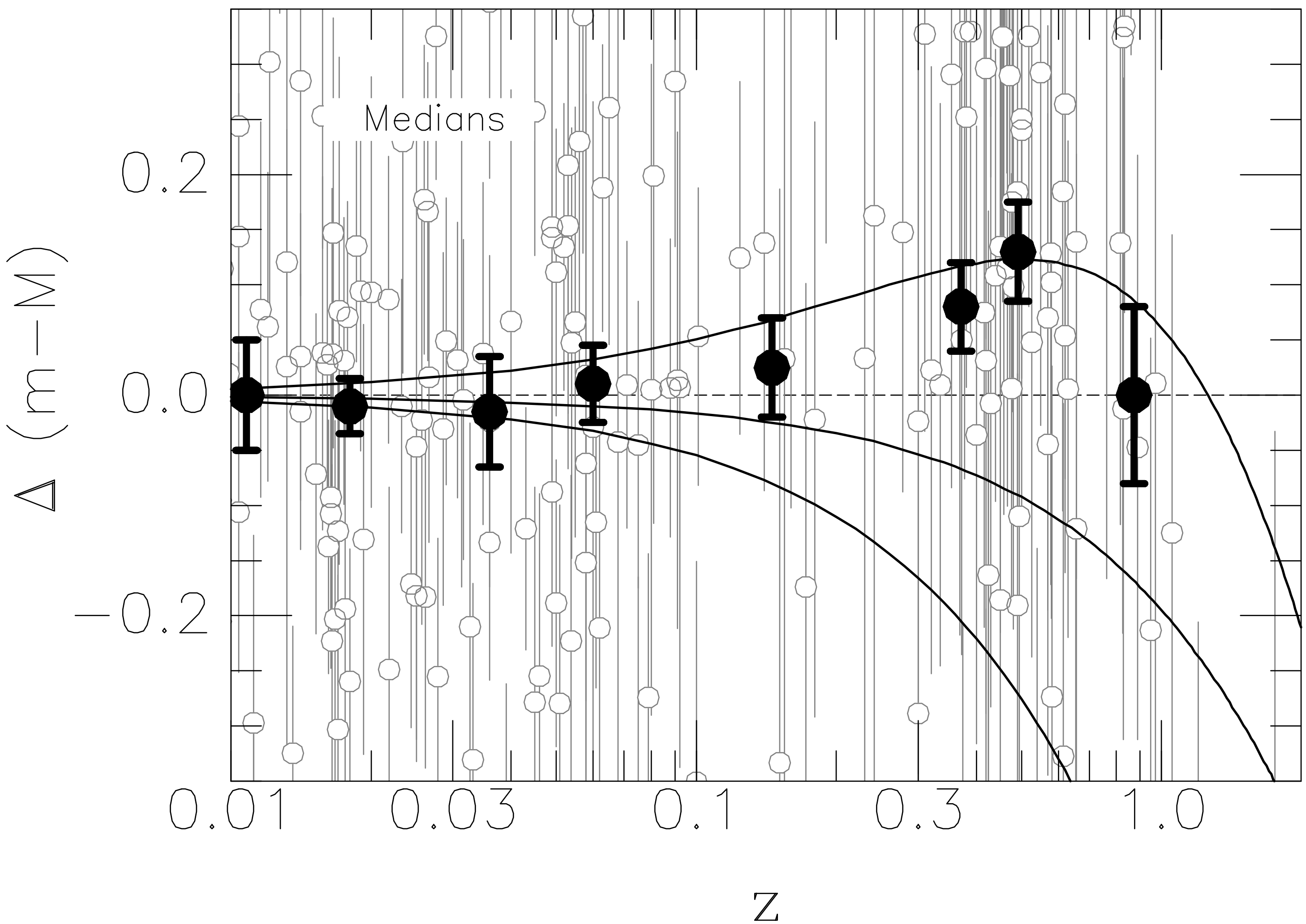
$$\begin{aligned}\cosh\eta \;&=\; \frac{-1}{2}+\frac{(1-\widetilde{\Omega}_0)(2+\widetilde{\Omega}_0)}{2\widetilde{\Omega}_0(z+1)}\\ &+\; \frac{\sqrt{\widetilde{\Omega}_0 z[9\widetilde{\Omega}_0 z-2\widetilde{\Omega}_0^2+16\widetilde{\Omega}_0+4]+(\widetilde{\Omega}_0^2+2)^2}}{2\widetilde{\Omega}_0(z+1)}\,.\end{aligned}$$

Supernova luminosity distance data

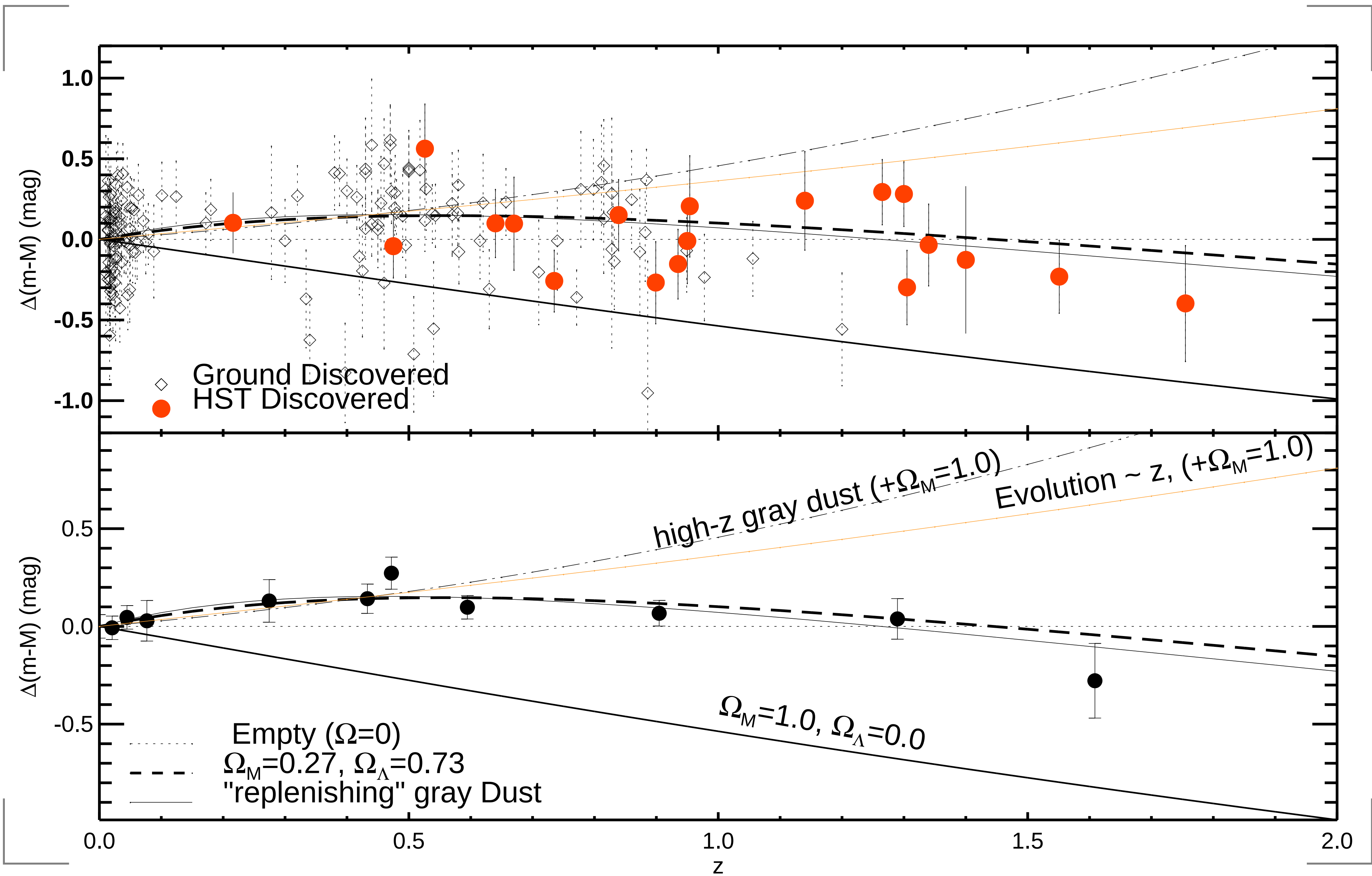


Cosmological constant data fi t: Tonry *et al*., ApJ **594** (2003) 1.

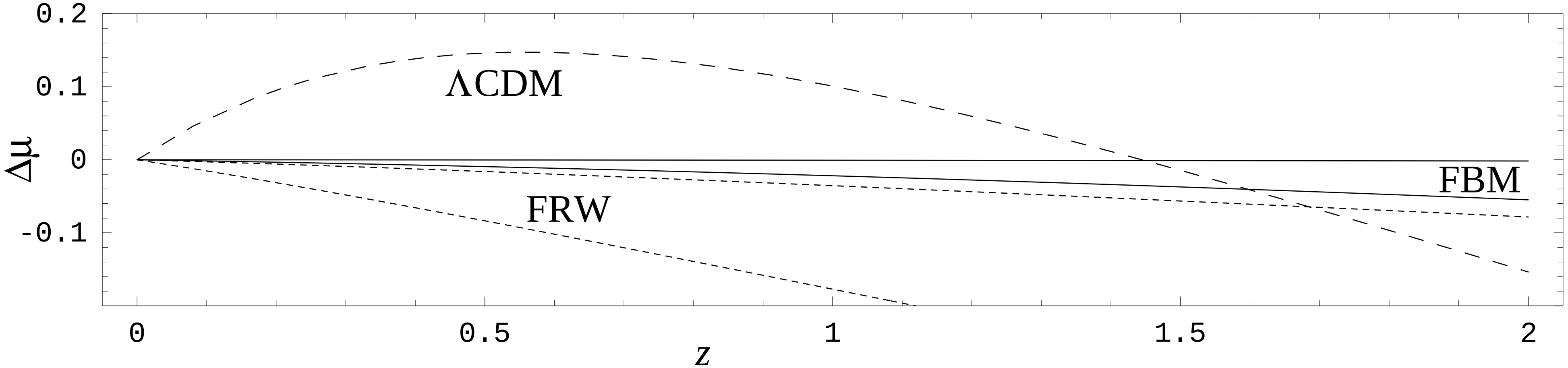
Tonry et al, 2003 supernova data



Riess et al, 2004 supernova data



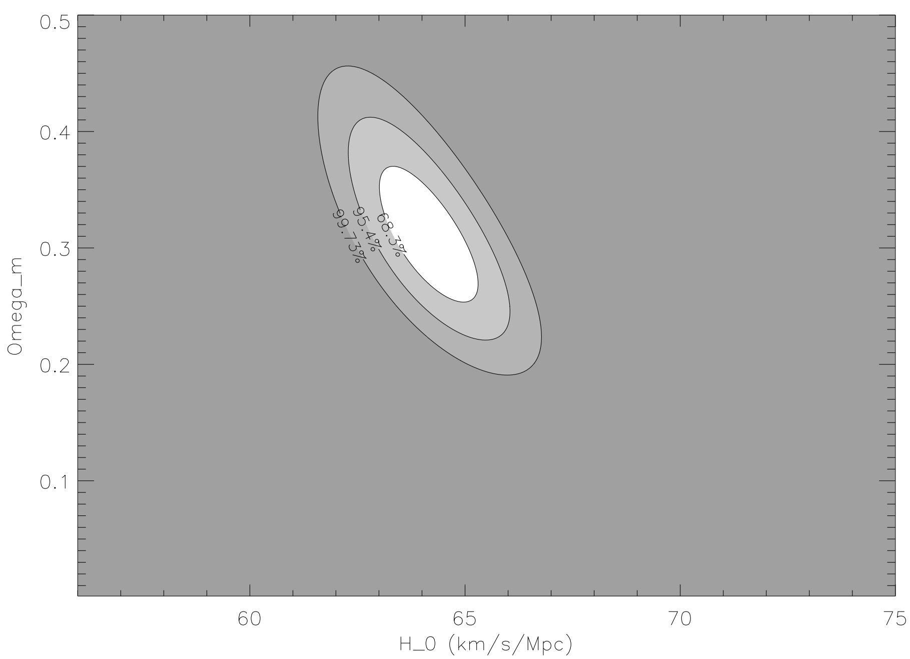
Supernova luminosity distance data



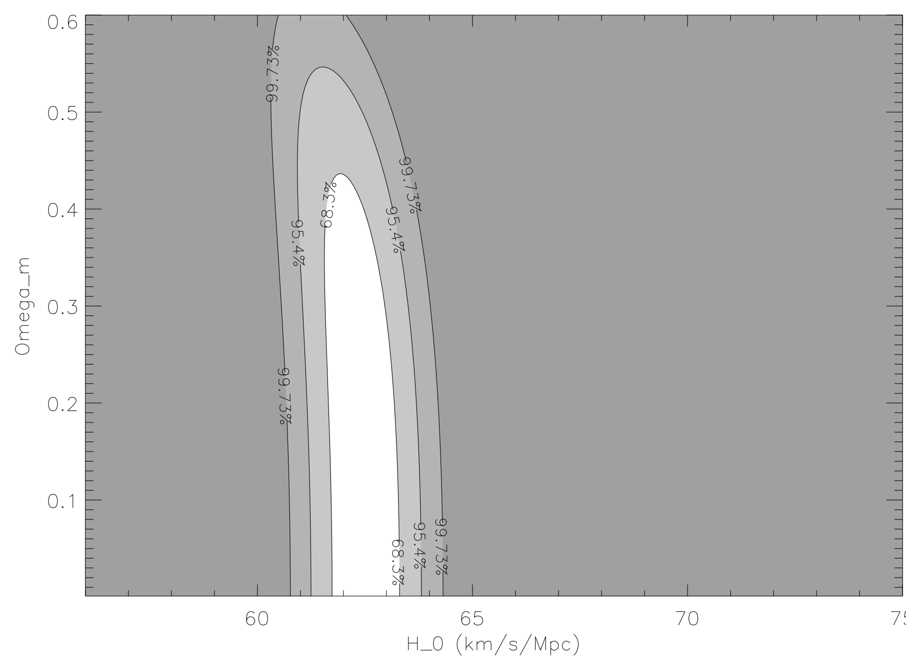
- Does the data actually go above the no acceleration line, as compared to an empty Milne universe, or FB model?
- Note – "acceleration" claim essentially depends on the large sample of data at $z = 0.4$–0.6.
- FB result is first approximation - ignores bubble "wall" versus void expansion rate differential.

Supernova data test, astro-ph/0504192

- B.M.N. Carter, B.M. Leith, S.C.C. Ng, A.B. Nielsen and DLW conducted analysis of all available data: using "Gold data set" (Riess *et al.*, 2004)



Flat ΛCDM model



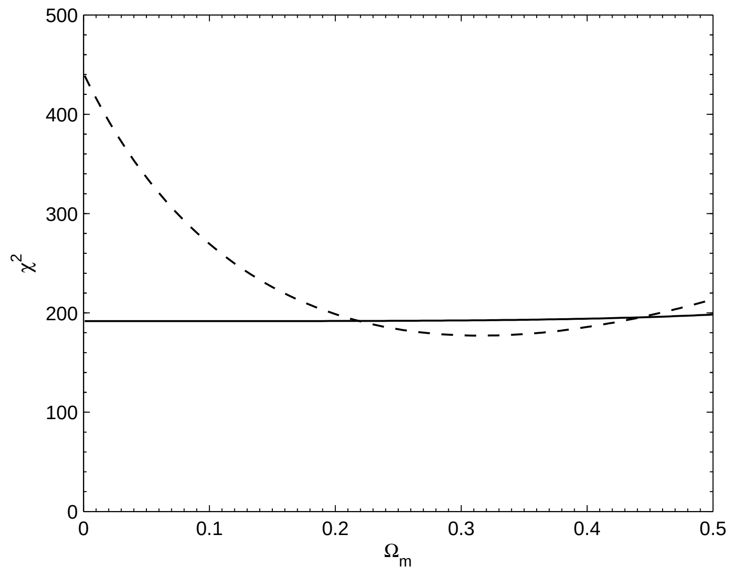
FB model

Supernova data test, astro-ph/0504192

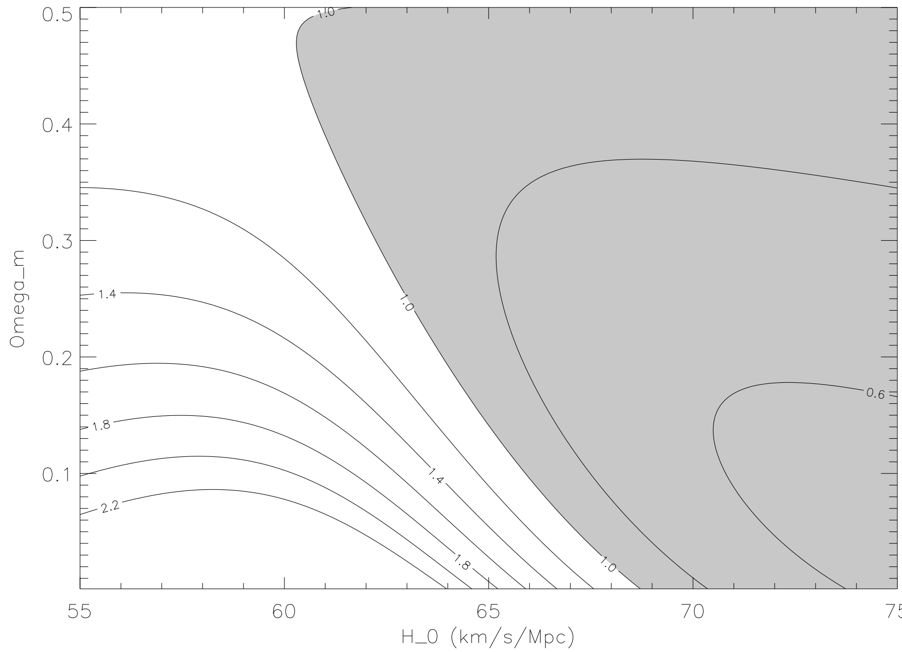
Ω_M prior	Model type	$B_{(\Lambda\mathrm{CDM}):(\mathrm{FBM})}$	Model favoured
0.01–0.5	Wide priors	396	ΛCDM
0.2–0.3	favoured CDM range	649	ΛCDM
0.3–0.5	high density CDM	1014	ΛCDM
0.01–0.2	low density CDM	0.53	FBM (slightly)
0.08–0.13	baryonic matter, low D/H	2.6×10^{-5}	FBM
0.02–0.06	baryonic matter, high D/H	1.3×10^{-14}	FBM

Bayes Factor comparison of a $\Omega_\Lambda = 1 - \Omega_M$ ΛCDM model versus the FB Model. Prior for $H_0 = 100h$ km sec^{-1} Mpc^{-1}, $58 \leq h \leq 72$, and varying priors for Ω_M.

Supernova data test, astro-ph/0504192



χ^2 comparison for best fit slices as function of Ω_M



χ^2 comparison for H_0, Ω_M parameter space

Baryon density

- Number density of baryons is estimated as

$$n_B = \frac{\Omega_B \rho_{\mathrm{cr}0}}{m_p} = \frac{33.7 f_B(\widetilde{\Omega}_0 + 2)\widetilde{\Omega}_0 h^2}{(2 + \widetilde{\Omega}_0^2)^2}$$

- Number density of CMBR photons unchanged

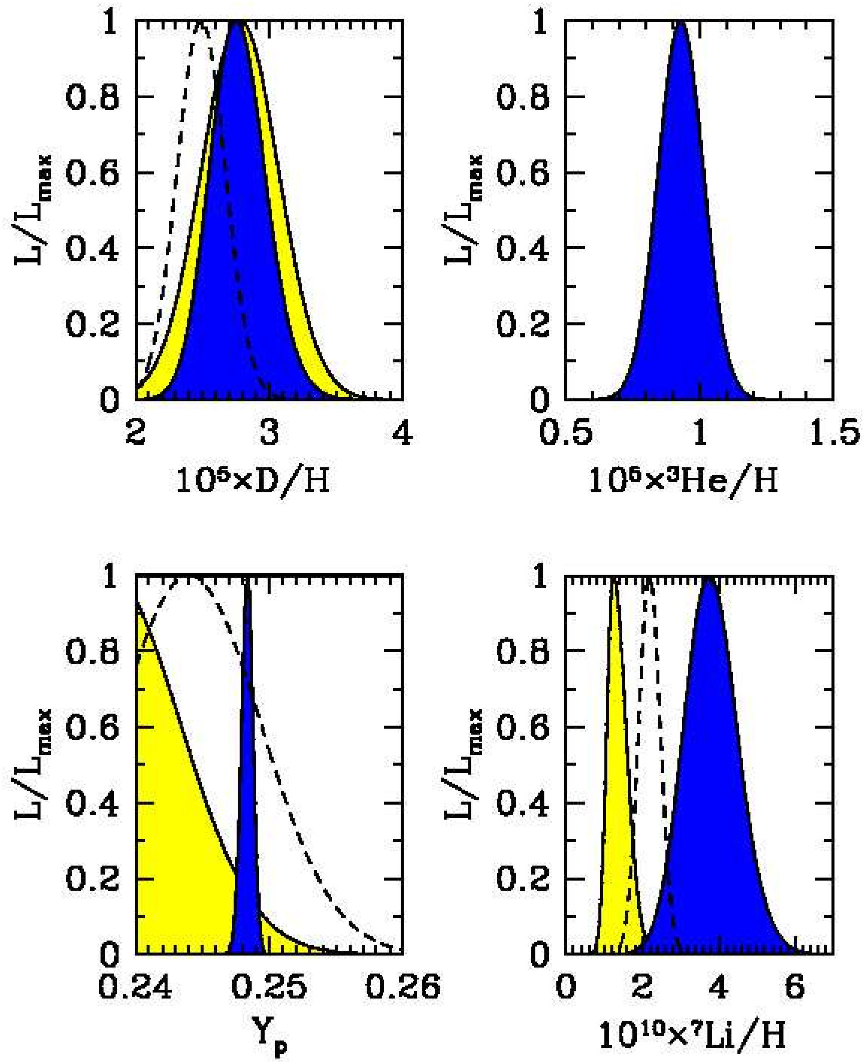
$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T_0}{\hbar c}\right)^3 = 4.105 \times 10^8 \,\mathrm{m}^{-3}\,.$$

- The present baryon to photon ratio is therefore

$$\eta_{B\gamma} = \frac{n_B}{n_\gamma} = \frac{8.209 \times 10^{-8} f_B(\widetilde{\Omega}_0 + 2)\widetilde{\Omega}_0 h^2}{(2 + \widetilde{\Omega}_0^2)^2}$$

c.f. standard $\eta_{\mathrm{std}} = 2.728 \times 10^{-8} f_B \Omega_M h^2$.

Nucleosynthesis bounds - post WMAP



[Yellow = observation; Blue = concordance model prediction]

Baryon density

- Pre Boomerang, WMAP Olive, Steigman and Walker (2000) quoted two possible ranges at the 95% confidence level: $\eta_{B\gamma} = 1.2$–2.8×10^{-10} or $\eta_{B\gamma} = 4.2$–6.3×10^{-10}, depending on higher or lower values of the primordial D/H abundance.
- If we take $H_0 = 62.7^{+1.1}_{-1.7}$ km sec^{-1} Mpc^{-1}, and use the low D/H (most accepted) bound range we obtain $\Omega_B = 0.08$–0.13.
- Baryon density is a factor three higher than otherwise allowed: more bayonic dark matter OK. Irrespective of whether there is additional non-baryonic CDM, the dark matter budget changes substantially.

Discussion

- More corrections to SneIa due to differential expansion of bubble wall interiors versus voids, (and maybe of different scale voids, c.f., Tomita, 2001), may explain "acceleration" as differential Hubble rates
- Any net asymmetry in void/wall distribution would give a CMBR dipole. The dipole anisotropy then contains both a piece due to our peculiar velocity and a piece due to a (possibly small) asymmetry in the void/wall distribution. Present dipole subtraction therefore includes an anomalous boost, which may account for low multipole anomalies and the "axis of evil".
- Hot big bang work in progress
- First Doppler peak: angular diameter distance not easy, averaging geodesic deviation over bubbles and voids not the same as averaging null geodesic lengths. . .

Conclusion

- If it works, a subtle recalibration of many quantities in cosmology is required. Other averaging issues to consider; higher order corrections.
- It would be ironic that Einstein's intuition about the irrevelance of Λ and the importance of Mach's principle might both be right and related to inflation.
- Even if it is not correct, it is clear that the process of averaging in a universe with matter as inhomogeneous as we observe has to be thought about carefully. Definition of average clocks is crucial.
- Physicists love to add wierd terms to the action; but maybe we just have to think about more complicated geometries and identification of observables in GR.